THE UNIVERSITY OF ALBERTA

DERIVATION AND EXPERIMENTAL EVALUATION OF A STABLE ADAPTIVE CONTROLLER

by

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A THESIS

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To My Parents

Abstract

The conventional, PID, feedback controller has been used by industry for years on a wide varity of control problems. However, there is an increasing number of applications where an adaptive controller would be desirable to tune the initial controller constants supplied by the user; to automatically retune the controller constants when process operating conditions change; and/or to compensate for slow changes in process parameters.

This thesis describes a robust Self-tuning Feedback Controller (SFC) that has the following characteristics:

1) Like most conventional feedback controllers, it is error driven and in its simplest form is structurally and mathematically equal to the discrete PID control law. Modifications such as feedforward control are easily included.

2) The problem formulation includes unmeasured noise and/or external disturbances requiring only that they be bounded.

3) It includes a quadratic performance index with polynomial functions to weight (filter) the error input and/or weight the control action. This weighting also makes it possible to control open loop unstable and/or nonminimum phase systems.

4) It includes an internal model of the setpoint and external disturbances so that it assumes the properties of a 'robust controller' and can asymptotically track arbitrary inputs despite changes and/or errors in system parameters.

5) A formal, mathematical proof of global stability is

included which guarantees that the I/O vectors are bounded and that the norm of the error between the optimal controller parameters and the current estimates of these parameters is a nonincreasing function.

6) Parameter adaptation is turned on or off automatically and normally off during periods of steady state operation.

The SFC algorithm is straightforward, can be coded in less than 50 lines of FORTRAN, and executes in a few milliseconds. It has been evaluated by simulation and experimental applications to the computer controlled evaporator at the University of Alberta which is equipped with conventional industrial instrumentation. The main emphasis during these evaluation runs was on an evaluation of the various design and controller parameters plus a comparison of the SFC(PID) versus Aström and Wittenmark's STR, Clarke and Gawthrop's STC, Martin-Sanchez's APCS and a conventional fixed-gain PID controller. In the experimental studies the PID version of the SFC performed better than conventional PID and, in general, was better than or equal to STR, STC and APCS. The substitution of an alternative parameter estimation algorithm would probably improve performance further.

The SFC is recommended for industrial applications because it can be interpretted as a self-tuning version of the conventional, discrete, PID feedback controller and, when the need arises and/or experience suggests, the SFC can be easily extended into a more sophisticated form.

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Table of Contents

List of Tables

List of Figures

1. Introduction and Objectives

1.1 Introduction

During the past decade or so, with the aid of digital technology and process control computers, significant progress has been made in process control. In particular, adaptive control algorithms have generated considerable interest and many successful attempts have been made to solve the control problems for plants with completely unknown parameters.

Of the various approaches to adaptive control, the self-tuning regulators (STR) [Aström and Wittenmark, 1973] and the model reference adaptive systems (MRAS) [Landau,1974b] have been the most widely discussed. The former were originally designed to solve stochastic control problems based on the certainty equivalence principle whereas MRAS were based on stability analysis, i.e. Lyapunov method (Parks,1966) and Popov's hyperstability (Landau,1969) and designed to solve the deterministic servo problem. In spite of their different starting points it has been recognized recently that the two approaches have strong connections as far as stability and convergence analysis is concerned [Egardt, 1979a; Ljung, 1977b] and there is a growing effort to unify these two approaches [Landau, 1982; Egardt, 1980; Narendra and Valavani, 1979].

Overall stability of the closed loop system is one of the most important properties of adaptive control systems

both theoretically and practically and is closely related to the analysis of convergence. For example, the convergence results presented by Ljung (1977b) required a stability assumption and for MRAS without disturbances the same assumption was required to prove convergence of the output error [Feuer and Morse, 1978; Narendra and Valavani, 1981]. Goodwin, Ramadge and Caines (1978, 1980) presented rigorous convergence proofs without any explicit stability assumption for the deterministic case and also for the stochastic system (1981). Martin-Sanchez, Shah and Fisher (1981c) have also proven stability and convergence of an adaptive controller in the presence of bounded disturbances by introducing an adaptation dead zone. This result has also been extended to a more general case by Martin-Sanchez (1983).

The development work undertaken as part of this thesis was restricted to adaptive controllers for which it was possible to prove global stability. At the present time it is possible that, for a given application, a different adaptive mechanism might give better performance even though it were not possible to prove theoretical stability for that particular controller. However, given the large amount of work currently being done on adaptive systems, it is expected that most adaptive controllers will have associated stability and convergence proofs and hence selection will be made based on performance characteristics rather than stability considerations.

Adaptive controllers have not been widely adopted in industry even though stability and convergence have been guaranteed theoretically and experimental results have demonstrated their advantages over conventional, PID controllers. One key problem may be the unfamiliarity and complicated structure of the adaptive controller. Since continuous or discrete PID controllers are widely used to solve industrial control problems in spite of the fact that periodic, manual retuning is required in many applications there is considerable motivation to develop an adaptive algorithm which automatically tunes the conventional, feedback controller. Some ad hoc self-tuning PID controllers have already been presented without stability and convergence proof [Gawthrop, 1982; Isermann, 1981; Cameron and Seborg, 1982]. In 1981 Silveira and Doraiswami proposed an adaptive servomechanism controller and presented stability and convergence proofs. Although they did not show how their controller could be structured as an adaptive PID system, the similarity was noted by the author and served as a starting point for the development of the SFC controller.

1.2 Objectives

Based on the above considerations, the main objective of this work was to develop a structually-simple, stable, adaptive controller which could be used in place of conventional, PID controllers and would eliminate the manual tuning effort and also compensate for changes in the

process. It was also hoped that the resulting adaptive controller would go beyond a simple replacement for the basic PID controller and serve as a basis for evolutionary introduction of features such as general performance indices, internal models, etc. into industrial applications.

The second objective was to investigate the effects of various design parameters on overall system performance. This was to be done by simulation and experimental applications to the pilot plant evaporator at the University of Alberta.

The third objective was to compare the derived adaptive controller theoretically and experimentally with the self-tuning controller (STC), the adaptive predictive control system (APCS) and the conventional, PID controller.

Although it is not documented as part of this thesis, this project involved a considerable amount of practical 'computer control engineering' due to the concurrent changeover in the departmental DACS Centre from an IBM1800 computer to a distributed network of HP and DEC computers. The author did most of the software changes necessary for the simulation and experimental evaluations as well as that required for support functions such as plotting.

1.3 Structure of Thesis

This thesis consists of eight chapters. The first three provide background information on adaptive controllers and the evaporator used to experimentally evaluate them.

Chapters four through six describe the STR/C, APCS and SFC controllers respectively. Each chapter has the same structure and contains the relevant theory plus simulation and experimental results. These chapters follow in logical order but can be read independently. Chapter seven focuses on factors that are common to all three classes and contains direct comparisons of the three adaptive controllers. It can also be read independently by those familiar with the field but is intended to supplement the material and the conclusions in the preceding three chapters. The overall conclusions and recommendations for future work are given in the last chapter.

2. Adaptive Control

2.1 Introduction

Adaptive control systems were first proposed in the late 1950s for use in autopilots to improve the performance of aircraft over a wide range of flight conditions. The early systems were unsuccessful because the hardware was poor and the associated theory was not adequate to fully analyse the system stability or performance in the presence of noise. Fortunately, in the 1960's, there were several important developments in the control area such as stochastic control theory, state space analysis, optimal control and stability theory, which are fundamental to the development of adaptive control systems.

Interest in adaptive control revived in the early seventies due to the improvements in control theory made in sixties and the dramatic progress in computer technology. A large number of adaptive controllers were developed [Aström and Wittenmark, 1973; Landau, 1973; Martin-Sanchez, 1974; Monopoli, 1974; Clarke and Gawthrop, 1975; Feuer and Morse, 1978; Goodwin et al, 1978; Narendra and Lin, 1980].

The objective of this chapter is to provide a broad overview of the structure and key characteristics of this adaptive controllers. For purposes of discussion the adaptive controllers have been classified as 'direct' and 'indirect' method. This method of classification has been used by others [Narendra and Valavani, 1979; Kreisselmeier,

1982] in their articles and clearly identifies the two main approaches to adaptive control developed in the 1970's. However, the direct/indirect classification is not absolute without its shortcomings. Some authors, particularly those in Europe have used the terminology 'implicit/explicit' in place of 'direct/indirect' [Aström, 1981]. Furthermore, several investigators [Ljung, 1978; Egardt, 1979; Narendra, 1980] have shown that from the point of view of stability analysis this classification into direct and indirect methods is somewhat artificial since the stability analysis for both classes of controllers is very similar and in some cases identical. However, despite its shortcomings the direct/indirect classification has been used in the following section since it is historically accurate, intuitively appealling and clearly identifies some of the key concepts used in the later chapters. This is followed by a review of some of the key refernces dealing with parameter estimation and stability in adaptive systems.

Additional publications dealing with specific features of STR/STC, APCS and SFC are referenced in chapters four, five and six respectively.

2.2 Types of Adaptive Controllers

The direct and indirect approaches to adaptive control have a common starting point as an ordinary feedback control loop containing the process plus an adjustable regulator mechanism [Figure 2.1]. However, the direct and indirect

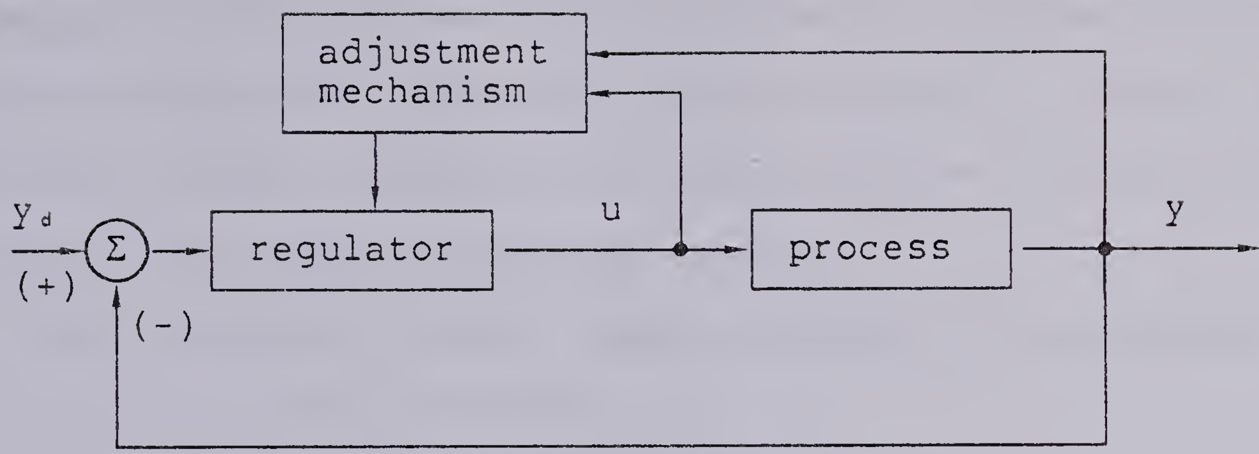


Figure 2.1 General Adaptive Control System

approaches to controller design are based on different methods. In the direct method, the controller is designed using stability principles, for example, Lyapunov's second method [Parks, 1966; Monopoli, 1974; Morgan and Narendra, 1978] and Popov's hyperstability [Landau, 1973]. In the indirect method it is designed based on the separation principle [Egardt, 1979; Aström, 1970].

2.2.1 Indirect Adaptive Control

The concept of adaptive control originated with Kalman (1958) who even attempted to implement it using a special purpose computer. However, the theory and the technology were so poor that the controller performance was not very successful. As a result, the area of adaptive control was essentially dormant until the late 1960's. It was revived and extended to include stochastic aspects by Peterka (1970) but it was the paper by Aström and Wittenmark (1973) that generated widespread practical interest in the subject

[Narendra and Valavani, 1979]. Their work led directly to practical applications and classified the problems involved with the adaptive scheme. The book edited by Harris and Billings (1981) presents a good overview of the current state of this area of adaptive control.

The indirect method can be thought of as composed of two loops as shown in Figure 2.2.

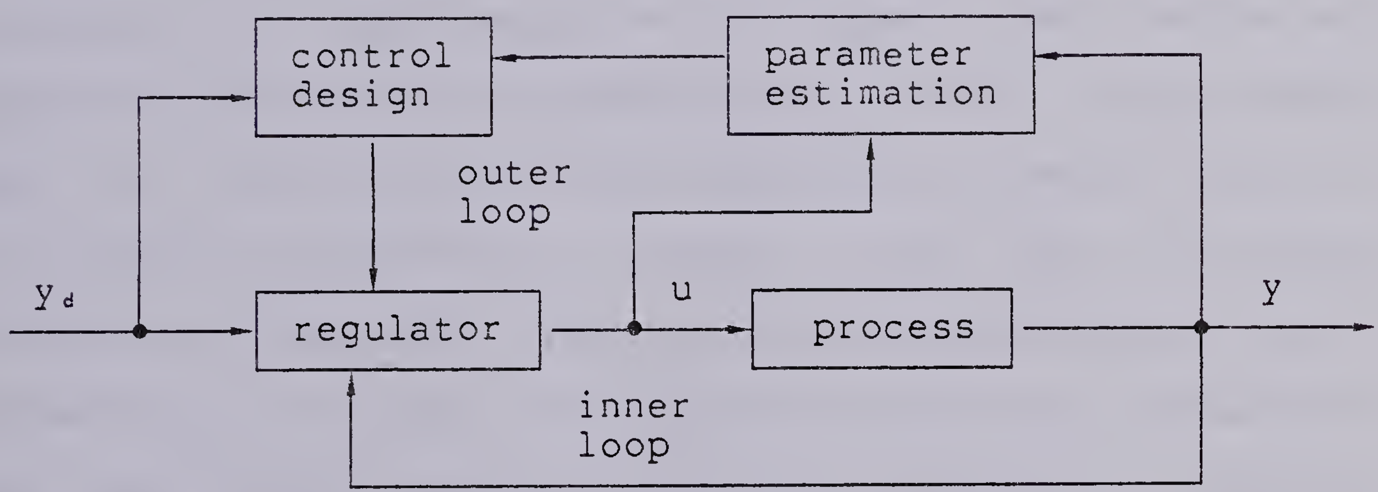


Figure 2.2 General Structure of Indirect Method

The inner loop acts like an ordinary, linear feedback loop. However, the parameters of the regulator are adjusted by a second, or outer, loop. The outer loop consists of parameter estimation and control design. In the parameter estimation routine a new set of process parameters for a linear model with a prespecified structure is recursively estimated based on the measured inputs and outputs of the process. The control design step provides a new set of coefficients for the feedback control law calculated using the parameter estimates. There have been many different "indirect"

algorithms proposed by different authors using different combinations of parameter estimation and a design methods. In fact, it is relatively easy to propose a new adaptive control algorithm in this way. However it is difficult to prove which combination is best or to establish the performance and stability characteristics of a given controller.

There are numerous schemes that have been used in the estimation of parameters of a linear model [Aström and Eykhoff, 1971; Eykhoff, 1976; Eykhoff, 1981]. Each method has its own strong and weak points and in general there is no absolute criterion to select the best parameter estimation algorithm. The selection should be not only a reflection of the type of process, the kind of disturbance and the control design but also consider convergence, convergence rate and computation effort. Several papers compare various estimation algorithms and try to give guidelines for choosing an algorithm [Saridis, 1974; Isermann et al, 1974; Graupe et al, 1980; Kurz et al, 1980, Isermann, 1980; Morris et al., 1982]. Some properties of parameter estimation schemes are briefly discussed in section 2.3.

The indirect method is very flexible with respect to control law design. Given an estimate of the model parameters the design block can incorporate almost any technique to generate new control parameters for the regulator block. The most commonly used design techniques

are minimum variance control [Aström and Wittenmark, 1973], linear quadratic Gaussian [Clarke and Gawthrop, 1975] and pole-assignment [Wellstead et al., 1979a, 1979b]. The minimum variance self-tuner is based on minimization of the measured output variance and has a structure and properties similar to a dead-beat controller. Since no account is taken of the control effort required, excessive control signals may be generated and in some cases the closed loop can be unstable. The idea of minimising a performance index with weighting on the output and input variables is introduced in the LQG self-tuner. This algorithm therefore includes tracking as well as regulatory control. In the third method the controller is designed so that the closed-loop poles are placed at prescribed locations while zeros are in their open-loop positions. This has been extended to cover the placement of zeros in arbitrary locations [Aström and Wittenmark, 1980].

The indirect method is also called the 'explicit' self-tuning regulator since the process is identified explicitly and then the identified model parameters are used as a basis for design of the controller. Similarly the direct method discussed in the next section, is sometimes called an 'implicit' method because the controller parameters or control action is calculated directly and the process parameters are implicit in the procedure, i.e. are not explicitly available.

2.2.2 Direct Adaptive Control

This method was suggested by Whitaker et al in 1958 to improve aerospace control applications. It was intended primarily for servo problems having time-varying properties. The early schemes based on sensitivity functions (MIT rule) were total failures in the sense of stability. In the mid-1960s Parks (1966) designed a controller based on Lyapunov's second method which also failed to establish the asymptotic stability and included differentiation of output error which is undesirable in practical applications due to the effect of noise. The first problem was solved by Morgan and Narendra (1978) and the second by Monopoli (1974) using the augmented error concept. But one of the most important contributions in this area was made by Landau (1974b), who introduced Popov's hyperstability concept into the design of adaptive mechanisms. The direct adaptive methods have already been extensively surveyed [Landau, 1979; Narendra and Monopoli, 1980; Parks et al, 1980]. The direct methods are generally considered as a significant advance over the indirect methods because some of the conditions on parameter convergence can be relaxed and because the implementation is simpler.

There are basically three different structures within the direct approach, i.e. parallel, series-parallel and series depending on the configuration of the reference model and the process [Landau, 1979]. The most popular structure is the parallel configuration, often call the "output error

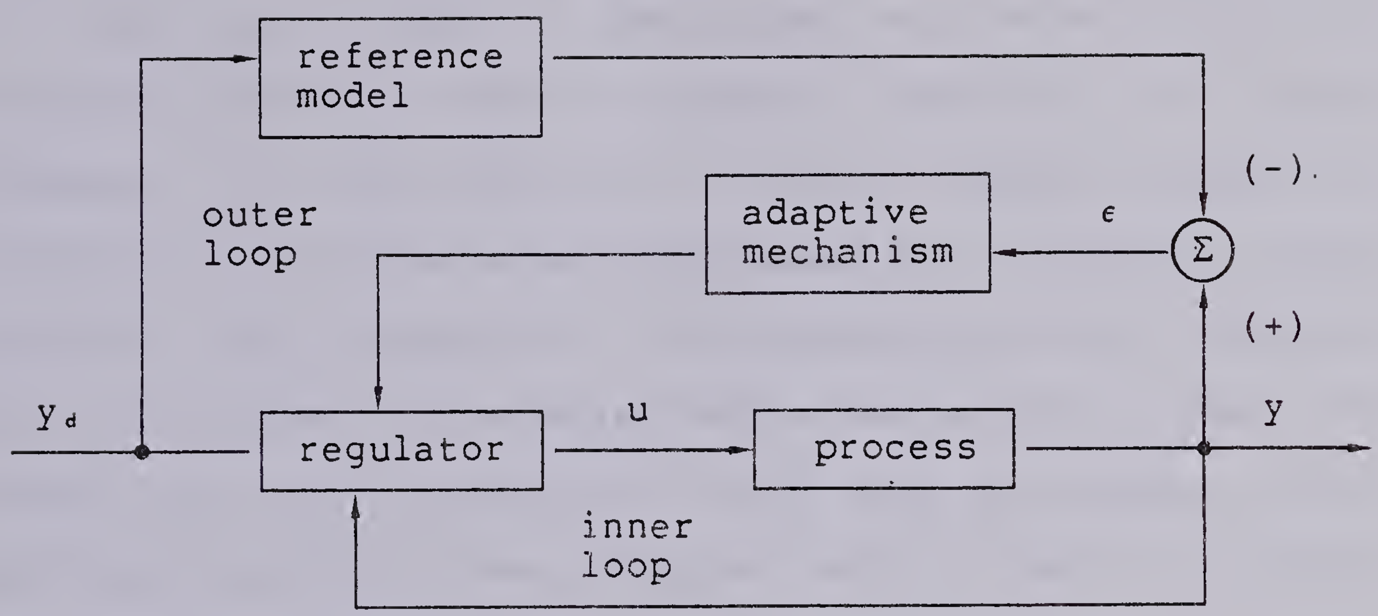


Figure 2.3 Structure of Parallel Adaptive Control

method" when used for identification. The parallel scheme as shown in Figure 2.3 consists of a feedback control loop (inner loop) and an adaptive mechanism (outer loop). The main features are: i) it has a reference model which defines the servo dynamics, ii) it directly calculates the regulator signal or parameters using the output generalized error which is the difference between the output of the reference model and the output of the process, iii) the input and the output of the process are not explicitly made available to the adaptive mechanism.

The reference model is specified by the user and receives the same desired input value as the feedback regulator. Its output is the reference signal that the process output should follow. The adaptive mechanism must be designed so as to make the generalized output error zero. Most work on direct adaptive control methods has been

concerned with designing the adaptive mechanism.

The early adaptive mechanisms were based on Lyapunov functions [Parks, 1966]. However, because of output feedback, it was difficult to prove complete asymptotic stability. In 1974 Monopoli introduced the augmented error concept and presented a differentiator-free adaptive controller based on the Meyer-Kalman-Yacubovitch lemma but could not prove stability. In the same year Landau (1974) used hyperstability theory [Popov, 1963] to design a stable MRAS [Landau, 1979].

2.3 Parameter Estimation

The estimation algorithm is the key to good performance and overall stability in any adaptive system. It also determines the convergence point and its rate which is very important when tracking variations in process dynamics and/or environmental changes.

The field of parameter estimation and identification has developed rapidly during the past two decades and there are a multitude of papers discussing its aspects [Eykhoff, 1974, 1981; Söderström et al, 1978; Landau, 1976; Ljung, 1977a, 1977b, 1981; Isermann, 1981]. The main aim of this section is to outline the properties and general problems of the estimation algorithms which are commonly used in adaptive systems.

The most important problems in parameter estimation are how to determine if the parameter estimates converge and how

to control or characterize the convergence rate. There are two methods generally used in analysing the stochastic convergence of parameter estimators; the ordinary differential equation method of Ljung (1977a, 1977b) and generalized martingale convergence method of Solo (1979). In 1977 Ljung proposed a set of ordinary differential equations which describe the trajectory of parameter estimates and showed that only stable, stationary points of the differential equations are possible convergence points of the estimator. He also showed that positive realness of the system noise equation is a necessary condition for convergence. One disadvantage of this method is that the set of differential equations can not be solved analytically. However it can be used to find the stable and unstable parameter region by numerical search [Dumont and Belanger, 1978]. The martingale convergence theorem was used by Sternby (1977), Gawthrop (1980) and Goodwin et al (1981). Sternby proved consistent convergence of the least squares estimator using the martingale convergence and Gawthrop used it to find the stability and convergence conditions for a self-tuning algorithm [Gawthrop, 1979].

The analysis of parameter convergence for a stochastic process is much more complicated when the process input is generated by an adaptive feedback loop. Only a few authors have proven the stability and the parameter convergence of stochastic systems [Goodwin et al., 1981; Martin-Sanchez et al., 1981c]. Detailed discussions are given in section 2.4.

The most widely used parameter estimation algorithms have the following form;

$$\theta(k+1) = \theta(k) + K(k)\cdot\xi(k+1) \tag{2.1}$$

$$K(k+1) = \frac{P_t(k)\Phi(k+1)}{\lambda(k+1)+\Phi^t(k+1)P_t(k)\Phi(k+1)} \tag{2.2}$$

$$P_t(k+1) = \frac{1}{\lambda(k+1)}\left[P_t(k) - K(k+1)\Phi^t(k+1)P_t(k)\right] \tag{2.3}$$

$$\lambda(k+1) = \lambda_0\lambda(k) + (1-\lambda_0) \tag{2.4}$$

Where $\theta(t)$ is a vector of parameter estimates calculated from the process inputs and outputs; $\xi(k)$ is the prediction error calculated using the estimated model; $K(k)$ is the estimator gain vector; $\Phi(k)$ is a vector containing the process input, output and the prediction error sequences; $P_t(k)$ is the covariance matrix and $\lambda(k)$ is a forgetting factor. If an ordinary RLS is applied to identify processes having correlated or coloured noise the estimated parameters are biased. This bias can be avoided by using recursive generalized least squares (GLS), recursive extended least squares (ELS), recursive instrumental variable (RIV), recursive maximum liklihood (RML), etc. In addition to bias, there are other problems associated with parameter estimation for adaptive control purposes. Basically, estimation and identification theory assumes persistent excitation of the input signal to the process in order to estimate the necessary parameters. In controlled systems or

low noise systems, e.g. chemical processes, there is no guarantee that the process will be perturbed enough to permit valid parameter estimation. More specifically, for systems with low input excitation the norm of the covariance matrix $P_t(k)$, and hence the gain $K(k)$, tends towards zero much faster than the parameters converge towards the true or optimal values when the forgetting factor is unity ($\lambda(0)=1$ and $\lambda_0=1$). Therefore, the estimate $\theta(k)$ tends to a constant vector even if there is a large error. This can be avoided by introducing an extra perturbation signal, e.g. PRBS, or by inflating the covariance using the forgetting factor.

A constant forgetting factor is very useful when estimating time-varying parameters since it is necessary to discount old data. However, if it is used for a constant parameter system great care should be taken when choosing the forgetting factor. The use of a forgetting factor will give fast convergence during the initial stage of parameter estimation even for time-invariant processes. However, when the process is well controlled by the converged parameters no information about the process can be obtained from the input and output data. During this period the covariance $P_t(k)$ from equation (2.3) and hence the gain vector $K(k)$ will grow exponentially. This phenomenon results in "blowup" or "bursting" of the parameter estimator. Large gains will then lead to large changes in the parameter estimates even though the prediction error is small and the closed loop system may become unstable. In some cases, the estimator

windup can result in the covariance matrix $P_t(k)$ losing positive definiteness and consequently its significance. There are several methods to avoid this estimator blowup problem. The first way is to modify the covariance matrix at each iteration such that it holds its positive definiteness [Morris et al, 1982] or to put limits on each element of the covariance matrix. The second is to use a variable forgetting factor as in equation (2.4) [Cordero and Mayne, 1981; Fortescue et al, 1981]. Note the $\lambda(k)$ in equation (2.4) converges to unity. The third option is to freeze the parameter adaptation when the deviations in the input and output variables are small. Using a constant scalar in place of the covariance is also one possibility, which permits the combination of equation (2.1) and (2.2). Such a simplification will reduce the computation time considerably at the expense of convergence rate.

The initial parameters, $\theta(0)$, are very important in the sense that they determine the performance during the startup stage, convergence time, the convergence point and in some cases the closed loop stability. When the process is unknown, the initial values for the parameter estimates may contain significant errors. Moreover the adaptive controller design is based on the certainty equivalency principle, that is, the control algorithm simply accepts the current estimates, which might have little value for purposes of control calculation, and ignores their uncertainties [Aström, 1981; Harris and Billings, 1981]. The initial

unknown parameters may result in the initial control action being undefined or the variations in the process I/O variables being unacceptable [Isermann, 1981]. In the actual application of adaptive control this can be avoided by using parameters obtained by off-line identification or background parameter estimation done while the process is operating under a non-adaptive control system.

2.4 Stability

The block diagram for most closed loop adaptive control systems can be simplified to a block diagram containing only a linear time-invariant feedforward block and a nonlinear time-varying feedback block (Figure 2.4).

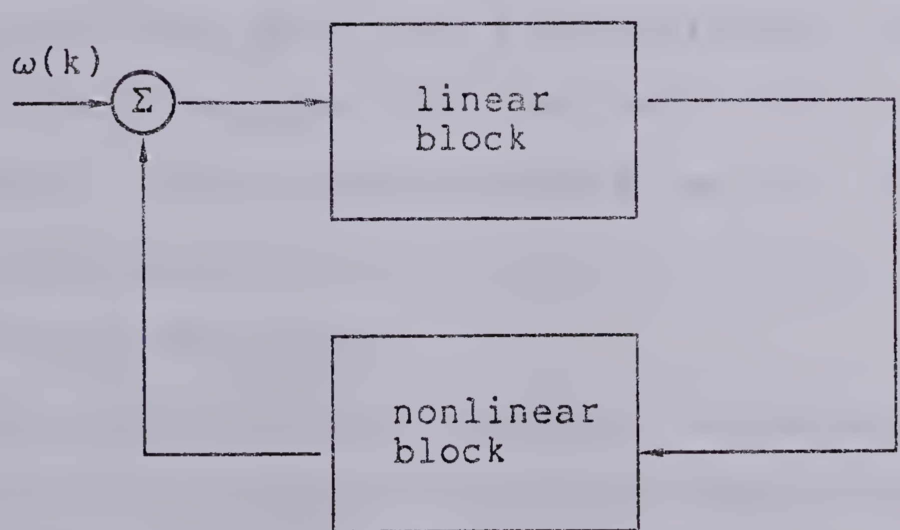


Figure 2.4 Simplified Adaptive Control Feedback System

The stability analysis of this kind of system, called a Letov-Luré problem, has been of great interest and several stability theories have been developed to analyse this nonlinear system, for example, Meyer-Kalman-Yacubovitch

(MKY) lemma and Popov's hyperstability theorem (1963), input-output stability [Zames, 1966a; 1966b]. The traditional way of analysing the stability of adaptive systems is Lyapunov's second method which makes use of MKY lemma. In this analysis disturbances are excluded [Parks, 1966; Monopoli, 1974]. Asymptotic stability was not treated rigorously until Narendra and Lin (1980) who established the global asympotic stability for the deterministic discrete adaptive system.

Input-output stability methods [Willems, 1970, 1976; Desoer and Vidyasagar, 1975] are more appropriate than Lyapunov's method in the sense that disturbances can be considered and systems corrupted with noise can be analysed. Two theorems are involved in the analysis; the small-gain theorem and the passivity theorem [Youla, 1959; Bikart and Prada, 1971; Estrada and Desoer, 1971; Desoer and Vidyasagar, 1975; Martin-Sanchez et al., 1981b]. Gawthrop (1979) used this method to exploit the special properties of a self-tuning controller.

The hyperstability concept was introduced to design a model reference adaptive system by Landau (1969, 1972, 1973, 1974a, 1979). He designed adaptive systems such that the transfer function of a linear system is strictly positive real and the nonlinear system satisfies the Popov integral inequality or passivity condition. Therefore, the resulting closed loop system becomes hyperstable.

There are some important details to be considered in the stability analysis of parameter adaptive systems. The control objective of reducing the contol error or tracking error to zero must be achieved using finite control input. Hence the stability analysis has to show that the process inputs and outputs are bounded for all time and since these are functions of the adapted parameters the stability proof is complex. This problem is even more difficult when the system is exposed to stochastic noise. Ljung (1979) showed that the positive realness is the key condition for the parameter convergence of a stochastic system but the boundedness of input and output variables was not considered. This difficulty remained unanswered for several years. Very recently rigorous and complete stability proofs were given by Narendra and Lin (1980), Goodwin et al. (1978, 1981) and Martin-Sanchez et al. (1981c) and Martin-Sanchez (1982). Narendra and Lin developed a stability analysis for model reference systems under the assumption of no unmeasurable disturbances. However, the latter two cases provide stability proofs for parameter adaptive stochastic as well as deterministic systems. Goodwin et al (1981) assumed that the disturbances were colored noise (more precisely the output of a linear stable filter whose input is a martingale difference sequence). The martingale convergence theorem [Solo, 1979] and the positive-real functions [Hitz and Anderson, 1969] were used to establish that the inputs and the outputs are sample mean square

bounded and the control objective is also mean square bounded. Martin-Sanchez required only a rather flexible and practical assumption on the stochastic disturbances to establish stability and convergence, i.e. all that was required is that the disturbances be a bounded sequence. Under this assumption Martin-Sanchez et al (1981c) proved that the control error asymptotically converges within that disturbance uncertainty (the control error converges to zero for the deterministic system) with the input and the output bounded. They also showed that the norm of the estimated parameter error vector is a nonincreasing function. In other words the point of parameter estimates in vector space never moves away from the true point.

2.5 Conclusions

One of the prime objectives of this work was to develop a practical, adaptive controller for which it would be possible to derive general properties, such as stability, rather than base conclusions purely an application-dependent results. Therefore because of previous work at the University of Alberta, and because at the time this work started it was the only approach that permitted a stability proof for stochastic systems, it was decided to study and extend the APCS approach.

The STR/C, APCS and SFC systems are discussed in chapters four through six respectively. The next chapter

describes the computer controlled pilot plant used to evaluate the different control schemes and its performance using conventional PID feedback control.

3. Background for Experimental Runs

This chapter describes the process equipment and control instrumentation that was used to experimentally evaluate the different adaptive controllers.

3.1 Description of Equipment

The process equipment used in this study was the double effect pilot plant evaporator in the Department of Chemical Engineering, University of Alberta. Its fifth-order, linear, state space model was also used for simulation studies.

The double effect evaporator has been described in detail in [Andre, 1966; Jacobson, 1970; Newell, 1971; Fisher and Seborg, 1976]. The schematic flow diagram of the equipment is shown in Figure 3.1. The symbols and steady state operating conditions are given in Appendix A. The unit operates at a normal feedrate of 2.27 kg/min of three percent acqueous triethylene glycol (TEG) solution. The first effect is a natural circulation calandria type unit with 32 eighteen inch by 3/4 inch OD tubes and the second effect is an externally forced circulation long tube unit with three one inch OD by six foot tubes. The second effect is operated under vacuum and utilizes the first effect overhead vapor as a heating medium to concentrate the first effect bottom stream.

The evaporator is equipped with industrial electronic instrumentation for about fourteen control loops and the

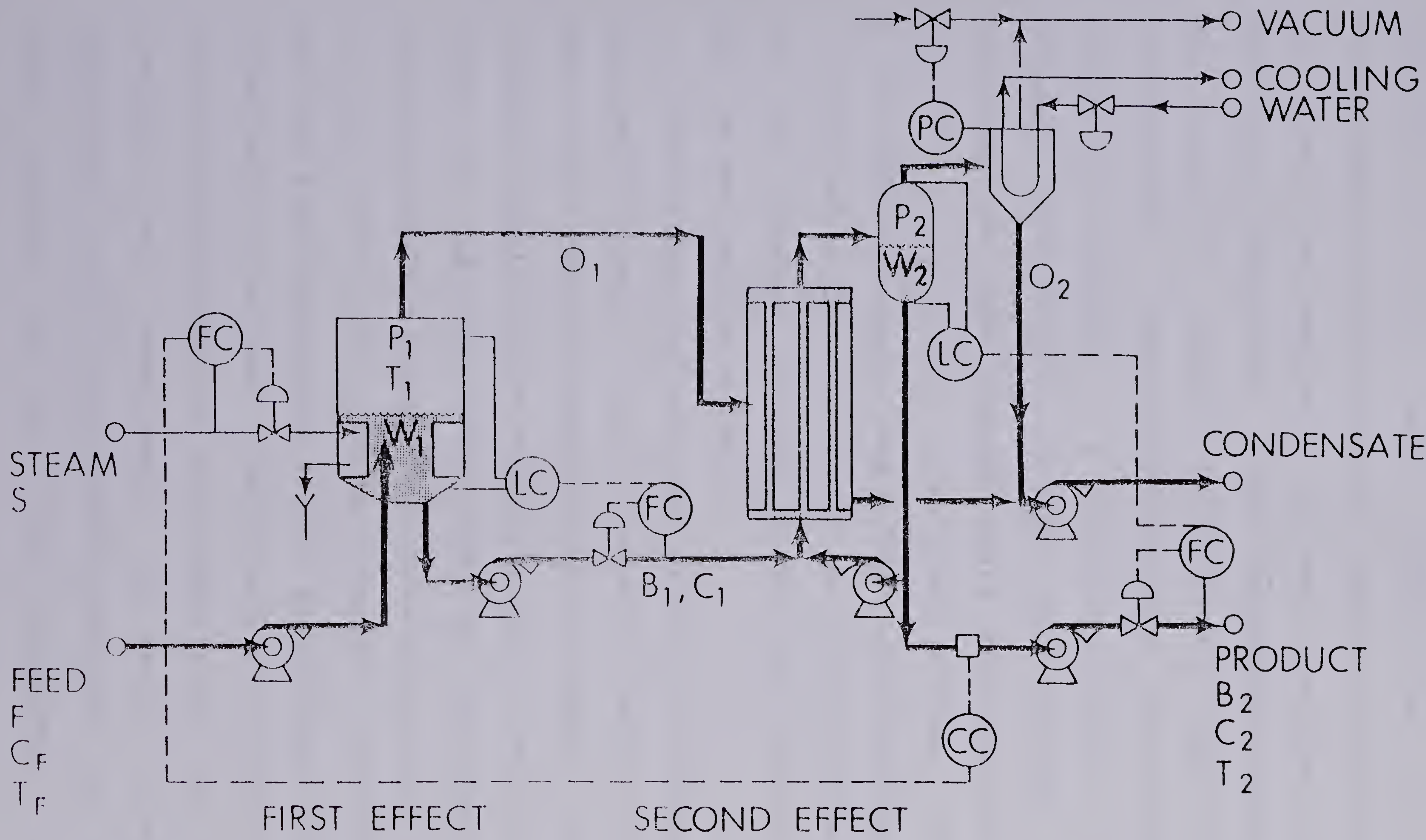


FIGURE 3.1 Schematic Diagram of the Double Effect Evaporator Showing Multiloop Control Configuration

recording of over twenty temperatures. An in-line refractometer is used to measure product concentration in real time. The evaporator had been interfaced to the IBM 1800 control computer but in 1978 the computer system of the Data Acquisition, Control and Simulation (DACS) center in the Department of Chemical Engineering at the University of Alberta was changed to a distributed network of digital computers. The network includes three HP/1000 digital computers, two disk storage units, and several LSI-11's. Since 1978 the evaporator has been monitored and controlled by the HP/1000 computer and an LSI-11 microprocessor which is interfaced to the process. All the utility programs that had been developed for the IBM 1800 were rewritten to fit into the new computer system.

During normal operation the evaporator is monitored and regulated by a means of eight computer control loops and seven local analog controllers. The computer controlled variables are the product concentration C2, the first effect holdup W1, the second effect holdup W2, the water flowrate and the triethylene glycol (TEG) solution flowrate. As shown in Figure 3.1, C2, W1 and W2 are cascaded to the steam flow, the first effect bottoms B1 and the second effecf bottoms B2 respectively. The feed flowrate and its concentration are ratio-controlled using the water flowrate and the solution flowrate. The conventional, PID control strategy is implemented through the Distributed Simulation and Control (DISCO) package developed in the Department of Chemical

Engineering under Dr. Fisher's supervision [Brennek, 1978]. The control loop of primary interest in this work is the cascaded C2/S loop which was used to evaluate the adaptive controllers. The other loops were closed using a conventional, PID control algorithm.

3.2 Evaporator Model

Several models of the double effect evaporator have been developed in previous studies [Newell,1971; Wilson, 1974]. They range from a tenth-order, nonlinear state space model to a first-order transfer function model. The models are fully described in [Fisher and Seborg, 1976]. In the simulation portion of this work the evaporator model used was the fifth-order, linear, stochastic state space model derived from the linearization of material and heat balance equations. The stochastic noise term was obtained by a time series analysis of experimental data [Kogekar, 1977]. The discrete, stochastic model is given in Appendix A.

A first or second order evaporator model in the form of a transfer function between the steam flowrate and the product concentration is desirable to calculate the initial parameters that are required to start the various adaptive controllers. The transfer function model has the form;

$$\frac{C2(s)}{S(s)} = \frac{K \exp(-T_d s)}{(T_1 s + 1)(T_2 s + 1)} \tag{3.1}$$

Three different models were developed based on experimental, open loop, step response data from the evaporator.

1) First order model

$$G(s) = \frac{2.965}{46.95s + 1} \tag{3.2}$$

2) First order plus time delay model

$$G(s) = \frac{2.24 \exp(-2.5s)}{28.5s + 1} \tag{3.3}$$

3) Second order model

$$G(s) = \frac{2.965}{(46.93s + 1)(.0044s + 1)} \tag{3.4}$$

The model parameters of (3.2) and (3.3) were identified by the nonlinear regression procedure of Deshpande and Ash (1981) and those of (3.3) were obtained from the process reaction curve analysis. Since the second time constant of the second order model is close to zero the second order model is virtually the same as the first order model without time delay. Note that the process gain and the time constant of (3.3) are significantly different from those of equation (3.2). However, figures 3.2, 3.3 and 3.4 show the agreement between these models and the experimental data. Similar variations of the process gain and time constant have been observed in the development of a simple evaporator model by Nieman (1971). Note, as indicated by Nieman, the process

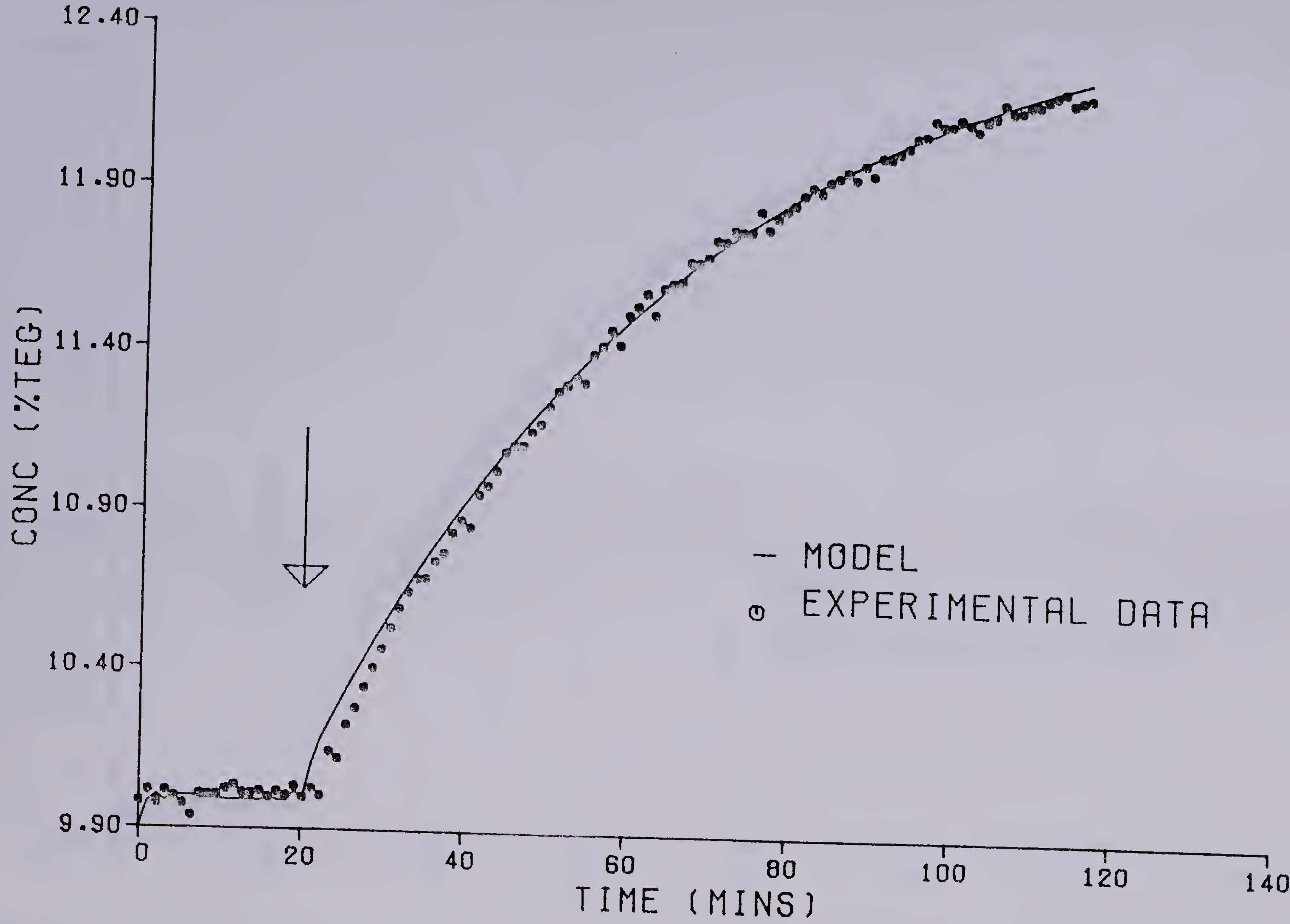


FIGURE 3.2 Identification of the Evaporator Using a First Order Model

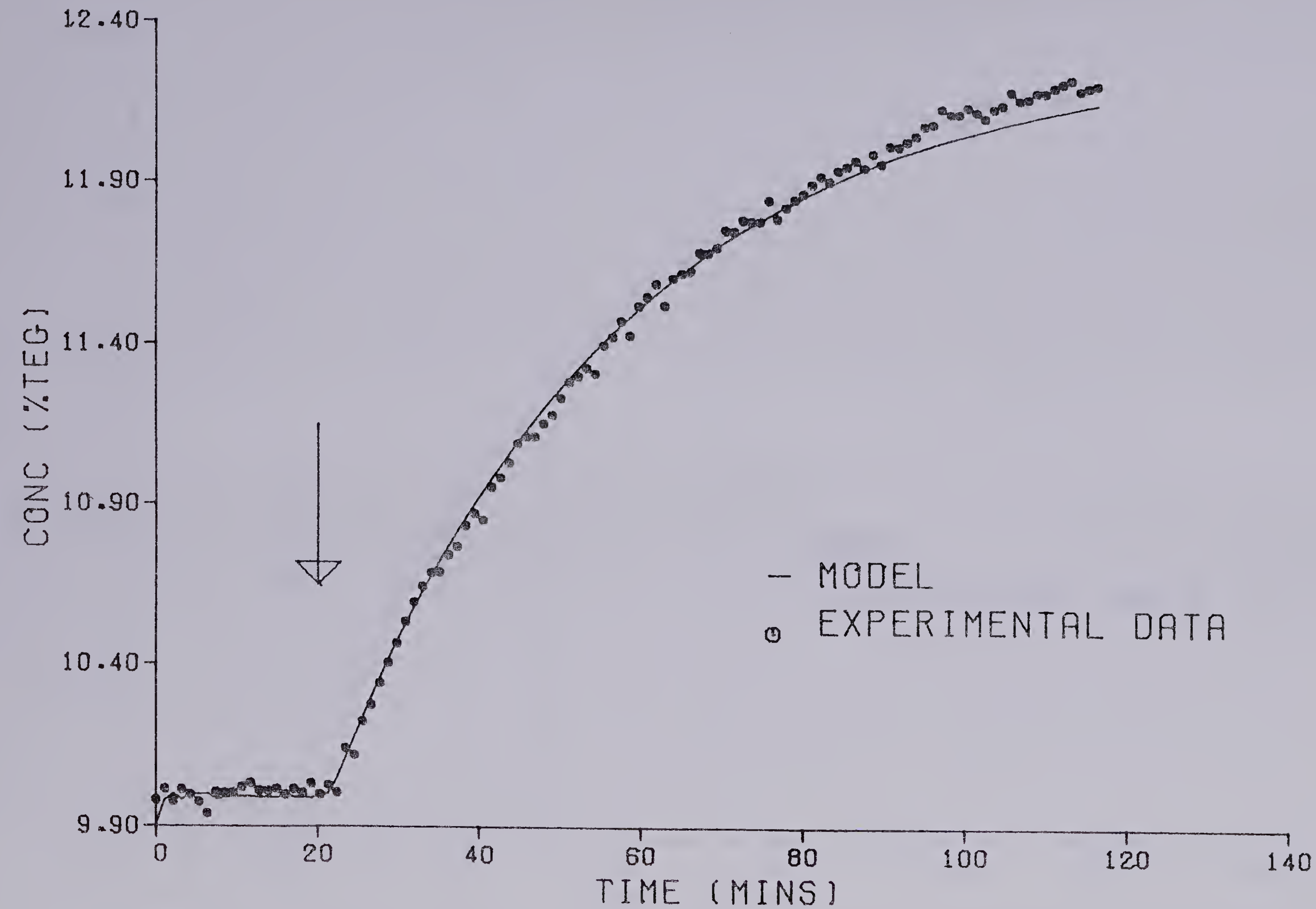


FIGURE 3.3 Identification of the Evaporator Using a First Order Model Plus Time Delay

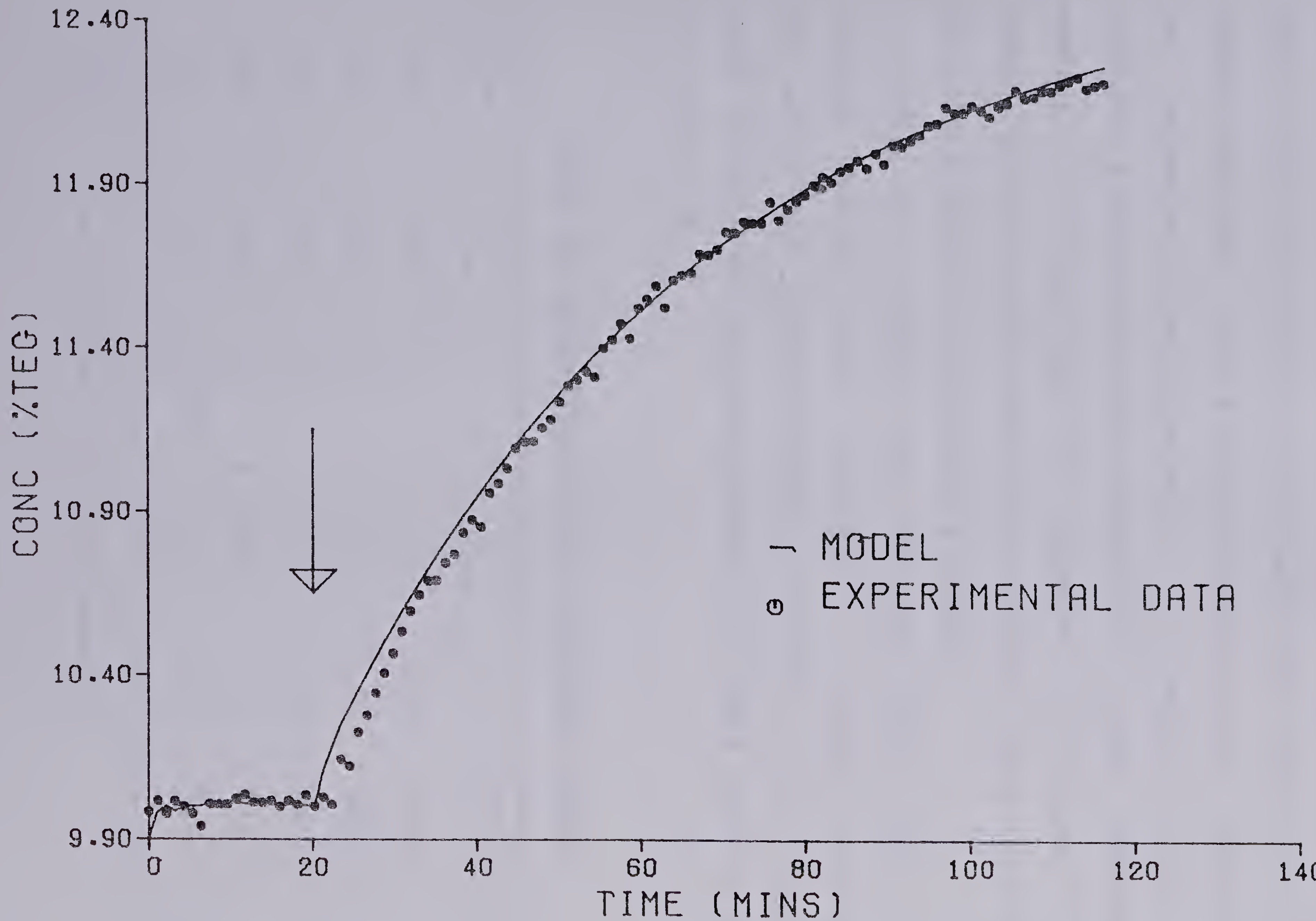
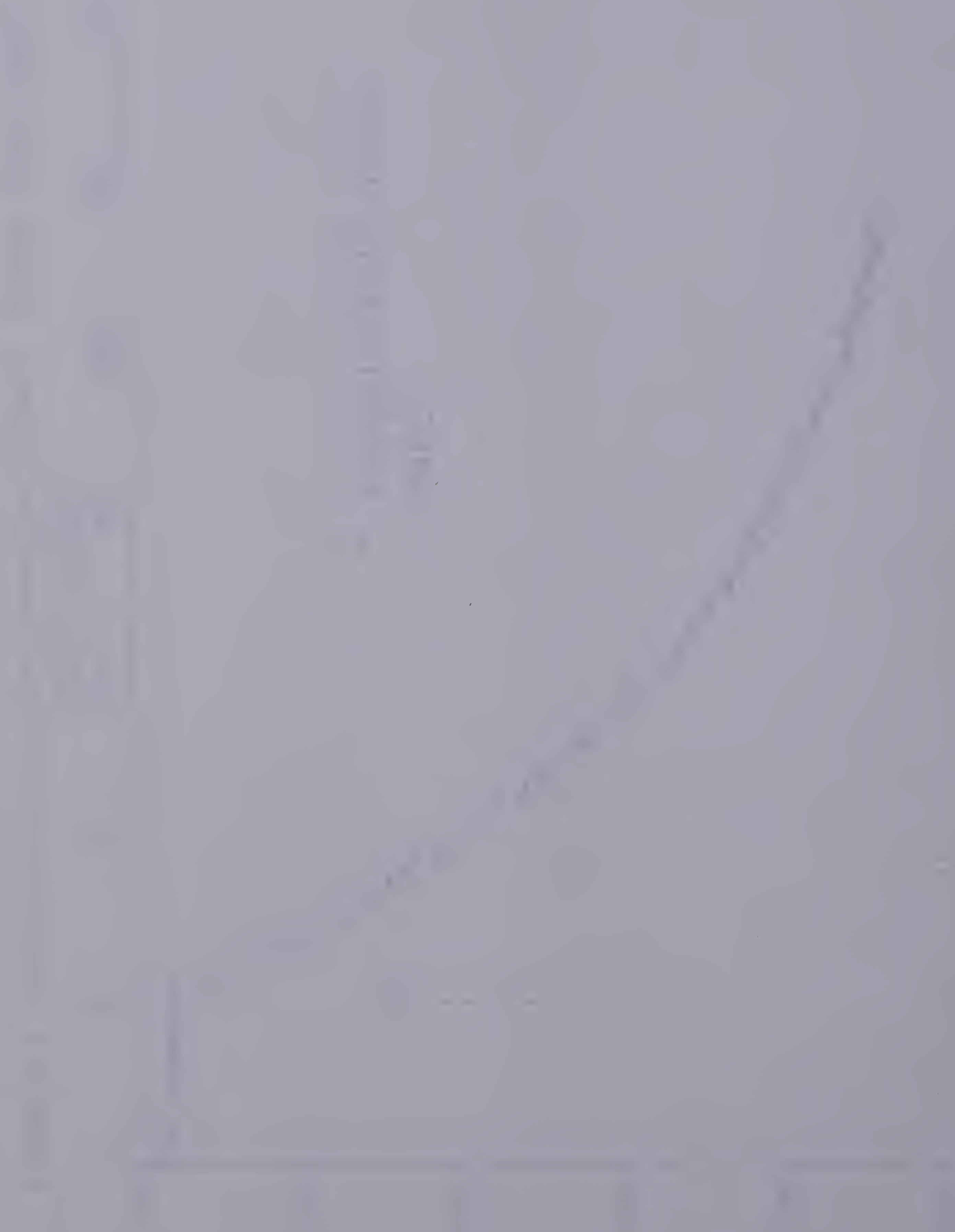


FIGURE 3.4 Identification of the Evaporator Using a Second Order Model



gain of the evaporator is not linear and the above models may only be valid over a limited operating range (e.g. for this identification the steam flowrate was increased by about eight percent of its steady state value.). If more steady state data (i.e. a longer duration runs) were used for parameter identification it is possible that the estimated steady state gains would be in closer agreement. However, in this study of adaptive controllers it was considered particularly important to have a good fit over the initial part of the process transient.

Table 3.1 compares the derived models above with previous models of the same kind with respect to process gain and process time constants. All models are expressed in terms of variables nomalized around the steady state values, e.g. $(X-X_{ss})/X_{ss}$.

Table 3.1 Comparison of the evaporator models

Order	T_1	T_2	T_d	K	reference
1st	25.0	0.0	0.0	2.04	Newell
1st	28.5	0.0	2.5	2.24	Eqn.(3.3)
1st	47.0	0.0	0.0	2.97	Eqn.(3.2)
2nd	44.0	1.85	0.0	2.62	Nieman
2nd	46.9	0.0044	0.0	2.97	Eqn.(3.4)

Note : K is the dimensionless process gain

The second order model in equation (3.4) is quite comparable with the Nieman's model. In this work the first order with time-delay model and the second order model were used to initialize the parameters of the adaptive controllers. When these models failed to give a satisfactory response, the time series model given by Kogekar(1977) and the discrete transfer function model obtained from the fifth-order state space model [Chang, 1975] were also considered when selecting the initial conditions for STR and APCS (unweighted) algorithms.

1) Time series model

$$\frac{C2(z^{-1})}{S(z^{-1})} = \frac{0.0272z^{-1} + 0.01639z^{-2}}{1 - 1.7z^{-1} + 0.702z^{-2}} \qquad (3.5)$$

2) Discrete transfer function model

$$\frac{C2(z^{-1})}{S(z^{-1})} = \frac{0.014z^{-1}-0.0002z^{-2}-0.009z^{-3}}{1-2.32z^{-1}+1.71z^{-2}-0.388z^{-3}} \qquad (3.6)$$

3.3 Sampling Interval

The choice of sampling interval plays a very important role in discrete control algorithms. It influences the time-delays and locations of the discrete system poles and zeros and thereby the closed loop control performance. A general rule for choosing an optimal sampling time is very difficult to formulate since it should reflect the process dynamics, the external disturbance characteristics, the desired control performance and so on. In general, a long

sampling time impairs the overall control performance mainly due to the loss of system information. On the other hand, a short sampling time usually gives better performance at the expense of large excursions of the manipulated variable. For adaptive systems the performance of the estimation algorithm must also be considered.

There are several rules for selecting the sampling time, suggested in the current literature [e.g. Isermann 1981]. If the frequency spectrum of the error signal is known, the sampling time can be chosen in accordance with Shannon's sampling theory

$$T_s \leq \pi / \omega \tag{3.7}$$

where ω is the maximum frequency of the error signal that can be detected by the sampled data controller. For the low frequency process the following range can be used.

$$1/16\omega_n < T_s < 1/8\omega_n \tag{3.8}$$

where ω_n is the eigenfrequency or natural frequency of the closed system in cycles/time [Isermann 1981].

Another criterion for an overdamped process with time-delay, T_d, is given by

$$T_d/8 < T_s < T_d/4 \tag{3.9}$$

or in terms of setting time

$$T_{st}/12 < T_s < T_{st}/6 \tag{3.10}$$

where T_{st} is the 95% settling time of the step response [Isermann 1981]. Note that these last two rules do not seem to be good for the evaporator. When the dominant time constant of the process, τ, is known, the following range has been suggested to ensure the satisfactory performance [Verbruggen et al., 1975].

$$T_s < \tau/10 \tag{3.11}$$

As mentioned before, for sampled data control sytems, a long sampling interval deteriorates the control performance. Thus when the control performance is of primary importance the sampling time should be as small as possible. However, it can not be arbitrarily small because a small sampling time relative to the dominant time constant of the process may result in the overall system being very oscillatory or in extreme cases actually unstable. This mainly stems from the fact that the locations of poles and zeros of a discrete transfer function are determined by the sampling time. In other words, a small sampling interval produces small

numerator coefficients in the transfer function and results in zeros close to the unit circle in z-plane, which is the set of the critical stability points. A small leading coefficient in the numerator of the discrete transfer function gives rise to another problem in the application of the minimum variance type adaptive controllers. The following example illustrates the effect of sampling time on the locations of poles and zeros in z-plane.

Example 3.1: Consider a process described by the second order with time-delay.

$$G(s) = \frac{K \exp(-T_d s)}{(T_1 s + 1)(T_2 s + 1)} \tag{3.12}$$

Z-transformation, assuming zero-order-hold, would produce the following form.

$$G(z^{-1}) = \frac{(b_0 + b_1 z^{-1})\ z^{-k}}{(1 + a_1 z^{-1} + a_2 z^{-2})} \tag{3.13}$$

The coefficients were calculated with different sampling times for the Nieman's evaporator model where K is 2.62 and T_1, T_2 and T_d are 44.0 mins, 1.85 mins and zero respectively. Table 3.2 shows the effect of sampling time on the discretization and hence poles and zero of the model.

Table 3.2 Effect of sampling time on the discretization

T_s	a_1	a_2	b_0	b_1	zero	pole	
.5	-1.7519	.7546	.0037	.0034	-.9105	.9887	.7632
1	-1.5600	.5694	.0134	.0111	-.8291	.9775	.5824
2	-1.2948	.3242	.0456	.0314	-.6889	.9556	.3392
3	-1.1317	.1846	.0880	.0506	-.5748	.9341	.1976
4	-1.0282	.1051	.1359	.0656	-.4825	.9131	.1151
5	-0.9596	.0598	.1865	.0761	-.4079	.8926	.0670

Note : sampling time T_s is in minutes

This shows that b_0 gets smaller and the poles and zero move closer to the unit circle as sampling time decreases. The first coefficient of the numerator, b_0, is closely related to sensitivity in some adaptive controllers, e.g. STR and APCS. In these controllers, the gain is proportional to the inverse of b_0 and hence small values of b_0 may generate an excessive control signal which may cause closed loop oscillation and/or stability problems in an actual application. Therefore, in some cases increasing sampling time helps stabilize the overall system response. However, excessively long sampling times will give sluggish or poor control due to loss of process dynamics.

A sampling time for adaptive control of the evaporator was chosen based on the above guidelines and then confirmed by experimental tests. The experimental results showed that a 64 sec sampling time, as recommended by Newell(1971) and used in most of previous control study, was satisfactory but that 128 sec was also reasonable as far as the output performance was concerned. On the other hand sampling times longer than 180 sec resulted in sluggish control without improving closed loop stability. Some experimental results are included in later chapers along with the evaluation of each adaptive controller.

3.4 Experimental Procedure

In order to compare the experimental results of the adaptive controllers STC, APCS and SFC and the conventional, PID controller the operating conditions were kept the same for all experimental runs. Before starting each experimental run the evaporator was operated at the normal steady state using multi-loop PI control. The supervisory control program, i.e. ADCON, was initialized after an appropriate period of time (usually 20 minutes) and a feed disturbance or a setpoint change in the product concentration was introduced. ADCON, which contains four control algorithms STC, APCS, SFC and discrete PID, was initialized by reading a data file which contained all the necessary control parameters. After initialization ADCON performed the specified control action every sampling interval as

determined by the scheduler segment of DISCO. The control signal calculated from the supervisory program ADCON was put into the setpoint of the DISCO activity (individual control loop), e.g. steam control loop. The experimental data, i.e the evaporator I/O variables and the adaptive controller parameters were stored into a disk data file by a separate data acquisition program for later plotting.

3.5 Conventional PID Control

The conventional, continuous PID control algorithm is part of the standard DISCO package and has been successfully used to control the major control loops of the evaporator. The main purpose of this part of the work was to investigate the dynamics of the pilot scale, double effect evaporator using PID controllers and also to obtain conventional, PID control results which can be used for comparison with the performance of the adaptive controllers.

As a first step in tuning the PID settings, the first order model with deadtime, equation (3.3), was used and the corresponding PID constants were calculated based on the IAE technique [Miller et al., 1967], where the PID parameters are chosen such that the integral of absolute error (IAE) is minimized. The followings are the various PID settings.

	PID	PI	P
KC	5.042	4.413	4.037
τ_i	6.063	9.614	∞
τ_d	1.076	0.0	0.0

The controller output, u(t), is usually expressed in terms of the controller setting above and the control error e(t) by

$$u(t) = KC[e(t) + \frac{1}{\tau_i} \int e(t)\, dt + \tau_d \frac{de(t)}{dt}] \qquad (3.14)$$

The equivalent discrete PID controller can be easily derived by introducing discrete integration for the error integration and when the trapezoidal rule is used equation (3.14) can be expressed as follows.

$$u(k) = \frac{q_0 e(k) + q_1 e(k-1) + q_2 e(k-2)}{(1 - z^{-1})} \qquad (3.15)$$

The constant parameters, q_i, are expressed in terms of the continuous controller settings and the sampling time T_s by

$$q_0 = KC\,(1 + .5T_s/\tau_i + \tau_d/T_s)$$
$$q_1 = -KC\,(1 + 2\tau_d/T_s + .5T_s/\tau_i) \qquad (3.16)$$
$$q_2 = KC\,(\tau_d/T_s)$$

The equivalent discrete PID constants for the evaporator are therefore given as follows;

	PID	PI	P
q_0	10.88	4.64	4.037
q_1	-15.48	-4.18	-4.037
q_2	5.42	0.0	0.0

From equation (3.16) when τ_d is set to zero the controller becomes PI and so on. These discrete controller constants were used as starting values for PID tuning and also as the initial parameters for adaptive control, i.e. SFC.

In this study feed flowrate changes equal to ±20% of its steady state value were introduced as disturbances for regulatory control and product concentration changes equal to ±10% of the steady state operating value were introduced for the servo control experiments. These external disturbances are the ones traditionally used in control studies on the evaporator so the performance of the adaptive controllers can also be compared with the previous results. For the experimental tests using SISO adaptive controllers for control of C2, the first and the second effect holdups,

W1 and W2, were controlled by the conventional P and PI controllers with the following controller constants. Note that these holdups are cascaded to their own outlet flow rates as shown in Figure 3.1.

	W1	B1	W2	B2
KP	-0.08	-10.0	-0.40	-30.0
KI	0.0	0.03	0.0	0.08

The first effect holdup was not tightly controlled in order to cut down the interaction between the first effect holdup and the product concentration but the second effect holdup was controlled close to its steady state operating value because of the small size of the cyclone separator.

Before applying adaptive controllers to the control of the concentration an effort was made to find comparable PID control results. First of all, as part of the tuning procedure, proportional control was used to investigate the evaporator dynamics and also to check the previously obtained constants. For these purposes discrete, constant PID control was implemented as a part of the supervisory control program for the evaporator. Figure 3.5 shows the result when the proportional constant was 6.0. Note the large offset in C2. Thus, the constant was gradually

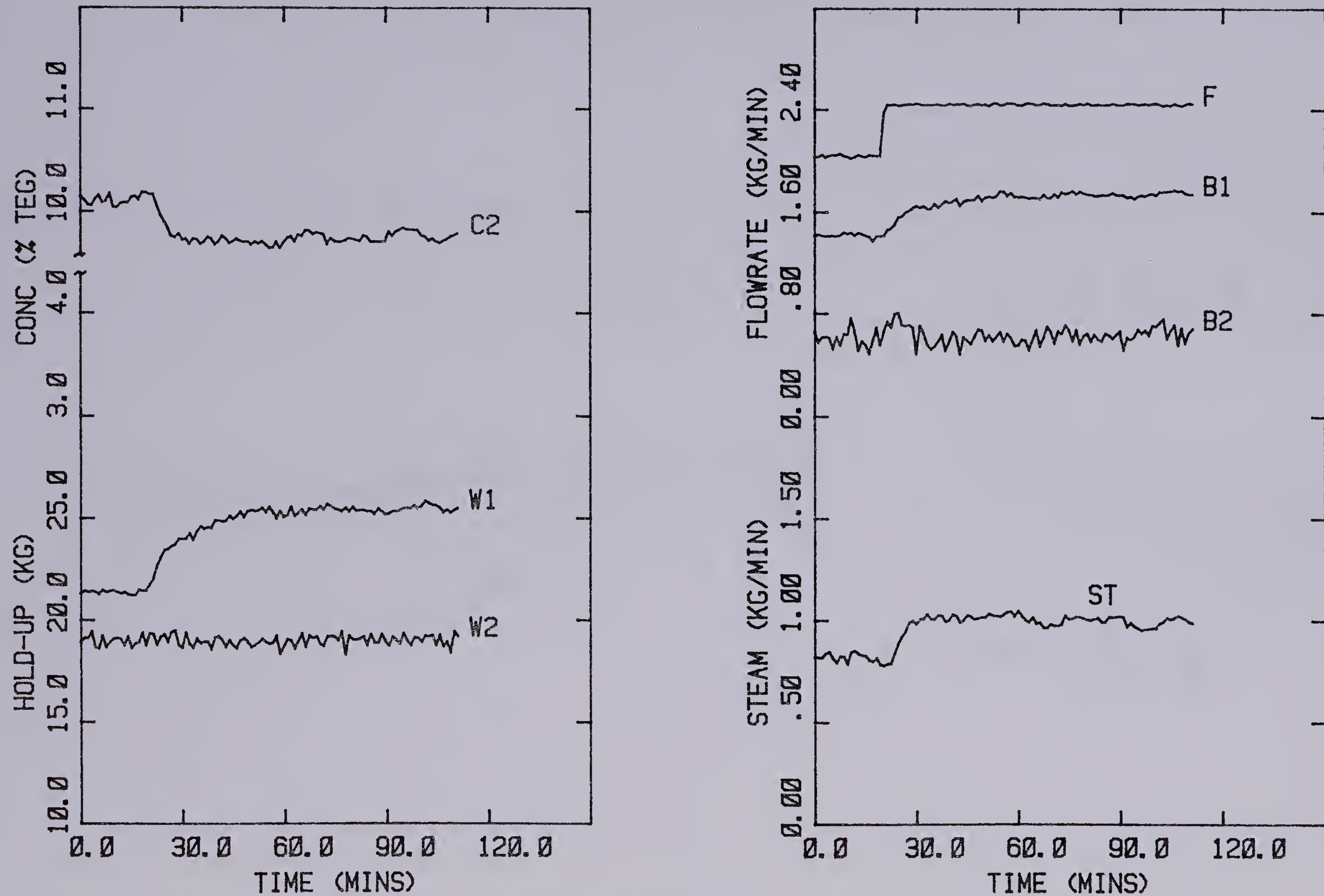


FIGURE 3.5 Evaporator Response Using Proportional Controller (KP=6)
(PID/RRPID2/KP 6/T64/P/ 20%FD/ PROPORTIONAL CONTROL cf.RRPID3)*

* refer to the nomenclature section

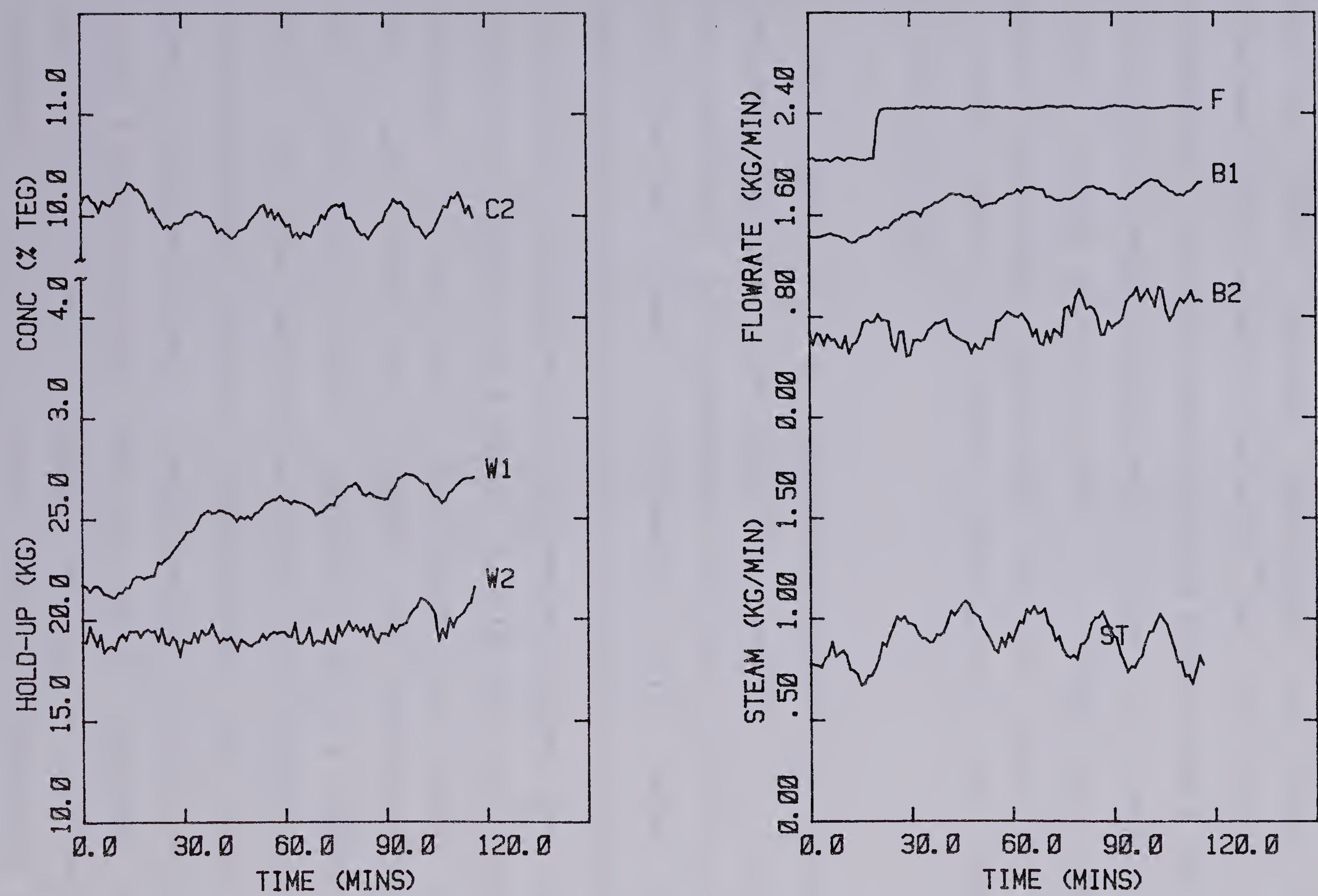


FIGURE 3.6 Evaporator Response Using Proportional Controller (KP=10)
(PID/RRPID3/KP10/T64/P/ 20%FD/ PROPORTIONAL CONTROL cf.RRPID2)

increased until the ultimate gain was found. Figure 3.6 shows the proportional controller performance when the gain was 10.0, which indicates the critical oscillation. This experimentally obtained ultimate gain was comparable to the corresponding ultimate gains obtained from the Bode plot of the evaporator models, equations (3.3) and (3.4), where the critical values were 9.1 and 11.1 respectively. These proportional control results also reveal the sensitivity of the double effect evaporator to the relatively small changes in controller gain.

The conventional PID tuning and experimental runs were done using DISCO, i.e. the continuous version of constant PID controllers was implemented and the control action calculated was based on engineering units rather than dimensionless values. During the tuning of PID constants it was found that inclusion of derivative action gave very oscillatory dynamics and made it difficult to tune the constants. (This may explain why the previous studies were mostly based on PI control [Newell, 1971; Kuon and Fisher, 1974; Oliver et al., 1974]) . Therefore, in this study, PI control was also used for the concentration/steam loop and the corresponding controller coefficients were carefully tuned to minimize the sum of absolute control error and also to give a robust response. Here, the old PI constants were used in the parameter tuning procedure as well as the constants in the previous section. The following values are

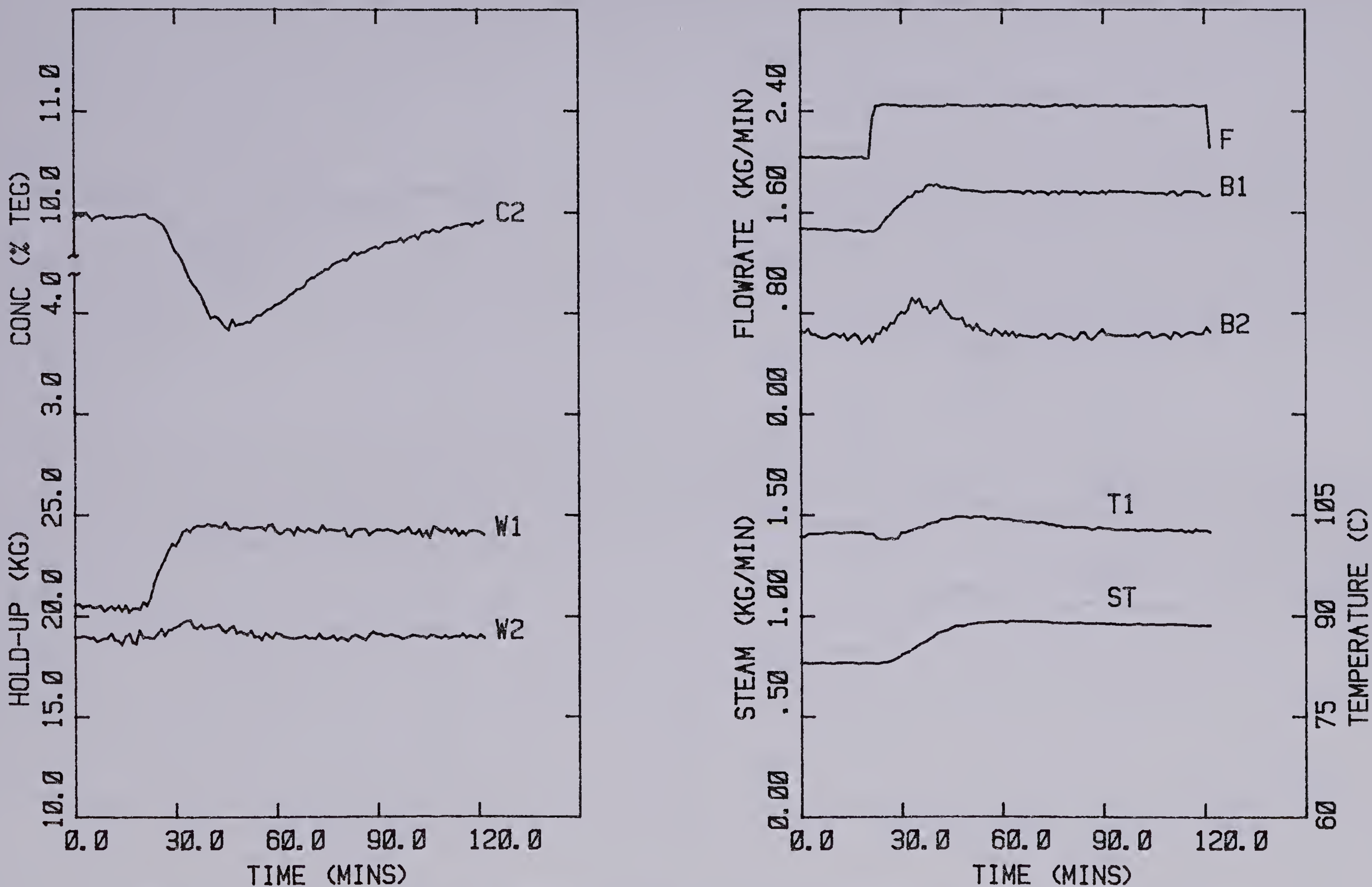


FIGURE 3.7 Evaporator Response Controlled by Conventional Fixed Gain PI
(PID/RRPID5/DISCO PI/T64/PI/ 20%FD/ TUNED PI RESULT, cf. RRPID4 RRPID6)

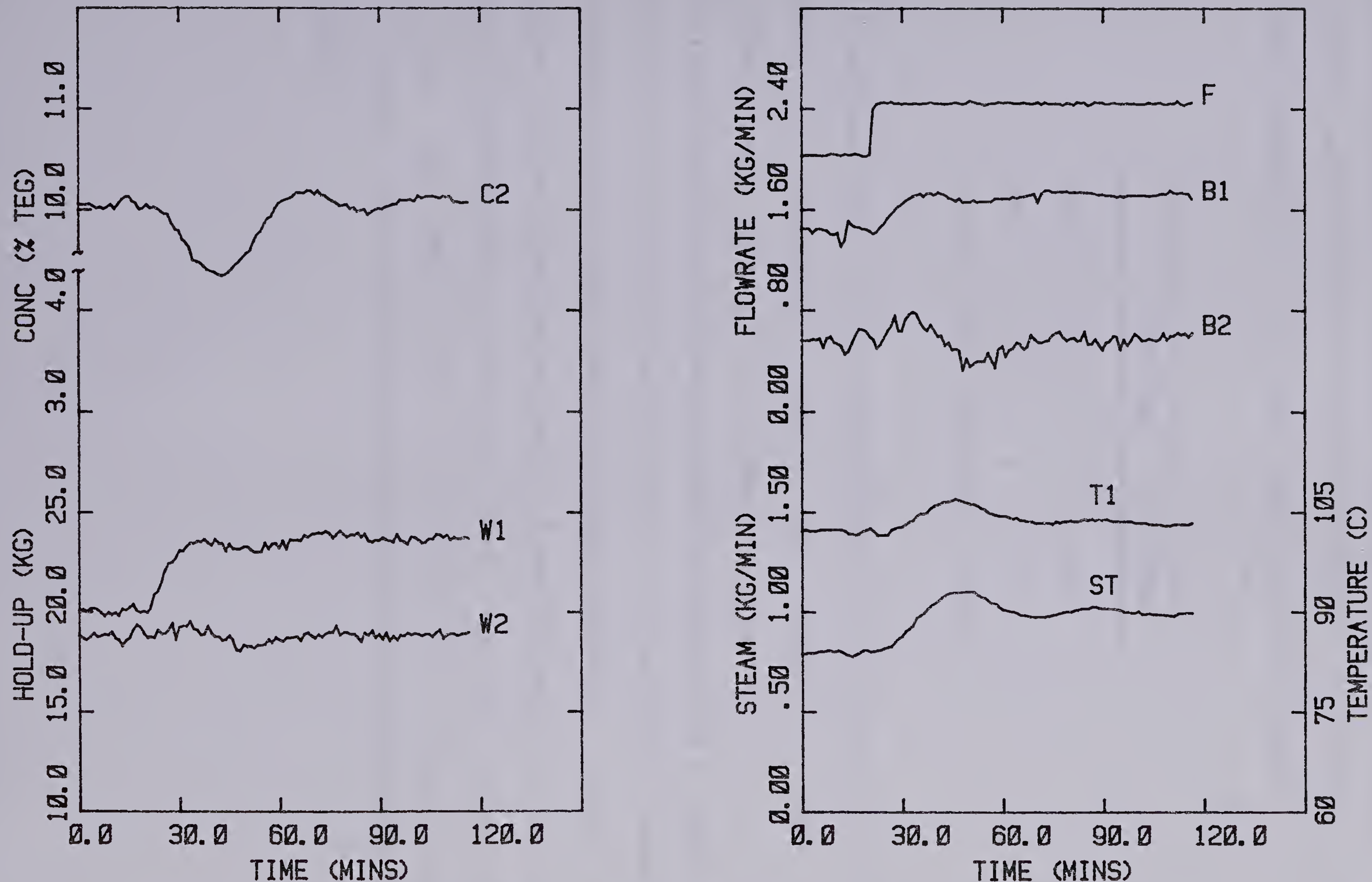


FIGURE 3.8 Evaporator Response Controlled by Conventional Fixed Gain PI
(PID/RRPID6/DISCO PI/T64/PI/ 20%FD/ SENSITIVE TO GAINS cf. RRPID4 RRPID5)

among the best for the master loop (C2) and the slave loop (steam) and Figure 3.7 shows the control performance using these values with a 20% step feed flow disturbance.

	C2-loop	S-loop
KP	0.120	10.5
KI	0.001	0.1

As mentioned before the evaporator was very sensitive to the choice of controller gain. For example, when the proportional gain was increased from .12 to .15 the corresponding C2 response started to oscillate as shown in Figure 3.8 and, furthermore, for a 10% setpoint change the response was not satisfactory. (Note the KP of .12 and .15 above differ from the values of 6 and 10 used in Figure 3.5 and 3.6 mainly due to the fact that they are used in a control law that works with normalized, dimensionless variables, i.e. the difference between the program ADCON and DISCO)

4. Self-Tuning Regulator and Controller (STR/C)

4.1 Introduction

The PID control scheme is one of the most widely used feedback strategies in the process industry. Typically, controller settings are set by 'experience' or 'tuned' after the control system has been installed using time-consuming, trial-and-error procedures. If process conditions change significantly, then the controller must be retuned in order to obtain satisfactory control. On the other hand adaptive control systems automatically adjust controller settings and thus 'self-tune' themselves to compensate for unanticipated changes in the process or the environment.

It is only during the last decade that such self-tuning or adaptive controllers have attracted significant attention. The idea of self-tuning or adaptive systems was conceived as early as 1958 by Kalman (1958) and later in a different form by Chang and Rissanen (1968) and Peterka (1970). An important step forward was made by Aström and Wittenmark (1973) who proposed the use of a minimum variance controller. Since then the method has been extended by Clarke and Gawthrop (1975) to include control costing or weighting. Significant number of other modifications have been made to render the algorithms more practical and robust.

In the following sections a literature review (in a limited form) has been undertaken to highlight the major

developments in the self-tuning control area. The practical aspects of implementing such controllers are discussed in the subsequent sections followed by an evaluation by simulation and experiment of these controllers on the pilot-scale double effect evaporator.

4.2 Literature Survey

This survey is done in an almost chronological manner highlighting the main results and methods that have appeared in the literature under the heading of STR/C.

The idea of self-tuning was originally proposed by Kalman(1958). The theory was revived and extended by Peterka(1970) who used the Aström-Bohlin model and recursive least squares (RLS) to identify its model parameters. Under some conditions he found that the parameters converged to the minimum variance controller. Aström and Wittenmark(1973) formulated the current self-tuning regulator and made it practical. They showed two important properties of the STR. First, if the parameter estimates converge then the autocovariance of the output and the crosscovariance between the input and the output tend to zero when the correlation time is greater than the system dead time, and secondly that the controller converges to the minimum variance controller that could be obtained from the known process model. Borrison(1975, 1979) extended the STR to MIMO systems. Some further improvements have been suggested by Keviczky and Hetthéssy(1977) and the corresponding MIMO controller has

been applied to a cement plant [Keviczky et al., 1978]. The objective of Aström and Wittenmark's STR is the minimization of the variance of the process output at each sampling instant. The objective can be justfied in many regulatory applications but it is not appropriate for general control problems because an excessive control action may be generated and cause closed loop instability. Also, nonminimum systems cannot be handled properly. In 1975 Clarke and Gawthrop presented a generalized self-tuner called the self-tuning controller (STC). Clarke and Gawthrop introduced a simple quadratic cost function and then derived an implicit self-tuner so that the controller parameters are estimated directly. The cost function includes a setpoint modifier as well as a term to penalize control effort. The control weighting solves two problems associated with the original self-tuning regulator. Firstly, it can be used to eliminate the large excursions in control actions and to control the transient response of the closed loop system. Secondly it can also be used to handle nonminimum phase systems. In the treatment of nonminimum phase systems Aström(1974) has proposed a simple cost function and Aström and Wittenmark(1974) have suggested the use of polynomial factorization to cancel out the nonminimum zeros. Gawthrop (1977) extended the earlier STC to a case where the weighting functions could be transfer functions instead of polynomial terms and made some interesting interpretations of this general STC. For example, it can be interpreted as a

model following scheme, conventional controller compensation (e.g. PID type Q-weighting), etc.

The STC of Clarke and Gawthrop has been generalized by Morris et al. 1977, 1981) to be robust, reliable and practical for actual applications. They introduced a discrete PID type compensator for the control weighting function and model following schemes for the setpoint tracking problem. This controller has been applied to the control of a distillation column [Morris et al., 1981]. Furthermore, they extended the controller to handle multivariable systems having the same number of inputs and outputs. In this scheme a multivariable system is reduced to a number of single loops. The interaction terms are treated like measurable disturbances and the system is decoupled using feedforward type compensation. This approach eliminates many involved matrix operations. Extension of Clarke and Gawthrop's STC for the mutivariable case was presented by Koivo in 1980. His work can also be considered as an extension of Borrison's self-tuner (1975, 1979) to handle nonminimum phase systems. In Koivo's approach the weighting functions included in the performance index are polynomial matrices and the controller parameters derived by Borrison's analysis are estimated by a RLS method in square-root form [Peterka, 1970]. An explicit MIMO self-tuning controller has been suggested by Wong and Bayoumi (1981), where the controller structure is the same as the Koivo's method but the process parameter matrices

instead of the controller parameter matrices are estimated to predict the process outputs. In this manner the requirement of the equal number of inputs and outputs is removed.

STR/C is based on optimal control law and predictive control theory which requires that the process time delay as well as the structual order of the model be specified a priori. It has been argued by Wellstead et al. that process time delays can be estimated as a part of the process dynamics and that the optimality of STR often results in large closed loop gains such that the control action becomes unacceptably large from the application point of view. They have designed an explicit self-tuner termed: a pole assignment self-tuning regulator. In this self-tuner the process dead time need not be given explicitly and the closed loop poles are forced to be placed at prescribed locations while the zeros remain at their open loop positions [Wellstead et al., 1979a, 1979b; Wellstead and Zanker, 1979]. Note that the minimum variance self-tuner is a stochastic analog of an optimal discrete dead-beat controller. The pole assignment scheme is a detuned controller which abandons optimality by forcing the poles to specified locations. However, in actual applications where excessive control action may cause stability problems it can be used to achieve moderately satisfactory and robust control performance. This pole placement scheme involves

more calculations than an implicit self-tuner. The pole assignment self-tuning regulator has also been extended to cover tracking and regulation [Wellstead et al., 1979b; Aström and Wittenmark, 1980] and mulivariable systems [Prager and Wellstead, 1981].

Most self-tuners are based on discrete models and discrete controller design. One disadvantage of discretization of continuous process is that even for minimum phase systems discretization can result in nonminimum phase characteristics due to a particular choice of the sampling time or fractional part of a time delay. To overcome this problem Gawthrop(1980) proposed the hybrid self-tuner which is the combination of a continuous-time model and a discrete-time adaptive controller. In this way the sampling rate can be fast for the identification of the continuous-time model and relatively slow for the control law. He derived a continuous form of self-tuning PID controller using the hybrid self-tuner under special conditions [Gawthrop, 1982].

There are numerous applications of STR/C for the control of various systems. The areas have been extensively covered by Lieuson(1980), Parks et al. (1980) and Harris and Billings(1981). More recent applications of STR/C can be found in the available proceedings of 1983 IFAC workshop on Adaptive Systems and the 1983 proceedings of the Yale

workshop on adaptive systems.

Stability and asymptotic convergence are desirable properties of an adaptive controller. In general, the performance of the parameter estimation algorithm depends on the feedback control law which inevitably introduces nonlinearity and time-varying characteristics to the analysis of the adaptive controller. Stability and convergence of STR have been heuristically discussed by Ljung and Wittenmark in the early stages of STR development (1974). Using ordinary differential equations to describe the parameter trajectory they showed that the STR does not converge for a general noise structure. The convergence problem related to self-tuning control has been discussed by Aström et al.(1977). In 1977 Ljung proved that positive realness for the noise equation is essential for convergence of STR but stability of the closed loop system has not been considered at all [Ljung, 1977a; Egardt, 1978]. Using input-output stability [Zames,1966a; Willems,1976; Vidyasagar,1978], small gain theorem [Desoer and Vidyasagar, 1975] and martingale theory, Gawthrop(1979) has derived stability conditions for STC and shown that stability implies convergence with probabilty one of the mean-square prediction error to the smallest value achievable by the control law. Rigorous, mathematical proof of stability and convergence of a class of STR has been established recently by Goodwin et al. (1978, 1981) and Martin-Sanchez et al.

(1981c) using rather simple adaptive mechanisms and some conditions on the stochastic disturbance.

4.3 Theory

In this section the underlying theoretical formulation of Clarke and Gawthrop's STC will be discussed.

4.3.1 Derivation of STC

Consider the following discrete, ARMAS representation of a SISO process.

$$A(z^{-1})y(k) = B(z^{-1})u(k-d) + L(z^{-1})v(k-q) + C(z^{-1})\xi(k) \tag{4.1}$$

where z^{-1} is the backward shift operator. A and C are monic polynomials in z^{-1} and the first coefficient of polynomial $B(z^{-1})$ is nonzero, $b_0 \neq 0$. The terms $y(k)$, $u(k)$, $v(k)$ and $\xi(k)$ are the process output, control input, deterministic disturbance and zero-mean white noise sequence respectively. The process is assumed to be of order na with a time-delay of d sampling intervals and a disturbance delay of q sampling intervals. It is also assumed that the unmeasurable disturbance is a stationary process with rational spectral density. The STC controller is designed to minimize the quadratic cost function.

$$J = E\{[P(z^{-1})y(k+d)-R(z^{-1})w(k)]^2+[Q'(z^{-1})u(k)]^2\} \quad (4.2)$$

where E{} is the statistical expectation operator. $w(\cdot)$ is the reference or setpoint sequence and P, R and Q are rational polynomials in z^{-1}, i.e.,

$$P(z^{-1}) = \frac{P_n(z^{-1})}{P_d(z^{-1})}, \text{ etc.}$$

When there is no weighting on the control action, i.e. $Q'(z^{-1})=0$, and the process output and the setpoints are weighted by unity, the cost function reduces to the variance of the error between the output and the setpoint and the resulting controller based on this cost function is the minimum variance (MV) self-tuning regulator of Aström. Here, a more general case, i.e., the self-tuning controller of Clarke and Gawthrop will be considered.

In order to be able to minimize the cost function J at time k, $P(z^{-1})y(k+d)$ must be expressed in terms of past and present process I/O data. Rewriting the process equation (4.1) in the form of weighted predicted output gives

$$Py(k+d) = \frac{PB}{A}\,u(k) + \frac{PL}{A}\,v(k+d-q) + \frac{PC}{A}\,\xi(k+d) \qquad (4.3)$$

For simplicity the argument (z^{-1}) has been dropped. The stochastic disturbance term can be expanded in terms of future disturbances and disturbances up to and including time k using the following identity [Aström, 1970].

$$\frac{P_n(z)C(z)}{P_d(z)A(z)} = G(z) + \frac{z^d F(z)}{P_d(z)A(z)} \qquad (4.4)$$

where na is the order of polynomial $A(z^{-1})$, etc., and polynomials $G(z^{-1})$ and $F(z^{-1})$ are defined as;

$$\begin{aligned} G(z) &= 1 + g_1 z + \cdots + g_{d-1} z^{d-1} \\ F(z) &= f_0 + f_1 z + \cdots + f_{ni-1} z^{ni-1} \\ ni &= \max\,(\,na + np_d\,,\; nc + np_n - d + 1\,) \end{aligned} \qquad (4.5)$$

Substituting equations (4.4) into (4.3) to separate past and future unmeasurable disturbances gives;

$$Py(k+d) = \frac{P_n \cdot B}{P_d \cdot A}\,u(k) + \frac{P_n \cdot L}{P_d \cdot A}\,v(k+d-q) + \frac{F}{P_d \cdot A}\,\xi(k) + G\xi(k+d) \qquad (4.6)$$

The past and present stochastic disturbances $\xi(k-i)$, $i \geq 0$ can be reconstructed in terms of known process I/O data from equation (4.1);

$$\xi(k) = \frac{A}{C} y(k) - \frac{B}{C} u(k-d) - \frac{L}{C} v(k-q) \tag{4.7}$$

Replacing $\xi(k)$ in equation (4.6) by (4.7) and using the identity equation (4.4) gives the weighted output of the process.

$$P(z^{-1})y(k+d) = \frac{F(z^{-1})}{P_d(z^{-1})C(z^{-1})} y(k) + \frac{B(z^{-1})G(z^{-1})}{C(z^{-1})} u(k)$$
$$+ \frac{L(z^{-1})G(z^{-1})}{C(z^{-1})} v(k+d-q) + G(z^{-1})\xi(k+d) \tag{4.8}$$

Defining the optimum prediction of the weighted output to be $y^*(k+d/k)$ the best prediction in the sense of a Wiener process can be obtained from the conditional expectation of equation (4.8).

$$y^*(k+d/k) = \frac{F(z^{-1})}{P_d(z^{-1})C(z^{-1})} y(k) + \frac{B(z^{-1})G(z^{-1})}{C(z^{-1})} u(k)$$
$$+ \frac{L(z^{-1})G(z^{-1})}{C(z^{-1})} v(k+d-q) \tag{4.9}$$

Equation (4.8) can now be written in terms of the predicted

output,

$$P(z^{-1})y(k+d) = y^*(k+d/k) + G(z^{-1})\xi(k+d) \tag{4.10}$$

Note that $y^*(k+d/k)$ and $G(z^{-1})\xi(k+d)$ are orthogonal because of the uncorrelation assumption and the prediction accuracy can be measured by the variance of noise;

$$E\{[G\xi(k+d)]^2\} = (1 + g_1^2 + \cdots + g_{d-1}^2)\sigma^2 \tag{4.11}$$

where σ^2 is the variance of the white noise $\xi(k)$,i.e.,

$$E\{\xi(k)\xi(k)\} = \sigma^2.$$

Note that the prediction accuracy decreases as the system time-delay increases.

Now, because the value of $y^*(k+d/k)$ can be predicted at time k the performance index J can be expressed in terms of present and past process I/O data.

$$J = E\{[y^*(k+d/k) - Rw(k) + G\xi(k+d)]^2 + [Q'u(k)]^2\} \tag{4.12}$$

Assuming the term $G\xi(k+d)$ is uncorrelated with the present and past values of $y(k-i)$, $u(k-i)$ and $v(k-i)$ for $i\geq0$ and using equation (4.11), then the above equation can be rearranged as;

$$J = E\{[y^*(k+d/k) - Rw(k)]^2 + [Q'u(k)]^2\}$$
$$+ \sigma^2(1 + g_1^2 + \cdots + g_{d-1}^2) \qquad (4.13)$$

The conditional performance function J can be minimized by setting its partial derivative with respect to the current control law u(k) to zero, i.e. $\partial J/\partial u(k) = 0$, or,

$$y^*(k+d/k) - R(z^{-1})w(k) + Q(z^{-1})u(k) = 0 \qquad (4.14)$$

where $Q(z^{-1}) = q_0'Q'(z^{-1})/b_0$, has been redefined. Now, convert this scalar function, equation (4.14), to a controller output function, $\Phi^*(k+d/k)$, as follows;

$$\Phi^*(k+d/k) = y^*(k+d/k) - R(z^{-1})w(k) + Q(z^{-1})u(k) \qquad (4.15)$$

Similarly the equivalent function for the actual weighted output can be defined as;

$$\Phi(k+d) = P(z^{-1})y(k+d) - R(z^{-1})w(k) + Q(z^{-1})u(k) \qquad (4.16)$$

and it can be written using equations (4.10) and (4.15) as follows;

$$\Phi(k+d) = \Phi^*(k+d/k) + G(z^{-1})\xi(k+d) \qquad (4.17)$$

By comparing equations (4.15) and (4.14) the control action at time k can be calculated such that the d-step-ahead

predicted controller output function, $\Phi^*(k+d/k)$, is set to zero.

$$u(k) = Q^{-1}(z^{-1})[R(z^{-1})w(t) - y^*(k+d/k)] \quad (4.18)$$

By substituting the predicted, weighted output of the process, equation (4.9) into equation (4.18) the control law establishes a stochastic controller for a known parameter process. For unknown parameter process $y^*(k+d/k)$ is assumed to be in linear regression form and its parameters are estimated by a least squares scheme using the process I/O information. Combination of the stochastic control law, equation (4.18) and an estimation algorithm for the parameters of the prediction model equation (4.9), forms the self-tuning controller [Clarke and Gawthrop, 1975,1979]. In other words STC is a controller that can be applied to control a process with a known model structure but unknown parameters. In order to simplify the analysis polynomials $E(z^{-1})$ and $D(z^{-1})$ are defined as follows;

$$E(z) = B(z)G(z) = e_0 + e_1 z + \cdots + e_{nj} z^{nj} \quad (4.19a)$$

$$D(z) = L(z)G(z) = d_0 + d_1 z + \cdots + d_{nd} z^{nd} \quad (4.19b)$$

where nj and nd are the order of polynomials E and D respectively, and vectors $\Theta(\cdot)$ and $X(\cdot)$ are introduced.

$$\Theta_0^t = [f_0,\cdots f_{ni}, e_0,\cdots e_{nj}, d_0,\cdots d_{nd}, c_1,\cdots c_{nk}]$$

$$X^t(k) = [y'(k),\cdots y'(k-ni), u(k),\cdots u(k-nj),$$
$$v(k+d-q),\cdots v(k+d-q-nd),$$
$$-y^*(k+d-1/k-1),\cdots -y^*(k+d-nk/k-nk)]$$

where $y'(k) = y(k)/P_d(z^{-1})$, i.e., the filtered process output and the superscript 't' denotes the transpose. Then the predicted, weighted output of the process can be expressed as;

$$y^*(k+d/k) = \Theta_0{}^t X(k) \tag{4.20}$$

Recalling equation (4.10), the actual weighted process output $P(z^{-1})y(k+d)$ can be written as

$$Py(k+d) = \Theta_0{}^t X(k) + G\xi(k+d) \tag{4.21}$$

If $G(z^{-1})\xi(k+d)$ is uncorrelated white noise, i.e., $G(z^{-1})$ is a constant, then RLS techniques can be applied to estimate the parameter vector Θ_0. In general this is not the case and the use of ordinary linear least squares may result in biased estimation. However, it has been shown by Clarke and Gawthrop (1975,1979) that the predicted, weighted output can be replaced by its estimated values in the actual calculation and the estimated, weighted output can be defined as;

$$\hat{y}^*(k+d/k) + \epsilon(k+d) = y^*(k+d/k) + G\xi(k+d) \tag{4.22}$$

where $\epsilon(\cdot)$ is assumed to be uncorrelated random sequence and the estimated weighted output is given as;

$$\hat{y}^*(k/k-d) = \hat{\theta}^t(k-1)\hat{X}(k-d) \tag{4.23}$$

Then the linear least squares identification techniques can be used. Rewriting equation (4.21) in terms of the estimated weighted output of the process gives;

$$P(z^{-1})y(k+d) = \hat{\theta}^t(k-1)\hat{X}(k-d) + \epsilon(k) \tag{4.24}$$

where $\hat{\theta}(k)$ is the estimation of Θ_0 and $\hat{X}(k)$ is the same as $X(k)$ but the elements of $y^*(\cdot)$ are replaced by their estimated values, $\hat{y}^*(\cdot)$.

Another way of implementation of STC is to express the scalar controller output function $\Phi^*(k+d/k)$ in linear form and then to combine it with the corresponding actual controller output function, $\Phi(k+d)$, to give an equation similar to (4.24) by using equations (4.15) and (4.17). Derivation of the regression form of the actual controller output function is very similar to the above and gives,

$$\Phi(k+d) = \hat{\theta}^t(k)\hat{X}(k) + (1-C)\Phi^*(k+d/k) + \epsilon(k+d) \qquad (4.25)$$

where $\Phi^*(k)$ and $\hat{X}(k)$ are of appropriate dimension and $\epsilon(k)$ is the estimation error which is a random sequence. The control law can be calculated by setting $\Phi^*(\cdot)$ to zero. Details are in Clarke and Gawthrop(1975,1979) or Morris et al.(1977).

Now, the model parameters, whether they are for the weighted process output or the controller performance function, can be estimated by means of RLS algorithms.

$$\hat{\theta}(k) = \hat{\theta}(k-1) + K(k)[P(z^{-1})y(k)-\hat{\theta}^t(k-1)\hat{X}(k-d)] \qquad (4.26)$$

$$K(k) = \frac{P_t(k)\hat{X}(k-d)}{\rho + \hat{X}^t(k-d)P_t(k)\hat{X}(k-d)} \qquad (4.27)$$

$$P_t(k+1) = [I - K(k)X^t(k-d)]\,\frac{P_t(k)}{\rho} \qquad (4.28)$$

where $K(k)$ is the estimator gain vector and $P_t(k)$ is the covariance matrix of the estimated parameter normalized with respect to the noise variance σ^2. It is a symmetric and positive definite matrix. I is the identity matrix. ρ denotes the forgetting factor for tracking slowly time varying process parameters.

The above RLS routine gives a new set of parameters at each control interval and the weighted output of the process can thus be calculated.

$$\hat{y}^*(k+d/k) = \hat{\theta}^t(k)\hat{X}(k) \tag{4.29}$$

The control law of equation (4.18) can be realized by replacing $y^*(k+d/k)$ by its estimate $\hat{y}^*(k+d/k)$.

4.3.2 Discussion of STR/C

In the minimum variance type STC, i.e. with $Q(z^{-1}) = 0$, the leading coefficient, e_0, of polynomial $E(z^{-1})$ plays a crucial role in the control performance and also in the rate of parameter convergence. For example, if it is very small the control action will be excessively large which will normally give fast parameter convergence but may result in an oscillatory or even unstable response. In the original STR of Aström the leading coefficient, called scaling factor β_0, was fixed reasonably close to its true value to provide parameter convergence to their true values and optimal control (and also to eliminate the possibility of division by zero). Fixing the scaling factor to an arbitrary value can be justified for the case when the desired output is zero since even if one parameter is fixed the control law can still achieve the control objective:

$$\beta \cdot \Theta_0{}^t X(k) = 0 \tag{4.30}$$

where β is any positive scalar constant. In other words if one parameter is fixed the other parameters will be scaled and identified accordingly and once the parameters have converged they would have a common factor. However, when the desired output is not equal to zero fixing one parameter may result in an offset unless it is fixed to its true value. Therefore, for the general case, such as STC and APCS no system parameters, including the first coefficient of polynomial $E(z^{-1})$, are fixed. The leading coefficient is adapted to account for the change of the process and disturbance dynamics. When this coefficient is very small, which means a large controller gain, its adaptation, however, may lead to serious stability problems. This effect has been illustrated in the simulated and experimental study of the evaporator in the following sections.

The RLS estimation algorithm, equation (4.26) through (4.28) with $\rho=1$, is derived based on the minimization of the loss function,

$$L = \sum_{i=0}^{k} \epsilon^2(i) \tag{4.31}$$

Where $\epsilon(i)$ is the equation error, and the recursive

calculations are started with initial values of $\Theta(0)$ and $P_t(0)$. The initial values of the parameters are frequently picked with no knowledge of the process and hence in order to increase the rate of parameter convergence the initial covariance matrix $P_t(0)$ is set to a very large diagonal matrix, say 1000I to 10,000I. The large covariance matrix denoting poor initial parameters gives rise to large variations in parameter estimates which in turn results in poor controller performance because of the certainty equivalence design principle of STC.

The performance of STR/C depends upon the effectiveness of the parameter estimator. For the RLS with a unit forgetting factor, if there is no persistent excitation in the process I/O data, the convergence of parameter estimates is usually much slower than the norm of the covariance matrix $P_t(k)$, and hence the estimator gain vector $K(k)$, tends towards zero. Thus, even if there is a large error in the parameters, the estimator can not adjust the parameters to their optimal or true values. For practical applications, the STR/C should also be able to perform the parameter tracking for slowly time-varying processes. To achieve this result, the covariance shrinking must be avoided. One simple method is to modify the basic RLS such that the data recently obtained are weighted more than the old data. This can be mathematically formulated by introducing an exponentially weighted loss function [Eykoff, 1974].

$$L = \sum_{i=0}^{k} \rho^{k-i} \epsilon^2(i) \ , \ 0<\rho<1 \tag{4.32}$$

The RLS based on this minimization ends up with the same set of equations (4.26) to (4.28). Here, a forgetting factor ρ of less than unity enables the estimator to forget or discount the old process information and also improves the convergence rate by inflating the covariance matrix at each sampling time. For instance, in order to calculate how many past data should be remembered before discounting to α% of its original value, the following relationship can be applied;

$$k = \frac{\log(\alpha/100)}{\log(\rho)} \tag{4.33}$$

The covariance will be inflated ρ^{-k} times. However, for time-invariant processes which are not properly excited or are operating at steady state the weighted RLS will gradually lose the valuable information collected in the past and be dominated by uncorrelated I/O data, i.e. noise. In this case the covariance matrix will gradually increase in value and finally the estimation algorithm will blow up (estimator windup) leading to a large variation of parameter estimates. Closed-loop instability as well as numerical problems caused by losing the positive definiteness of covariance matrix may immediately follow. To prevent the

estimator from winding up under low system excitation and disturbances, several ad hoc remedies have been suggested [Isermann,1981a]. One obvious way is to freeze the estimation algorithm during periods of low excitation depending upon the variance of the process output. Another way is to modify the covariance matrix at each control interval to retain its positive definiteness and/or to put upper and lower bounds for diagonal and off-diagonal elements on the matrix [Morris et al., 1982], in which case the resulting matix elements no longer stand for the parameter estimates' variance (diagonal) and covariance (off-diagonal). Third is to introduce a variable forgetting factor [Albert and Sittler, 1966; Fortescue et al., 1981], which is a modification of the fixed forgetting factor. In this scheme the forgetting factor is chosen in such a way that a prespecified information criterion is kept constant at each sampling time. When there is no change in the process variables the forgetting factor approaches unity so that no process information is lost. Although it may sometimes be impractical, introducing extra disturbance signal such as white noise or PRBS is another method of avoiding the estimator windup or parameter bursting phenomenon.

4.4 Implementation of STC

Just as with conventional controllers, the performance of STC is very strongly influenced by the choice of design parameters. It could be argued that most of the initial parameters for the STC are relatively easy to choose and the control loop is comparatively insensitive to their values. However, to give superior control performance and reliability a STC controller also needs to be 'tuned'. In the following section the parameters of the STC which must be known before the control algorithm starts, will be discussed from a practical rather than theoretical point of view.

4.4.1 Initial Parameters

The implicit STC is designed based on the known structure of the process model. The order of controller polynomials, ni, nj and nd is thus directly related to the model structure and can be given in terms of the number of model parameters:

$$\begin{aligned} ni &= \max(np_n + nc - d + 1,\ na + np_d) \\ nj &= nb + d - 1 \\ nd &= nl + d - 1 \end{aligned} \tag{4.34}$$

The total number of parameters to be estimated is as follows;

$$n\theta = ni + nj + nd + nk + 3 \text{ , if } nl \neq 0$$
$$= ni + nj + nk + 2 \text{ , if } nl = 0 \tag{4.35}$$

In fact the selection of a process model profoundly affects the control performance as well as the convergence of the parameters. When a model is sought there are, of course, many consideratons such as a priori knowledge about the process, its usage, complexity of the system, etc. Clearly, the model can not be chosen entirely arbitrarily. Furthermore, the real system is quite often far more complex than can be actually represented by a linear mathematical model. The 'best' description of a particular physical system can be found by trial and error. To put it somewhat differently, the practical problem in modelling reduces to that of finding an approximate description rather than that of determining the exact equation. It has been shown that if the exact structure model is employed the parameter estimates converge to their true values as the estimation time goes to infinity [Ljung, 1978] and that the STR achieves the optimal performance of the corresponding minimum variance controller [Ljung, 1977b]. However, the convergence of parameters to their optimal values may not be useful in practical situations where the chosen model is not able to describe the system dynamics for a given set of data. In this case it is usually more realistic to be content with a suitable, approximate model, e.g. 2nd or 3rd

order model. Since the approximate model converges to the local optimum the performance of STR/C with this model will become suboptimal instead of optimal.

Initial values of parameter estimates $\Theta(k)$ must be given before turning on the STR/C algorithm. The initial values are very important in the sense that they determine the trajectory of the estimated parameters and so the final stationary points [Ljung, 1977a]. If the process to be controlled is completely unknown the controller parameters are frequently initialized by zero values except the leading coefficient of polynomial $E(z^{-1})$, which should be given a reasonable value reflecting the process gain or dynamics. As has been discussed in section 4.3.2 a poor choice of the initial parameter set may give unacceptable I/O variations during the transient state and result in unstable closed loop response. In a practical application STR/C should thus not take any control action on the process during the initial stage when the start-up parameter values are poor. Well identified parameters should therefore be applied initially or background estimation should be done with the process under the control of a conventional controller such as PID before starting the self-tuner [Isermann, 1981a]. For control of the double effect evaporator the choice of initial parameter values was based on the open-loop response experiments described in chapter three.

4.4.2 Parameter Estimation Mechanism

The parameter estimation algorithm is the central part of all parameter adaptive control schemes. There are many different estimation techniques that have been used with adaptive control algorithms[Saridis, 1974; Isermann et al., 1974; Kurz et al., 1980; Morris et al., 1982].

The RLS method is one of the most popular and powerful techniques for parameter estimation or identification of unknown parameter systems. This technique is, of course, not perfect. It usually gives biased estimation when the system is exposed to nonwhite noise and also, as has been discussed in section 4.3.2, has numerical deficiencies when used as part of adaptive schemes for long term regulation of lightly excited or low noise systems. However, the RLS gives fast and stable parameter convergence compared to extended or generalized least squares and is simpler to implement than the maximum likelihood or other ad hoc variations of RLS such as factorization methods, instrumental variable technique, etc. The numerical problems, e.g. covariance shrinking and estimator windup can be prevented in most case by introducing a forgetting (discouting) factor and making other ad hoc variations. In this simulation and experimental studies the ordinary RLS estimator with a forgetting factor has been used.

4.4.3 Weighting Functions

1) The Q-Weighting Function

The original STR of Aström and Wittenmark (1973) takes the form of a discrete-time dead beat controller and the corresponding control signal often oscillates vigorously hitting the upper and lower physical bounds and in some cases there produces serious stability problems. However, penalizing the control effort by introducing Q-polynomial weighting improves the control performance and also the closed loop stability [Clarke and Gawthrop, 1975, 1979]. It is easily seen from equations (4.1), (4.10) and (4.18) that the closed loop dynamics can be modified by the choice of Q-polynomial, i.e. the closed loop characteristic equation is

$$Q(z^{-1})A(z^{-1}) + B(z^{-1}) = 0 \tag{4.36}$$

when $P(z^{-1})$ is unity. The location of closed loop poles can be manipulated by choosing Q to be a scalar constant. Although a scalar weighting factor can make the closed loop response stable the output of the process usually results in a steady state offset. This is apparent from the controller output fuction equation (4.16) when $P(z^{-1})$ and $R(z^{-1})$ are unity and $Q(z^{-1})$is a constant λ, i.e.

$$y(k) = w(k-d) - \lambda u(k) \tag{4.37}$$

The steady state offset can be eliminated by careful choice of $Q(z^{-1})$. The simplest way is to introduce a pure integrator, i.e. $Q(z^{-1}) = \lambda(1-z^{-1})$. However, this may impair the overall stability and deteriorate the transient response. A more useful design of $Q(z^{-1})$ weighting is one where its inverse takes the form of a discrete conventional PID compensator:

$$Q^{-1}(z^{-1}) = \frac{(a_0 + a_1 z^{-1} + a_2 z^{-2})}{(1-z^{-1})} \tag{4.38}$$

Then, from the control law equation (4.18), $u(k)$ is calculated according to the conventional PID law acting on the d-step-ahead control error.

$$u(k) = u(k-1) + (a_0+a_1z^{-1}+a_2z^{-2})e^*(k+d/k) \tag{4.39}$$

where $e^*(k+d/k) = w(k)-y^*(k+d/k)$. Because of the robustness of PID control this approach gives good self-tuning properties as well as avoiding the problem of steady state offset. However, the corresponding coefficients in the weighting function must be tuned before starting control action. This algorithm becomes a discrete conventional PID by simply putting the measured output of the process instead

of the predicted output.

2) The P-Weighting Function

The STR control law attempts to make the process output equal to the d-step-ahead reference value in a single step. If a sudden or step change in setpoint occurs, such control policy may result in large excursions of the process variables especially during the initial part of the transient. The transient response of the process to a sudden setpoint change can be improved by including in the control design a reference model which generates the optimal trajectory for the setpoint change [Gawthrop, 1977]. The output of the process is given from equations (4.18) and (4.10) if the Q-weighting is not considered.

$$y(k) = \frac{1}{P(z^{-1})} [Rw(k-d) + G\xi(k-d)] \tag{4.40}$$

In other words the output tends to follow the output of the reference model, $R(z^{-1})/P(z^{-1})$, whose input is the delayed setpoint $w(k-d)$. The P-weighting is quite comparable to the reference model of the MRAS, where the difference between the process output and the reference model output is used to design the adaptive control law [Landau, 1973]. Another important point to note with the P-weighting is that the unmeasurable system noise $G\xi(k)$ is also filtered by its inverse. Therefore, the design of the P-weighting should

avoid the possibility of unstable response caused for example by differenciating system noise. A typical design procedure would be as follows. If the plant to be controlled is assumed to be second-order with a dominant time constant T_1 then the reference model should also be of at most second order but with unity steady state gain and an open loop response that is faster than that of the process. The corresponding $P(z^{-1})$ can be found by discretizing the closed loop continuous transfer function with zero-order-hold, e.g. when $R(z^{-1})=1$,

$$\frac{1}{P(z^{-1})} = Z\{ \frac{1}{(Ts+1)^2} \} , T < T_1$$

If the time constant T is too large system information contained in the noise sequence will be filtered out and some difficulties will arise in the parameter estimation, e.g. slow convergence.

3) The R-Weighting Function

Another way of modifying setpoint changes to improve the transient response is to use the polynomial $R(z^{-1})$. This polynomial can be designed in conjunction with the $P(z^{-1})$ filter. As discussed in the previous section, $P(z^{-1})$ weighting modifies both the setpoint and the stochastic noise terms and as a result the parameter estimation may be degraded. This can be prevented, while still having the same

desired setpoint trajectory, by putting $R(z^{-1})$ equal to the desired model and setting $P(z^{-1})$ equal to unity.

The STC derived in section 4.3 has been implemented to apply real time processes reflecting the practical viewpoints discussed above.

4.5 Simulation Study

The objective of the simulation study was first to investigate the properties of STR/C in a series of simulated applications to the double effect evaporator and secondly to explore guidelines for the control of the pilot scale double effect evaporator at the University of Alberta. The experimental equipment and the control objectives are described in chapter three.

As discussed in section 4.4 the choice of initial and design parameters directly influences the control performance of STC. In the next section the effect of the following important parameters is demonstrated by simulation runs;

1) Model order
2) Choice of initial model parameters
3) Evaluation of RLS estimation law - particularly the effect of the covariance matrix and the forgetting factor on its performance
4) Choice of weighting functions, P and Q, in the performance index

A summary of the STR/C simulation runs is given in Table

Table 4.1 List of Simulation Runs Using STR/C

Figure No.	Run No.	Intial O(0)	Ts (sec)	Model order	cov matrix	for. factor	P wt	Q wt	Comments
4.2 4.8	ST2001	O_1	64	2	1000	1	1	0	zero initial parameters cf. SP2001
4.7 4.9	ST2002	O_1	64	2	10	1	1	0	effect of covariance on convergence
	ST2003	O_1	64	2	1	1	1	0	effect of covariance on convergence
4.13 4.14	ST2004	O_1	64	2	1000	.95	1	0	forgetting factor, oscilation
4.15	ST2005	O_1	64	2	1000	.99	1	0	forgetting factor cf. ST2004
4.6	ST2006	O_4	64	2	1	1	1	0	initial parameter from eq (3.5)
	ST2007	O_5	64	1	1	1	1	1	first order model eq (3.2)
4.4 4.11	ST2008	O_6	64	1	1	1	1	0	first order model plus delay
4.10 4.12	ST2009	O_6	64	1	10	1	1	0	effect of covariance cf.ST2008
	ST2010	O_6	64	1	1000	1	1	0	effect of covariance cf.ST2009
4.5	ST2011	O_0	64	2	1	1	1	0	second order time domain model (TDM)
4.19	ST2012	O_0	64	2	1	1	1	0	TDM, setpoint change
4.18	ST2013	O_0	64	2	1	1	$(1-.8z^{-1})$	0	P-wt, setpoint change
	ST2014	O_0	64	2	1	1	$(1+.5z^{-1})$	0	P-wt setpoint change cf.ST2013
	ST2015	O_0	64	2	1	1	$(1-.8z^{-1})$	0	P-wt cf. ST2011
	ST2016	O_0	64	2	1	1	1	$5-5z^{-1}$	integral Q-wt, oscillation
4.17	ST2017	O_0	64	2	1	1	1	PI	PI type Q-wt, smooth response
4.16	ST2018	O_0	64	2	1	1	1	PID	PID type Q-wt, smooth control
	ST2019	O_0	64	2	1	1	$(1-.5z^{-1})$	PI	P and Q wt, setpoint change
	ST2020	O_0	64	2	1	1	$(1-.5z^{-1})$	PI	P and Q wt feed and setpt.
	ST2021	O_2	64	2	1000	1	1	0	effect of covariance cf.ST2002
	ST2022	O_2	64	2	1000	1	1	PI	effect of Q-wt, cf. ST2021, SP2016
4.3	ST3001	O_1+0.0	64	3	1000	1	1	0	third order model cf. ST2001
	ST3002	O_0+0.0	64	3	1	1	1	0	third order cf. ST2011
	ST3003	O_0+0.0	64	3	1	1	$(1-.5z^{-1})$	0	P-wt, feed change cf. ST2015
	ST3004	O_0+0.0	64	3	1	1	$(1-.8z^{-1})$	0	P-wt, setpoint, cf. ST2013
	ST3005	O_0+0.0	64	3	1	1	1	PI	PI type Q-wt, cf. ST2017
	ST3006	O_0+0.0	64	3	1	1	1	PID	PID type Q-wt, cf. ST2018
	ST3007	O_0+0.0	64	3	1	1	$(1-.5z^{-1})$	PI	P and Q wt, cf. ST2019
4.1	ST3008	O_1-0.0	64	1	1000	1	1	0	first order model zero initial

Note (i):
O_0 =[.9775 -.000 .0664 .00027]
O_1 =[.00 .00 .05 .00]
O_2 =[.00 .00 1.00 .00]
O_3 =[.00 .00 .10 .00]
O_4 =[1.70 -.702 .0272 .01639]
O_5 =[0.9775 .0667]
O_6 =[0.9655 .039 .076 .0746 .037]

$PI = (1-z^{-1}) / (4.64-4.18z^{-1})$
$PID = (1-z^{-1}) / (10.88-15.48z^{-1}+5.42z^{-2})$

Note (ii): Measurement noise is added to all runs.

4.1. These simulation runs were designed to comparable to the APCS runs in chapter five and the SFC runs in chapter six. Note that these runs are not intended as a complete or independent evaluation of STR/C. The overall results based on simulated and experimental applications are summarized in the last section of this chapter.

4.5.1 Model Order

The choice of process model order for STR/C determines the controller structure and the number of parameters to be estimated and must be specified before implementation. In this simulation study three different process models were evaluated; first, second and third order models. Figure 4.1, 4.2 and 4.3 show simulated evaporator responses when the process model in STR/C was assumed to be first, second and third order respectively (The evaporator simulation is always based on the fifth order linear state space model). For each case the dead-time was assumed to be zero. First order approximation of the evaporator leads to large sustained fluctuations in the process variables. Note that the controller structure for this case corresponds to simple proportional control only and the results in Figure 4.1 suggest that the overall loop gain is too high. Use of third order model results in a more oscillatory response than the second order case. This can be explained as follows. As the model order increases, more controller parameters have to be estimated and it takes more sampling intervals to make them

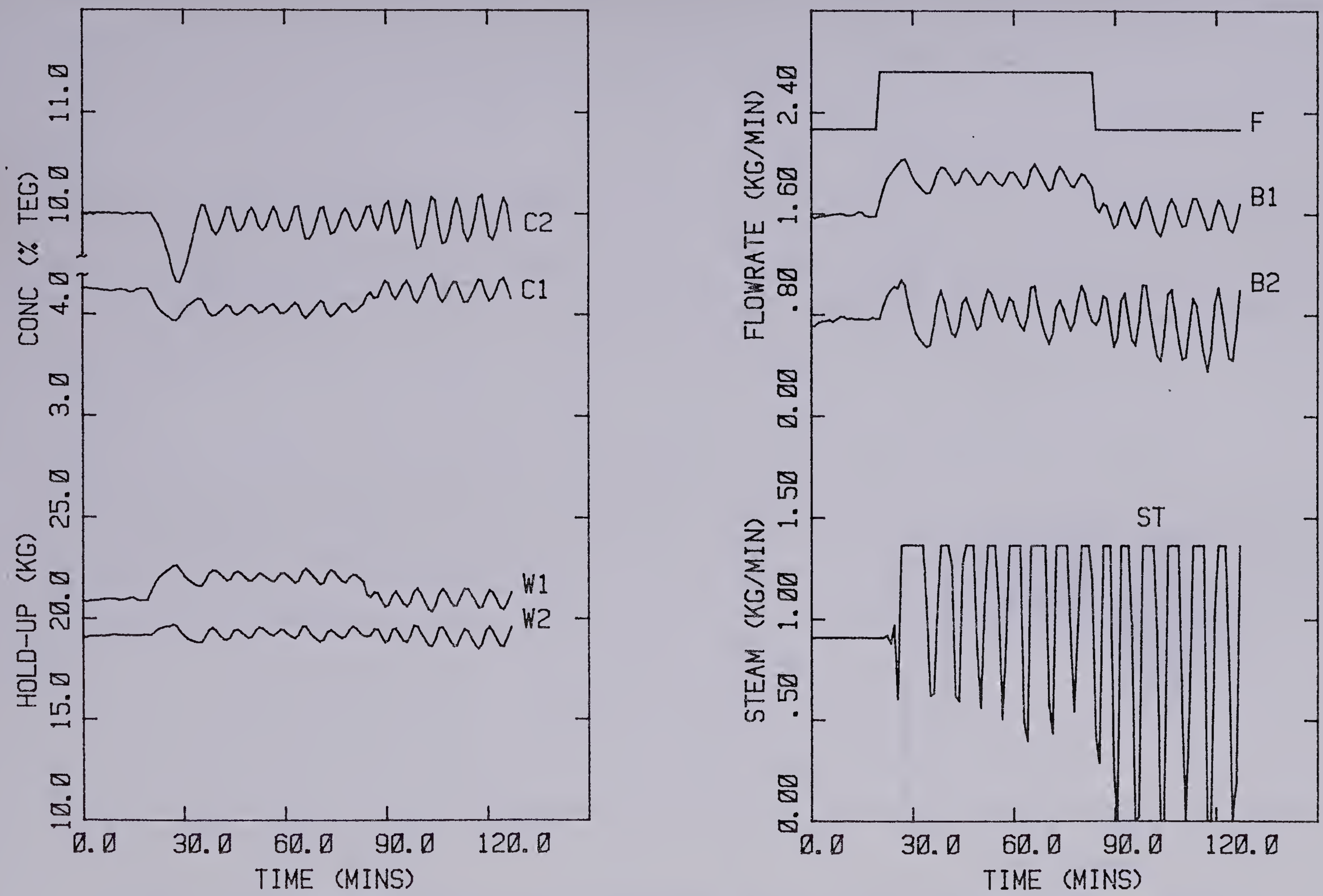


FIGURE 4.1 Simulated Evaporator Response Using STR with Zero Initial Parameters (STR/ST3008/I0/T64/M1/C1000/F1/P1/Q0/ 20%FD/ FIRST ORDER AND ZERO PARAMETERS)*

* refer to the nomenclature section

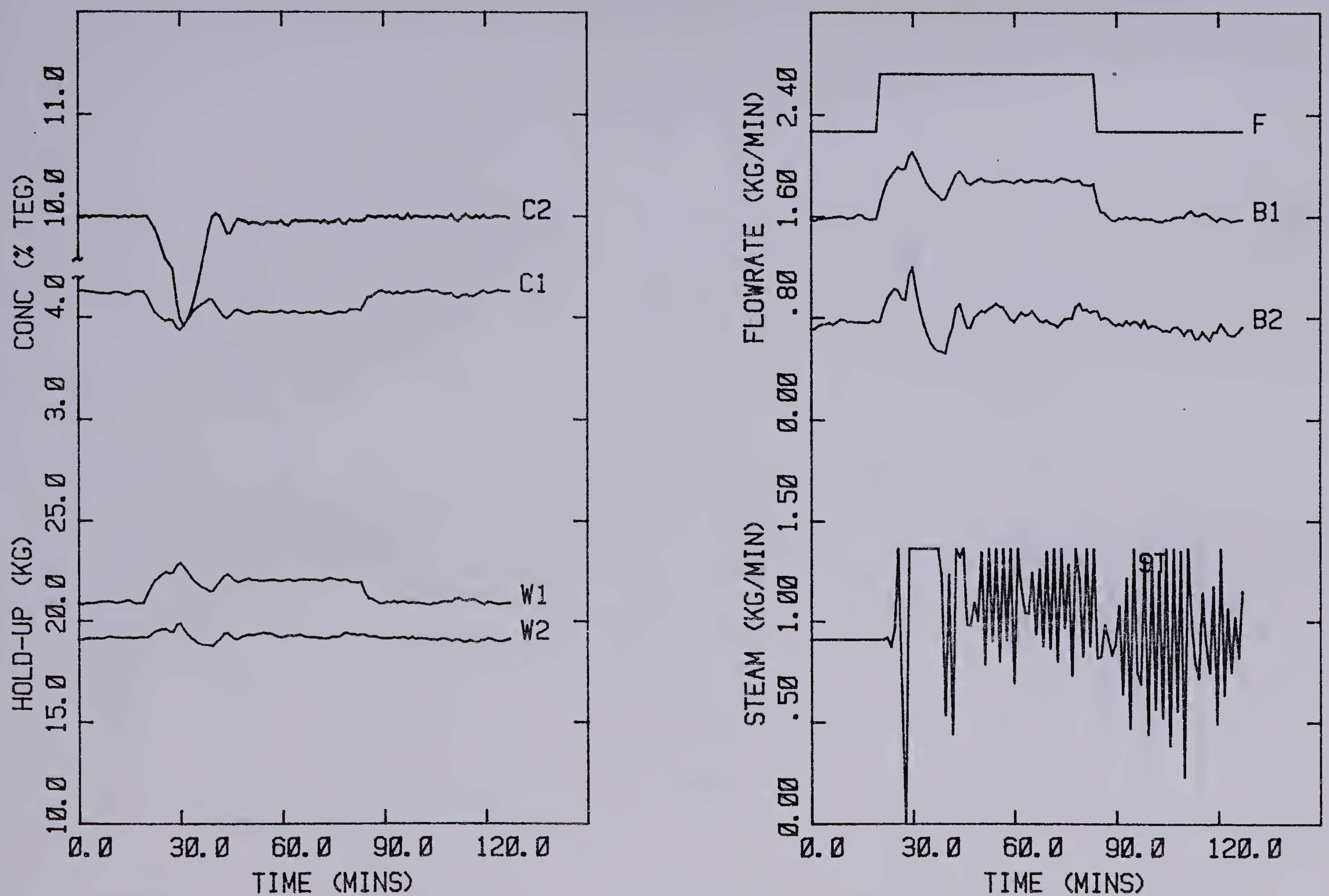


FIGURE 4.2 Simulated Evaporator Response Using STR with Second Order Model
(STR/ST2001/I0/T64/M2/C1000/F1/P1/Q0/ 20%FD/ SECOND ORDER ZERO PARAMETERS)

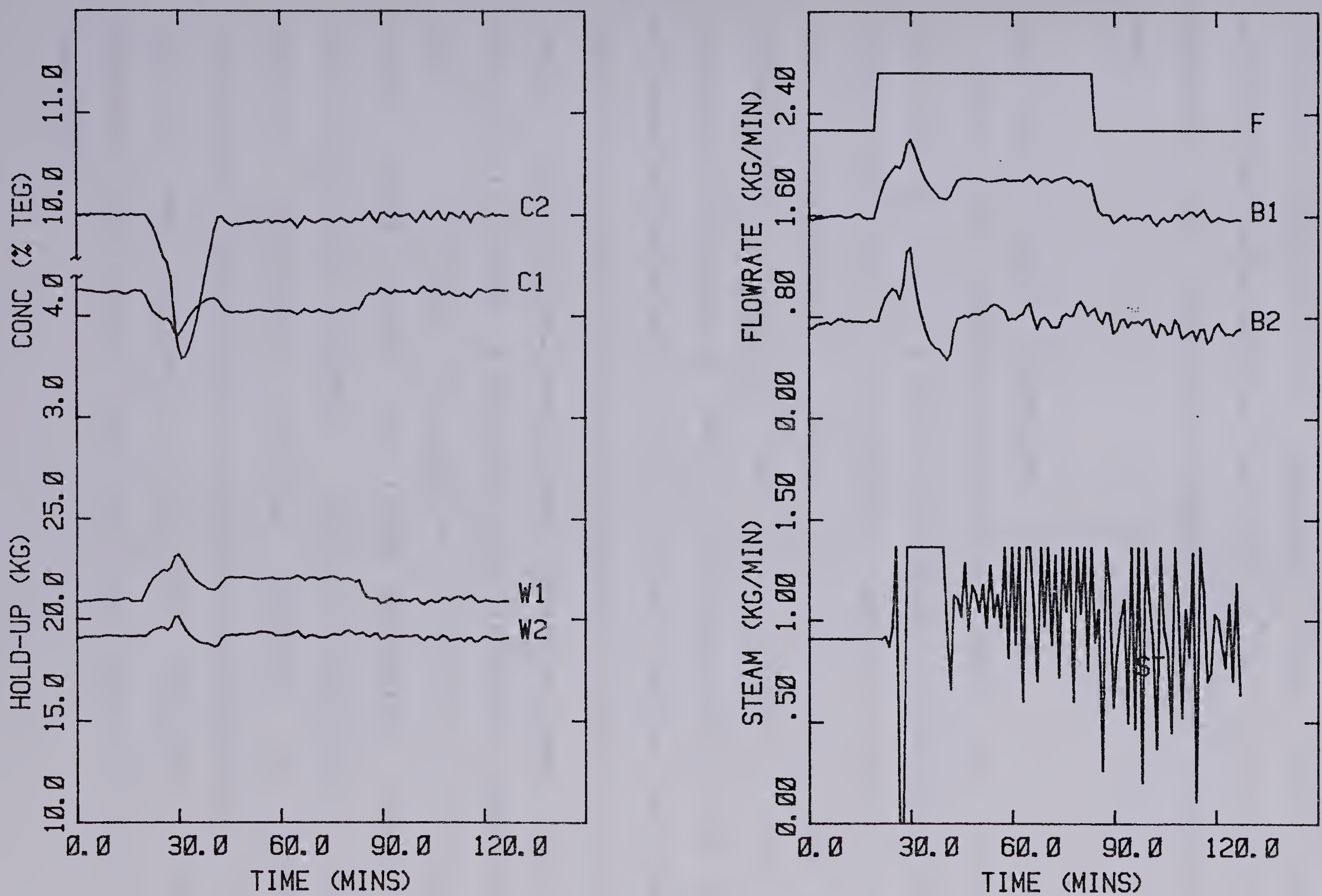


FIGURE 4.3 Simulated Evaporator Response Using STR with Third Order Model
(STR/ST3001/I0/T64/M3/C1000/F1/P1/Q0/ 20%FD/ THIRD ORDER & ZERO PARAMETERS)

converge. The convergence mechanism is more complicated which results in poorer control performance during parameter estimation period. The simulation study shows that the second order model was the best choice for controlling the evaporator in the sense of adaptability and suitability of model.

4.5.2 Initial Model Parameters

The initial model parameters of STR/C can be chosen with zero or little a priori knowledge of process dynamics. However, poor initial parameters usually result in poor control performance. Note that as the time since initialization (startup) of the adaptive controller increases the effect of the initial conditions on the system performance decreases, i.e. by its very nature an adaptive controller is more strongly influenced by recent performance than by 'old' initial conditions. Therefore some operators, particularly those starting up an industrial application, prefer to initialize the adaptive controller by running the process under manual (or fixed parameter) control and identifying the necessary 'initial parameters' for the adaptive controller on-line. However, in order to investigate a large number of factors experimentally in this study it was necessary to specify 'realistic' initial parameters and conduct relatively short runs, e.g. less than three hours.

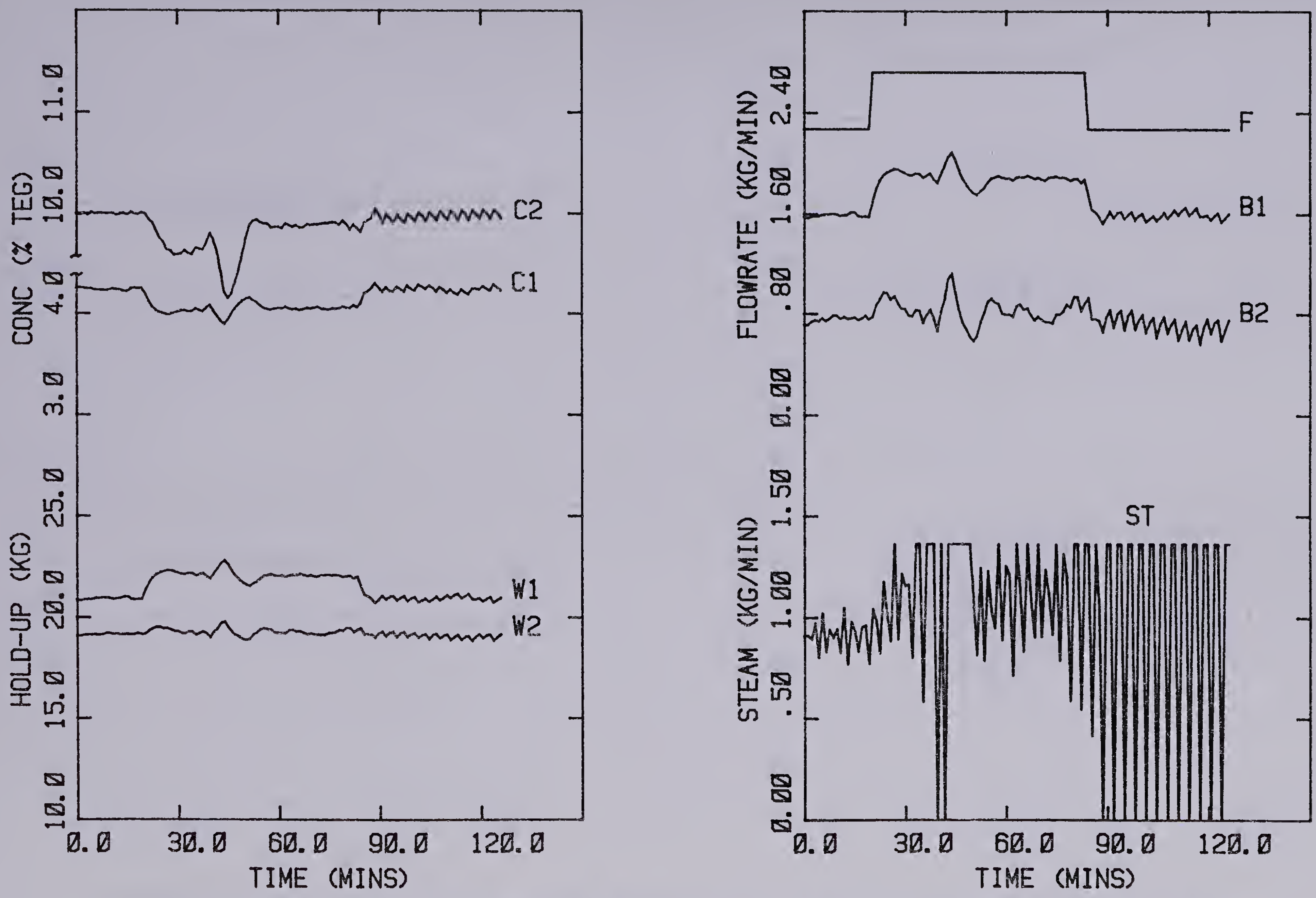


FIGURE 4.4 Simulated Evaporator Response by STR with First Order Model & Delay
(STC/ST2008/ITDM/T64/M1+d/C1/F1/P1/Q0/ 20%FD/ FIRST ORDER MODEL PLUS DELAY)

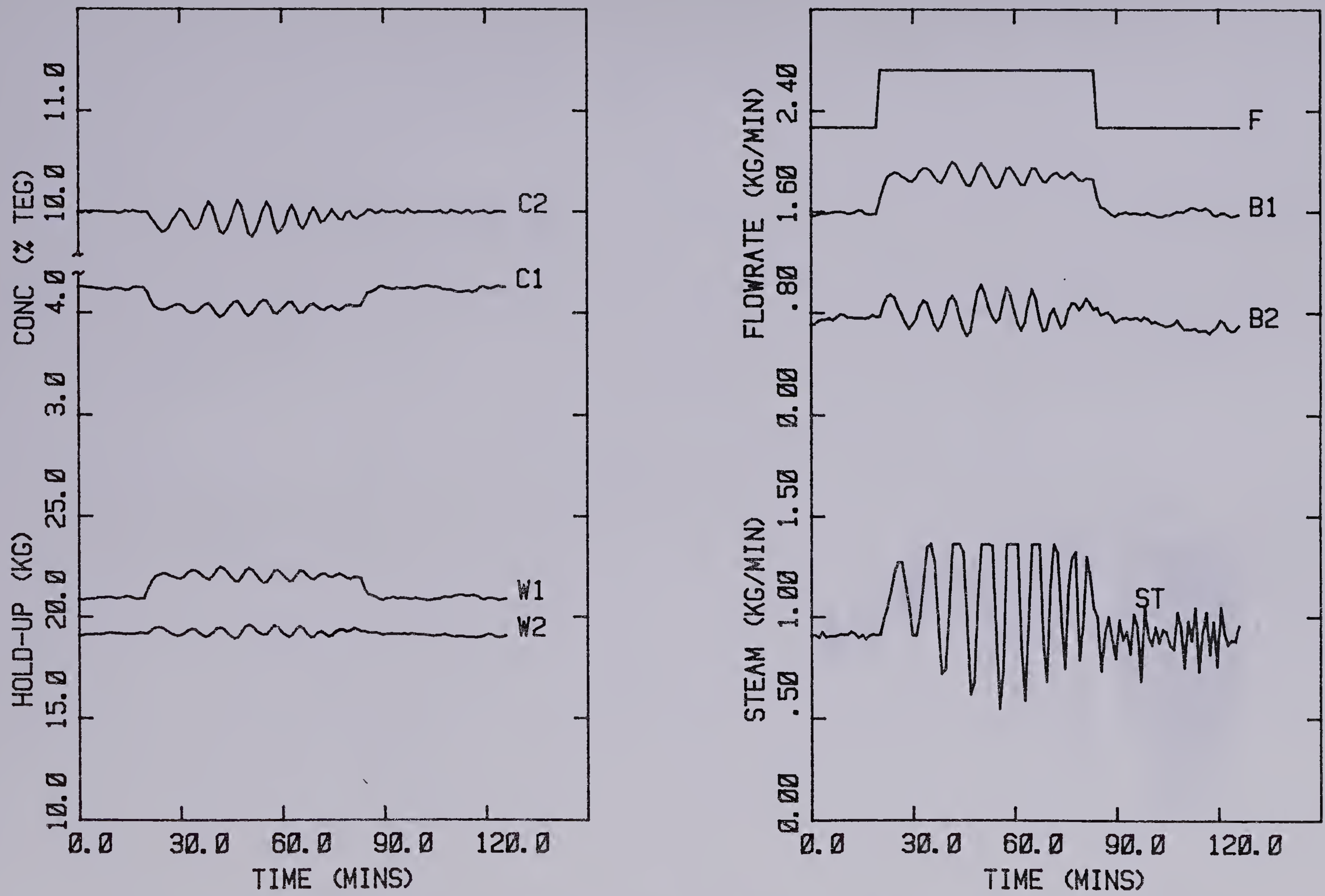


FIGURE 4.5 Simulated Evaporator Response by STR with Second Order Model
(STC/ST2011/ITDM/T64/M2/C10/F1/P1/Q0/ 20%FD/ INITIAL PARAMETER FROM TDM)

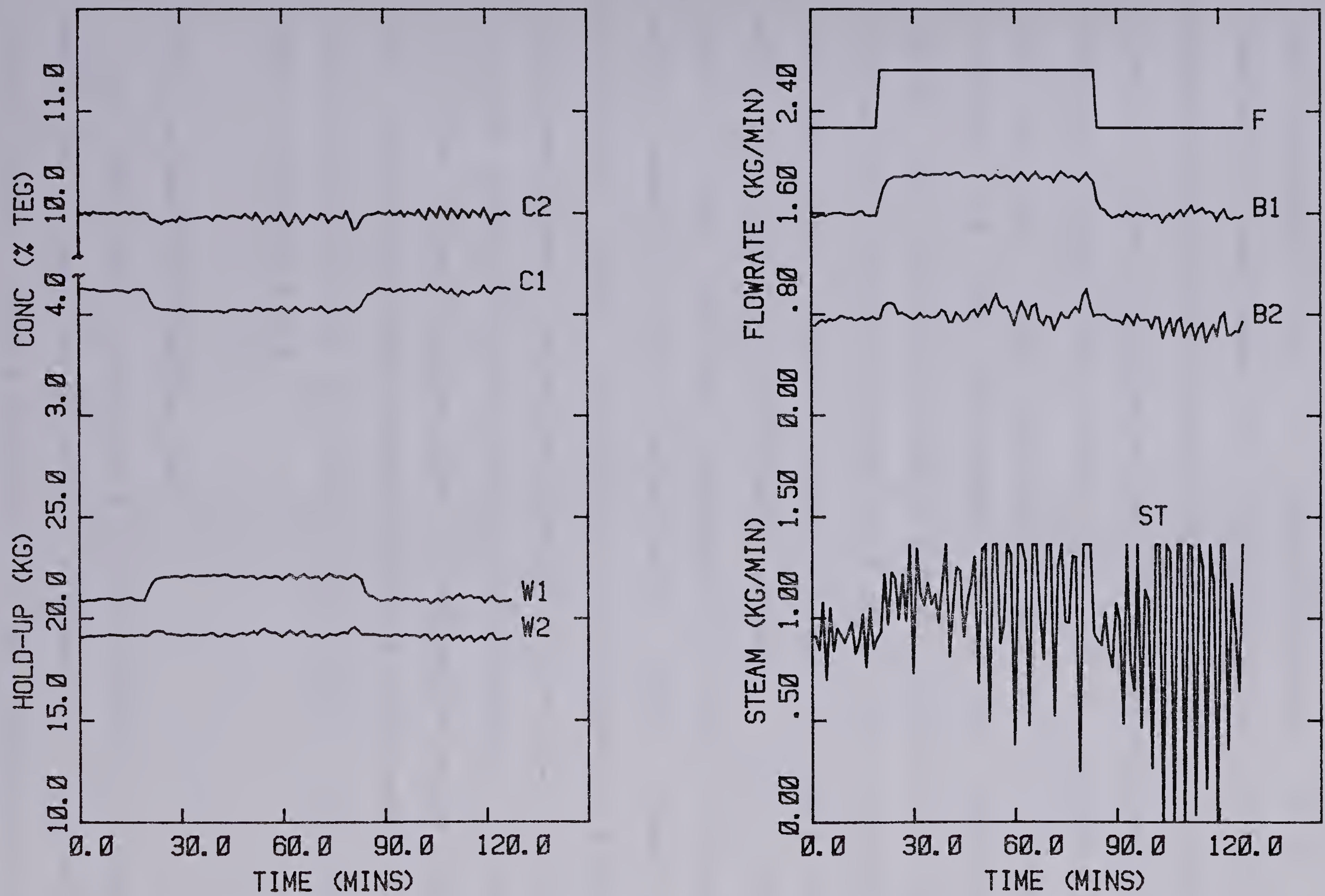


FIGURE 4.6 Simulated Evaporator Response Using STR with Time Series Model
(STR/ST2006/ITSM/T64/M2/C1/F1/P1/Q0/ 20%FD/ INITIAL PARAMETERS FROM TSM)

In the preceding section the initial parameters were all set to zero except for the first coefficient of the input polynomial, i.e. e_0. Large deviations in C2 were observed in each case. This is mainly due to the certainty equivalency principle. Several runs were made to reduce the effect of the disturbance on C2 and the variance of the manipulated signal. The choice of initial parameters as obtained from an open loop model, equation (3.2), is shown in Figure 4.4. The disturbance in C2 is significantly reduced but the fluctuations in control action still remain the same and the oscillations in C2 appear to increase in magnitude after about 90 minutes. Figure 4.5 and 4.6 use a second order model whose initial parameters were obtained from an open loop model, equation (3.4) and a time series model, equation (3.5) respectively. Both cases gave satisfactory output regulation but the control action is still unacceptable. Note that the control of C2 in Figure 4.5 was quite oscillatory while the feed was at its higher level. The control variance can be reduced by using detuned STR such as a pole assignment technique or by weighting the control action in the quadratic performance index as is shown in section 4.5.4.

4.5.3 Parameter Estimation Law

As described in section 4.4 the estimation or adaptive law is one of the most important elements of an adaptive control system. In this STR/C study a RLS scheme was chosen,

and the effect of the choice of the initial covariance matrix and forgetting factor were investigated.

(1) **Covariance matrix:** It is well known that if the initial parameters are poor then a large positive definite symmetric matrix as the initial choice of the covariance matix, e.g. 10,000I - 1000I results in fast parameter convergence. The choice of the initial covariance matrix denotes the degree of uncertainty in the choice of the initial parameters. Figure 4.2 shows the response when the initial covariance was 1000I and Figure 4.8 shows the corresponding parameter convergence. These results are compared with small initial covariance case, e.g. 10I in Figure 4.7 and 4.9 respectively. As expected, the large initial covariance matrix achieves parameter convergence in almost 20 iterations (cf. Figure 4.8) and the effect of the step down disturbance is not noticable (Figure 4.2). On the other hand when $P_t(0)=10I$ (cf. Figure 4.7) the parameters do not converge fast enough. Thus, the process I/O variables fluctuated, which gave very good dynamic process information for identification. However, the norm of the covariance matrix, and hence the gain vector K(k), are not big enough to update the parameters, which results in poor controller performance. Figure 4.4 and 4.10 show the effect of the covariance matrix on the identified initial parameters, where the covariance matrix is I and 10I respectively. The initial parameters are obtained from the second order open

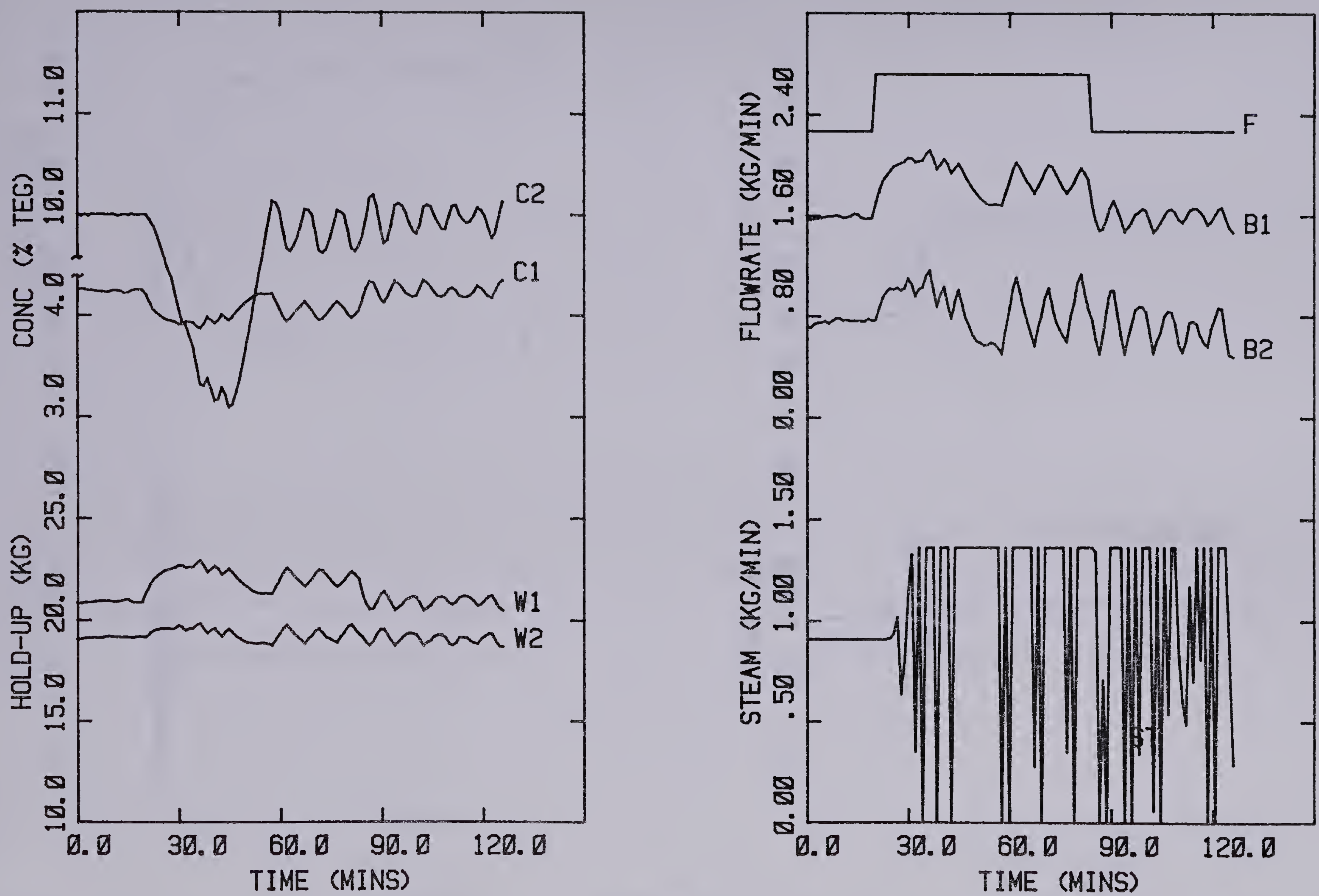


FIGURE 4.7 Simulated Evaporator Response by STR with COV 10I & Zero Parameters
(STC/ST2002/I0/T64/M2/C10/F1/P1/Q0/ 20%FD/ COVARIANCE AND INITIAL PARAMETERS)

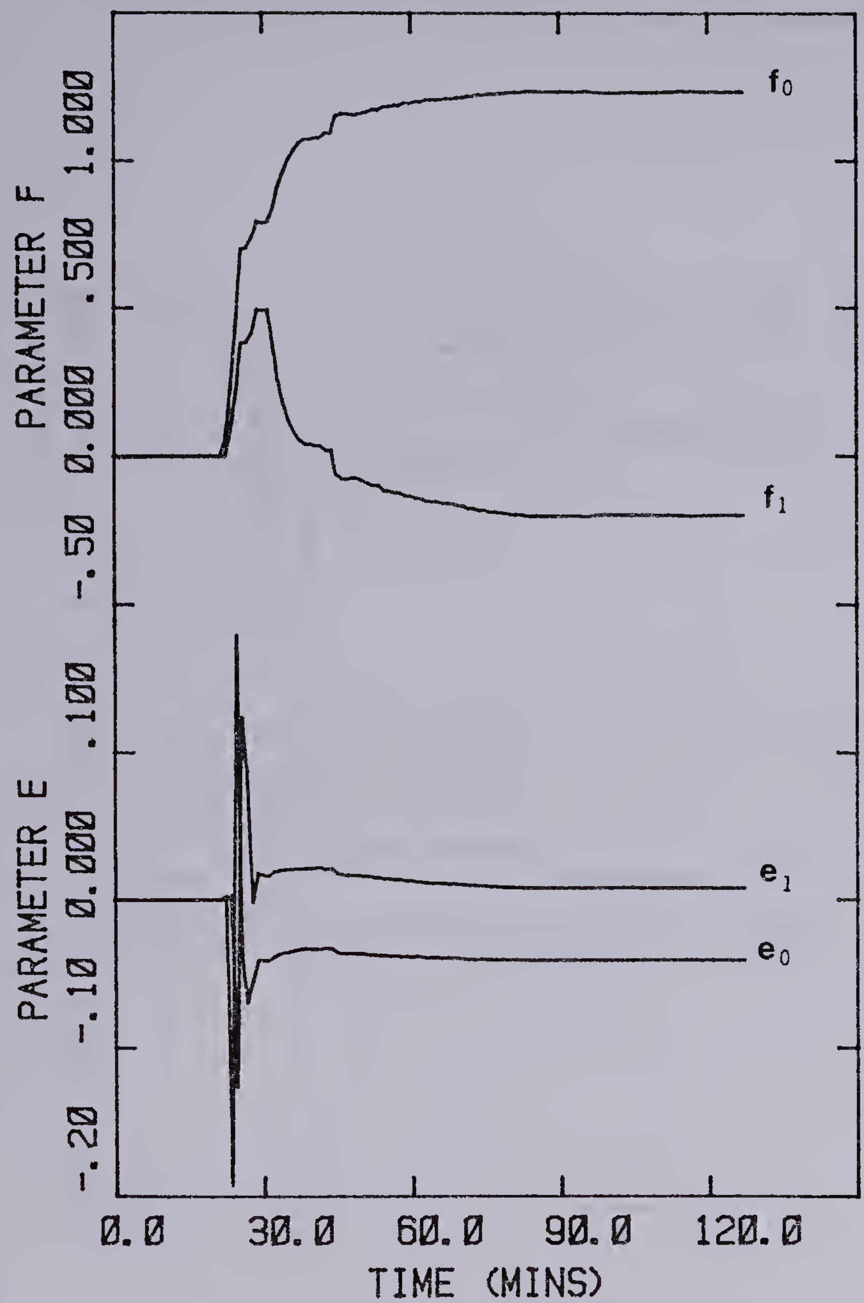


FIGURE 4.8 Parameter Trajectory for cov=1000I and Zero Initial Parameters (b_o=.05)

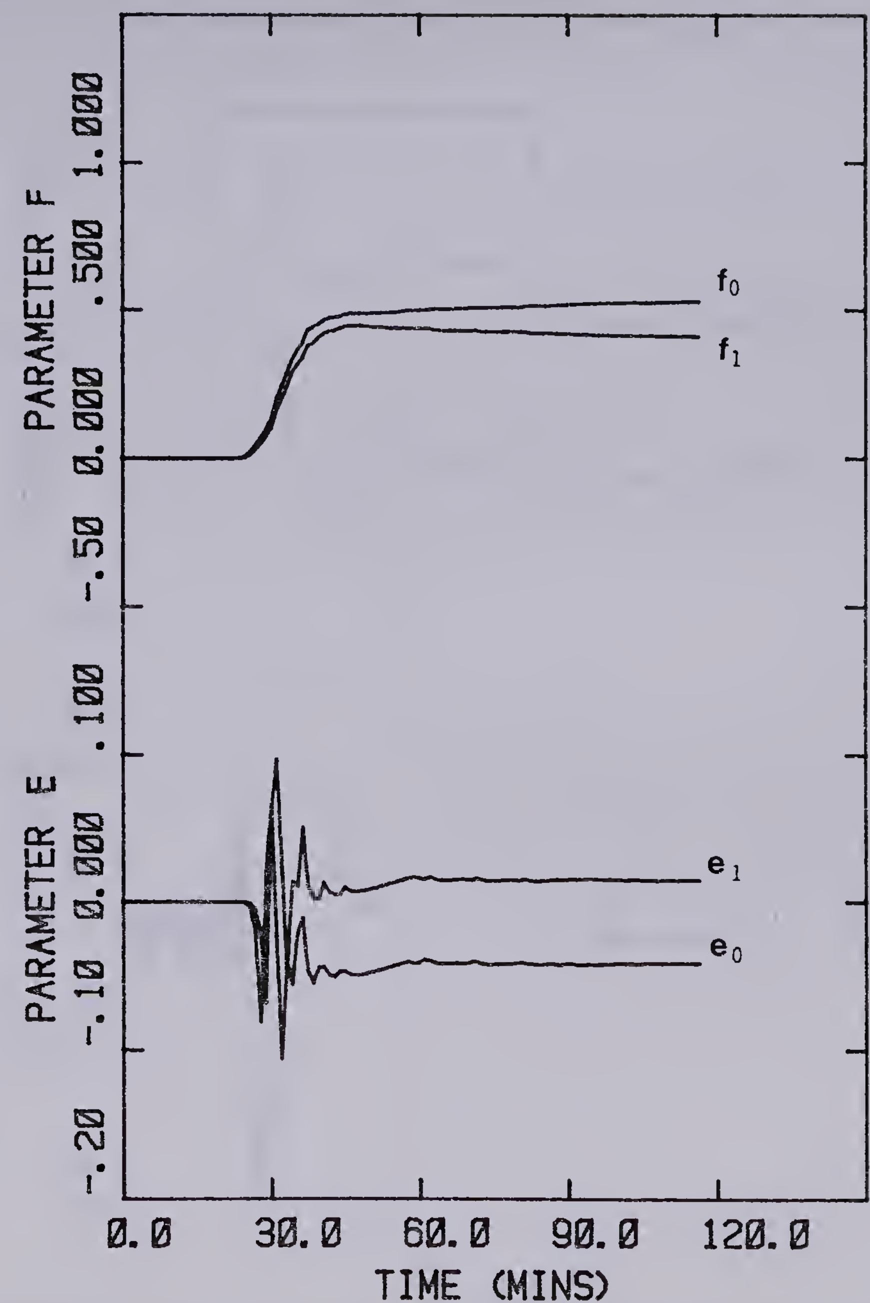


FIGURE 4.9 Parameter Trajectory for cov=10I and Zero Initial Parameters (b_o=.05)

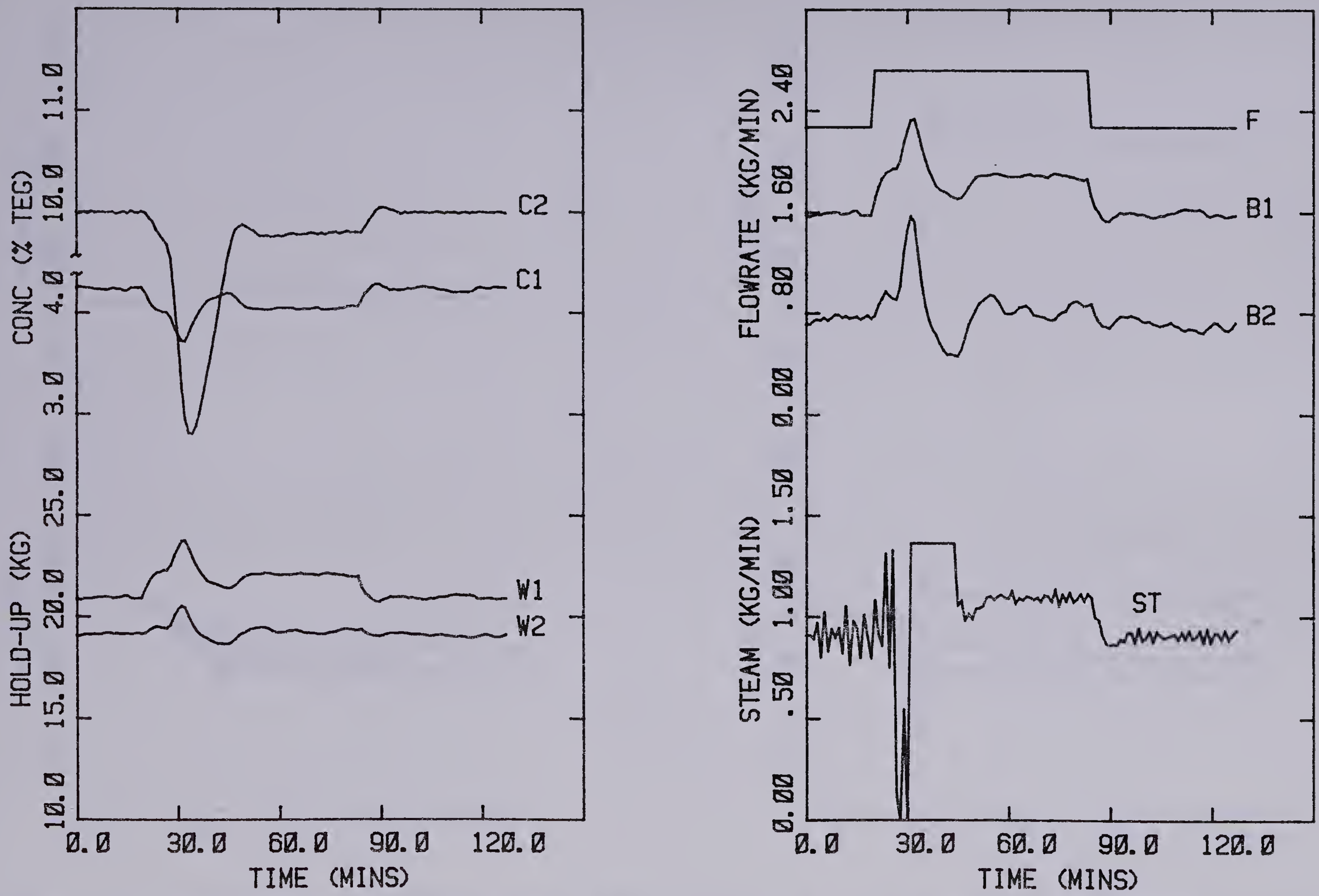


FIGURE 4.10 Simulated Evaporator Response by STR with First Order Model & Delay
(STC/ST2009/ITDM/T64/M1+d/C10/F1/P1/Q0/ 20%FD/ COV=10I cf. FIGURE 4.4)

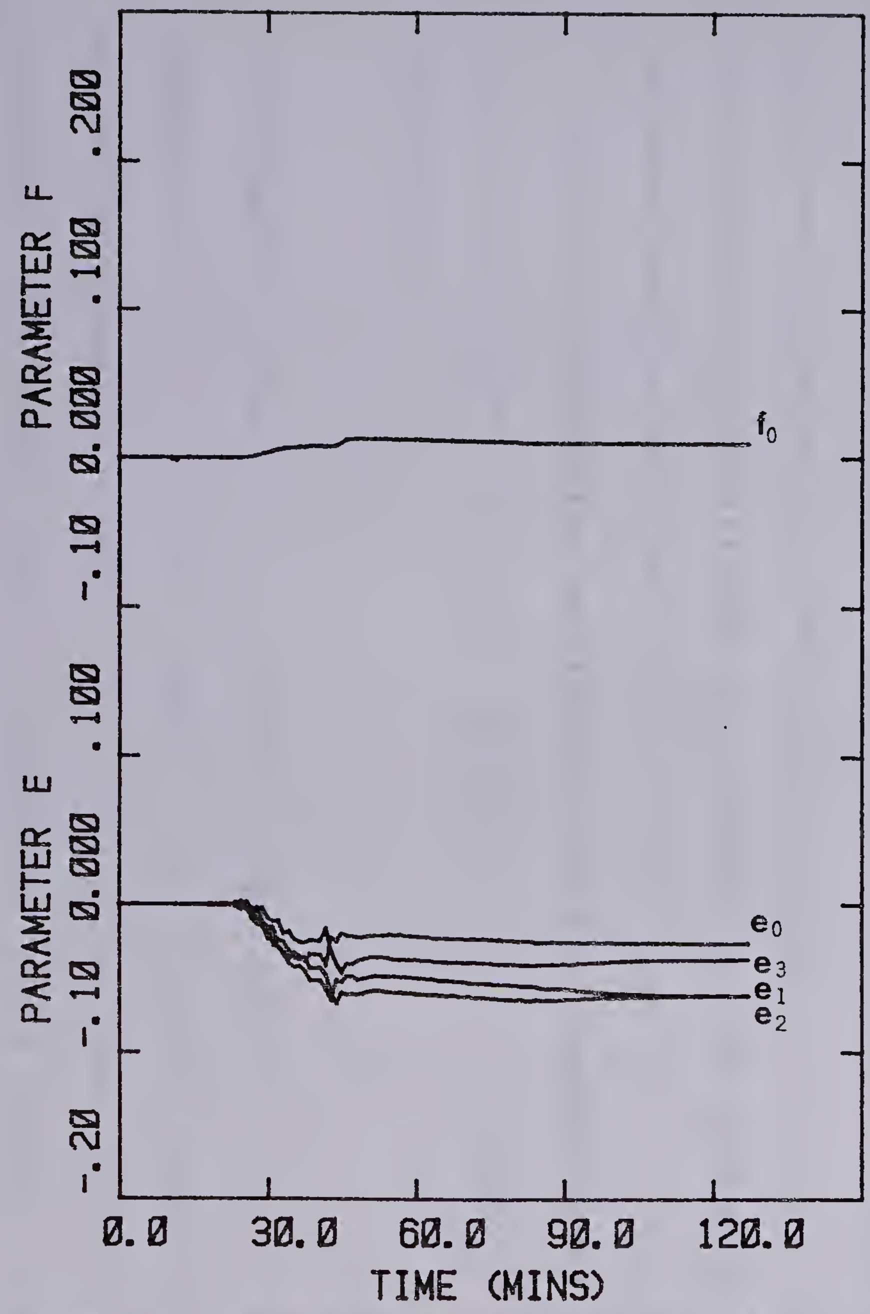


FIGURE 4.11 Parameter Trajectory for cov=I and Identified Parameters

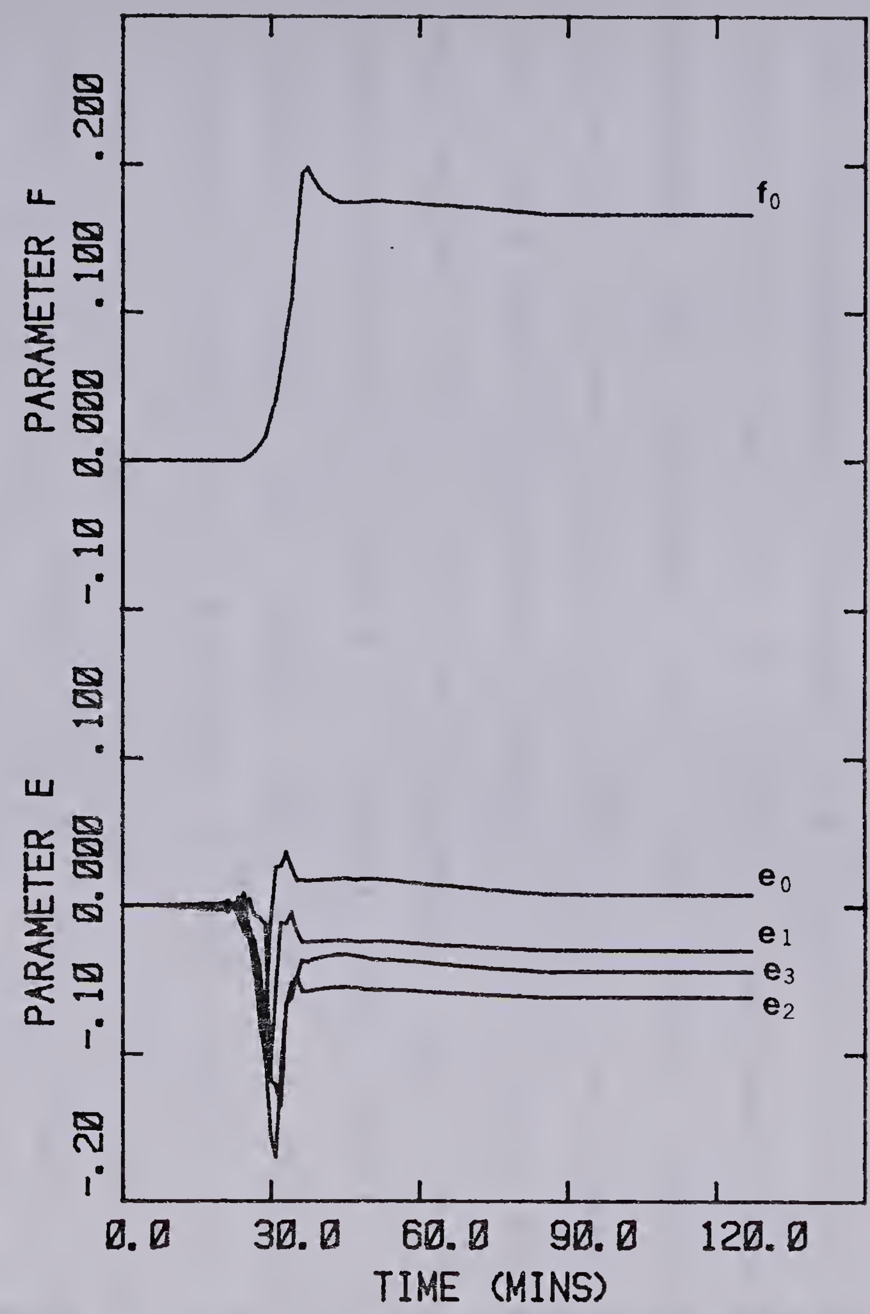


FIGURE 4.12 Parameter Trajectory for cov=10I and Identified Parameters

loop plant model, equation (3.3). In Figure 4.4 there is a slight offset due to the step disturbance in feed flowrate. Thus, the parameters did adapt to eliminate this offset (cf. Figure 4.11), i.e. f_0 is slightly increased and e_0 is decreased. The decrease in e_0 was very significant. Initially it was 0.0353 but after 90 minutes converged to 0.01. This small value of e_0 results in high gains and hence oscillations during the corresponding duration of the response. The run in Figure 4.10 shows the effect of the estimator windup. The large variations in process I/O variables in Figure 4.10 are due to the effect of large changes in the parameter estimates at approximately t=30 minutes as shown in Figure 4.12. The estimator windup results from large diagonal elements of the covariance matrix at that point. The highly inflated parameter estimates are no longer useful for predicting the evaporator output and consequently the process output has an offset. The estimator does attempt to reidentify the parameters but the covariance matrix has shrunk after the first disturbnace so that the parameter estimates can not change at all (cf. Figure 4.12) even in the presence of a negative step disturbance.

(2) **Forgetting Factor:** As has been discussed in the previous examples the covariance matrix of the RLS estimator can become very small in terms of its norm even before the parameter estimates have converged. This can be noted in

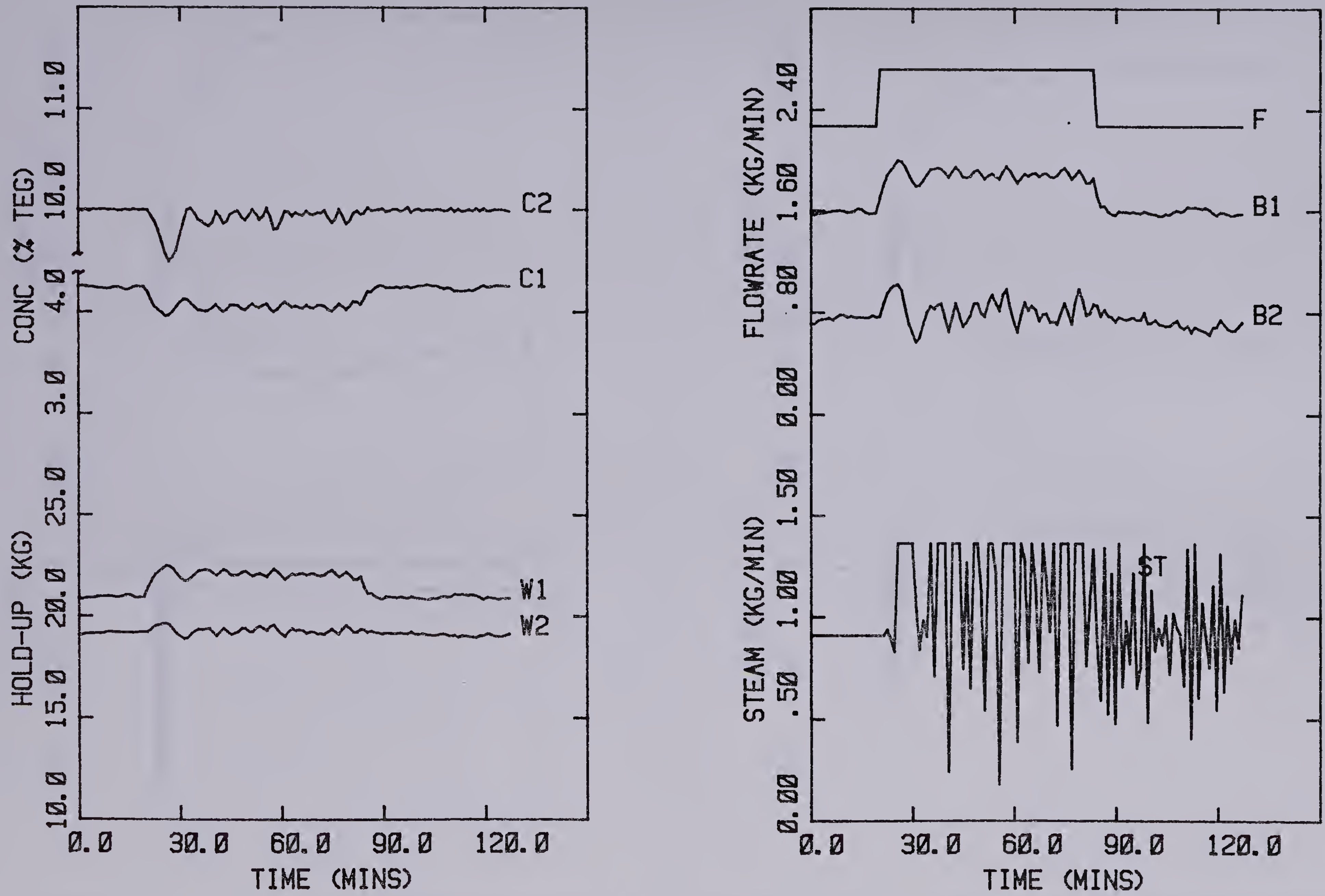


FIGURE 4.13 Simulated Evaporator Response by STR with Forgetting Factor 0.95
(STC/ST2004/I0/T64/M2/C1000/F0.95/P1/Q0 20%FD/ FORGETTING FACTOR cf. 4.15)

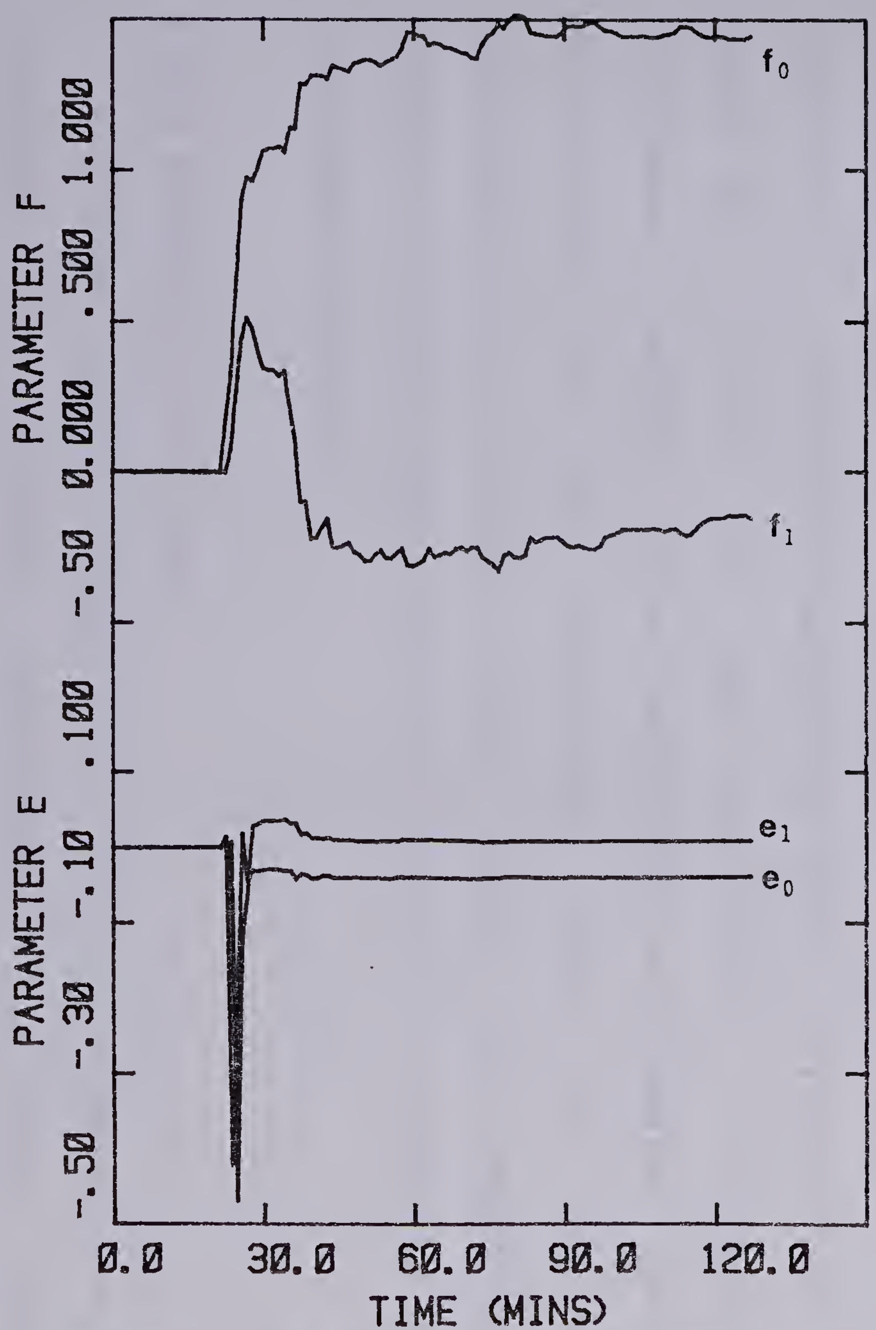


FIGURE 4.14 Parameter Variation with Forgetting Factor .95

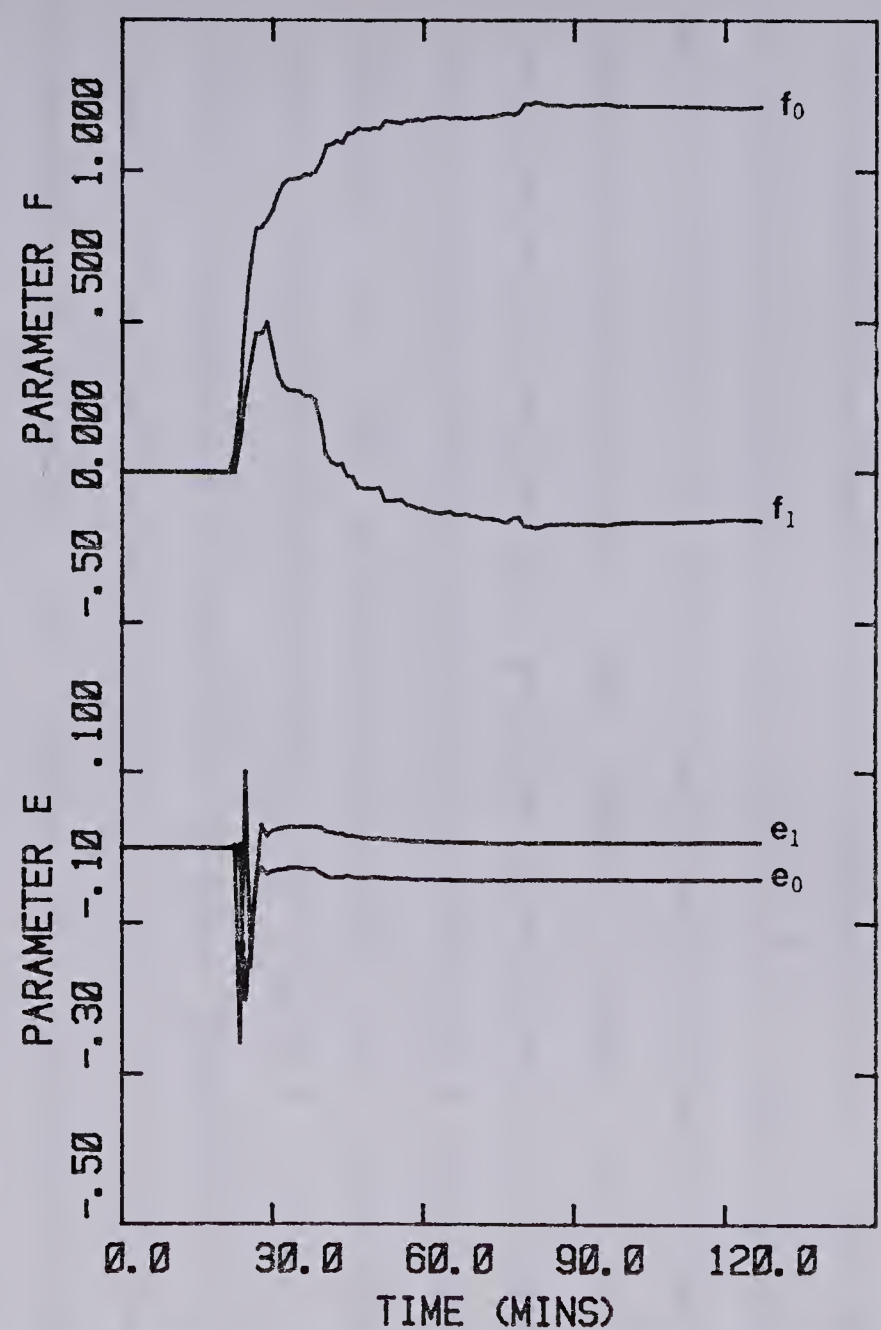


FIGURE 4.15 Parameter Variation with Forgetting Factor .99

Figure 4.2 where there is small offset in the output in the presence of a positive step disturbance in feed flowrate. The parameter estimator tries to compensate for this offset by adapting the parameters but the norm of the covariance matrix and hence the adaptive gain have become too small to have any effect during the initial phase of the feed disturbance. To solve this problem an exponential forgetting factor term of .95 that continuously discounts old data was introduced in the parameter estimation law (cf. equations 4.26 to 4.28). The result is shown in Figure 4.13 where the offset is eliminated at the cost of more fluctuations in the process I/O variables and large variations of the parameter estimates (cf. Figure 4.14). Figure 4.15 shows the parameter deviation when the forgetting factor is set to .99. As the forgetting factor is decreased the process variables become more oscillatory. For the evaporator application it was found that a choice of forgetting factor slightly less than unity, say .99 to .995 was sufficient to prevent the covariance matrix from shrinking or loosing positive definiteness. If the forgetting factor is .99 then approximately 160 data points should be remembered to discount the first data point to 20% of its original value (cf. equation (4.33)).

4.5.4 Weighting Functions

In the present simulation study it has been shown that although the desired performance of the product

concentration, C2, can be achieved using the STR in the presence of ±20% step changes in feed flowrate, the variance in the manipulated variable is undesirably large and in fact the control action is almost of a bang-bang type. This excessive control action causes closed loop stability problems when applied to the actual evaporator. In this section the use of weighting functions, $P(z^{-1})$ and $Q(z^{-1})$, in the quadratic performance index is investigated to resolve the problem of vigorous control action without imparing the output performance.

Several $Q(z^{-1})$ functions were tried including constant weighting and pure integral action. Pure integral weighting still gave oscillatory response but the control action was smoothed significantly. The reason for the oscillations is explained in the experimental section to follow. Finally $Q(z^{-1})$ in form of a PID weighting term was considered. This achieved the desired control objective satisfactorily. Figure 4.16 and 4.17 show the effect of the $Q(z^{-1})$ weighting. As can be seen from the figures the control signal is dramatically smoothed with the reasonable C2 performance. These results are directly comparable with those obtained without Q-weighting in the preceding section, e.g. Figure 4.5, 4.6, etc.

The $P(z^{-1})$ and $R(z^{-1})$ weighting is for servo control and the effect ofthe $P(z^{-1})=(1-.8z^{-1})$ is shown in Figure 4.18, which can be compared with the base case, i.e. $P(z^{-1})=1$ in Figure 4.19. The variation of steam in the base

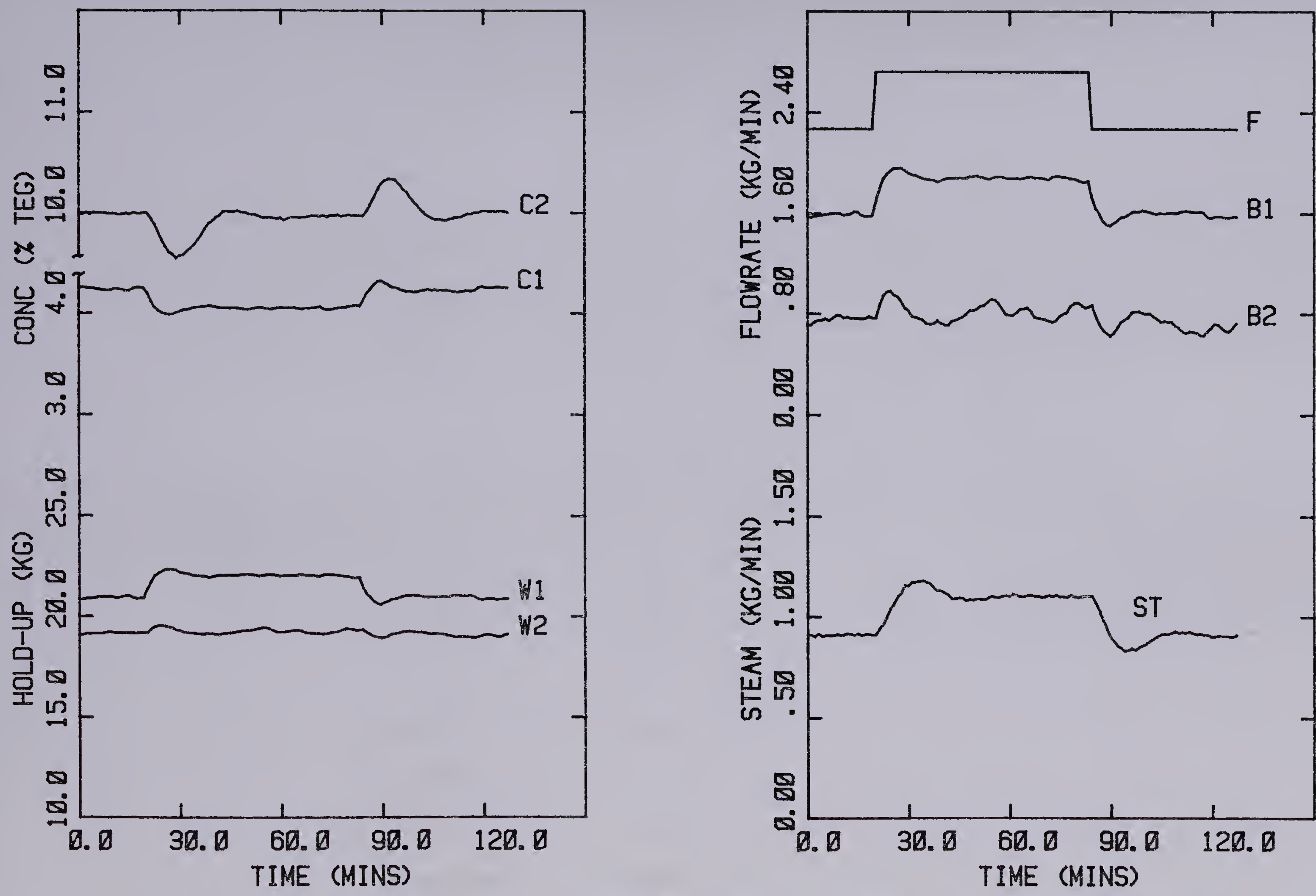


FIGURE 4.16 Simulated Evaporator Response by STC with PID Type Q-Weighting
(STC/ST2018/ITDM/T64/M2/C1/F1/P1/Q PID/ 20%FD/ STC WITH Q-WT cf. FIG 4.5)

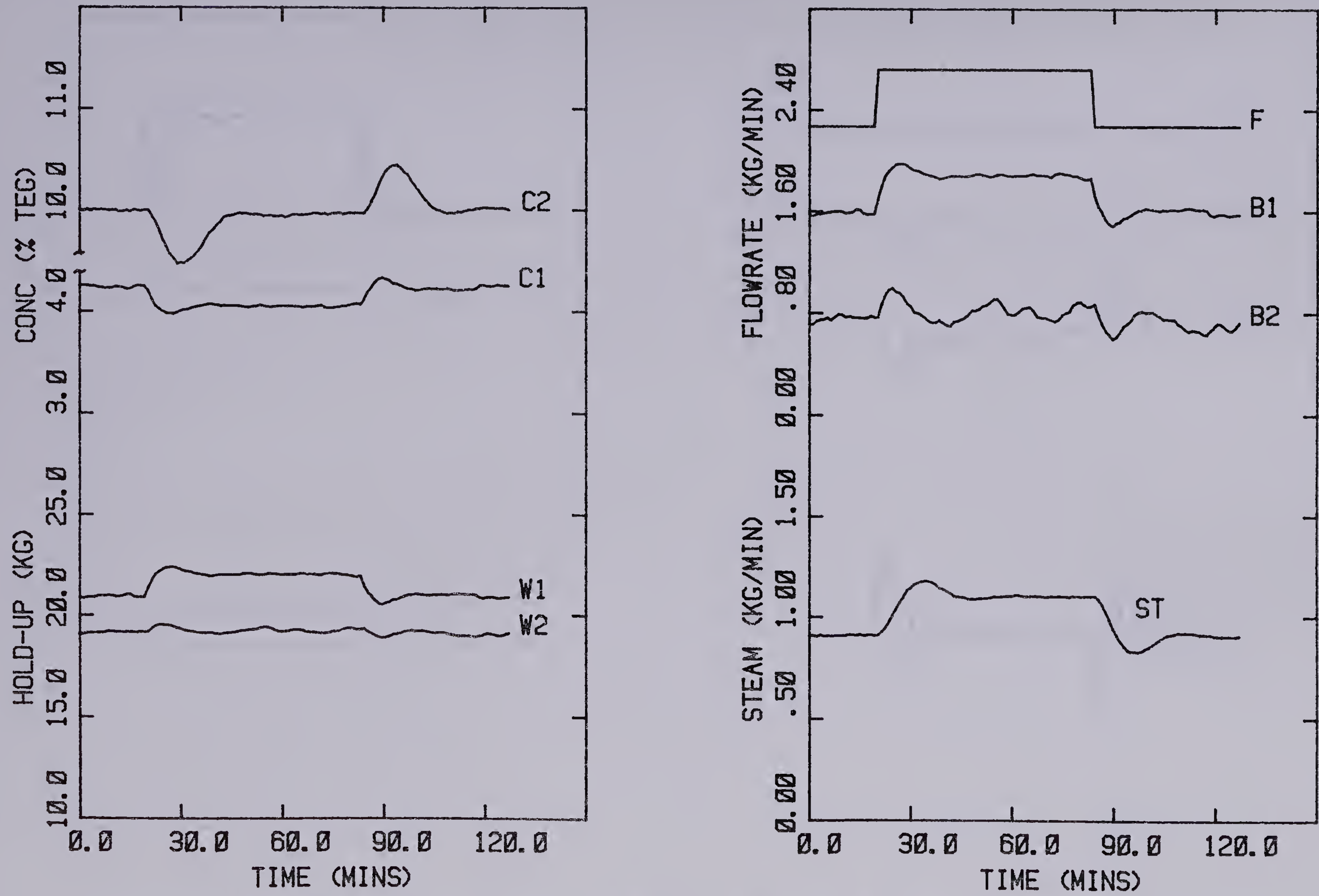


FIGURE 4.17 Simulated Evaporator Response by STC with PI Type Q-Weighting
(STC/ST2017/ITDM/T64/M2/C1/F1/P1/Q PI/ 20%FD/ STC WITH Q-WEIGHTING)

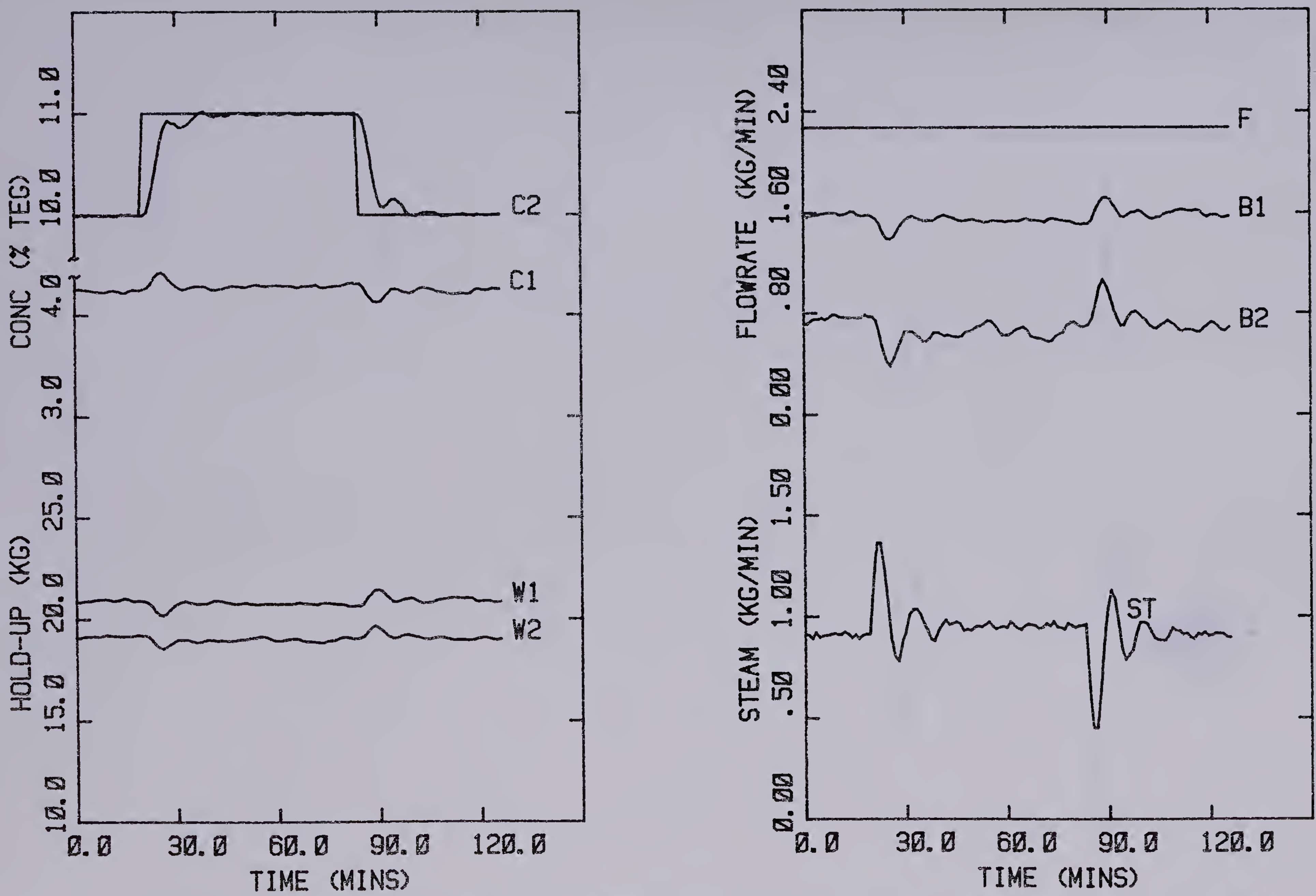


FIGURE 4.18 Simulated Evaporator Response by STC to Setpoint Changes with P-wt
(STC/ST2013/ITDM/T64/M2/C1/F1/P(1-.8z)/Q0/ 10%FD/ SERVO CONTROL WITH P-WT)

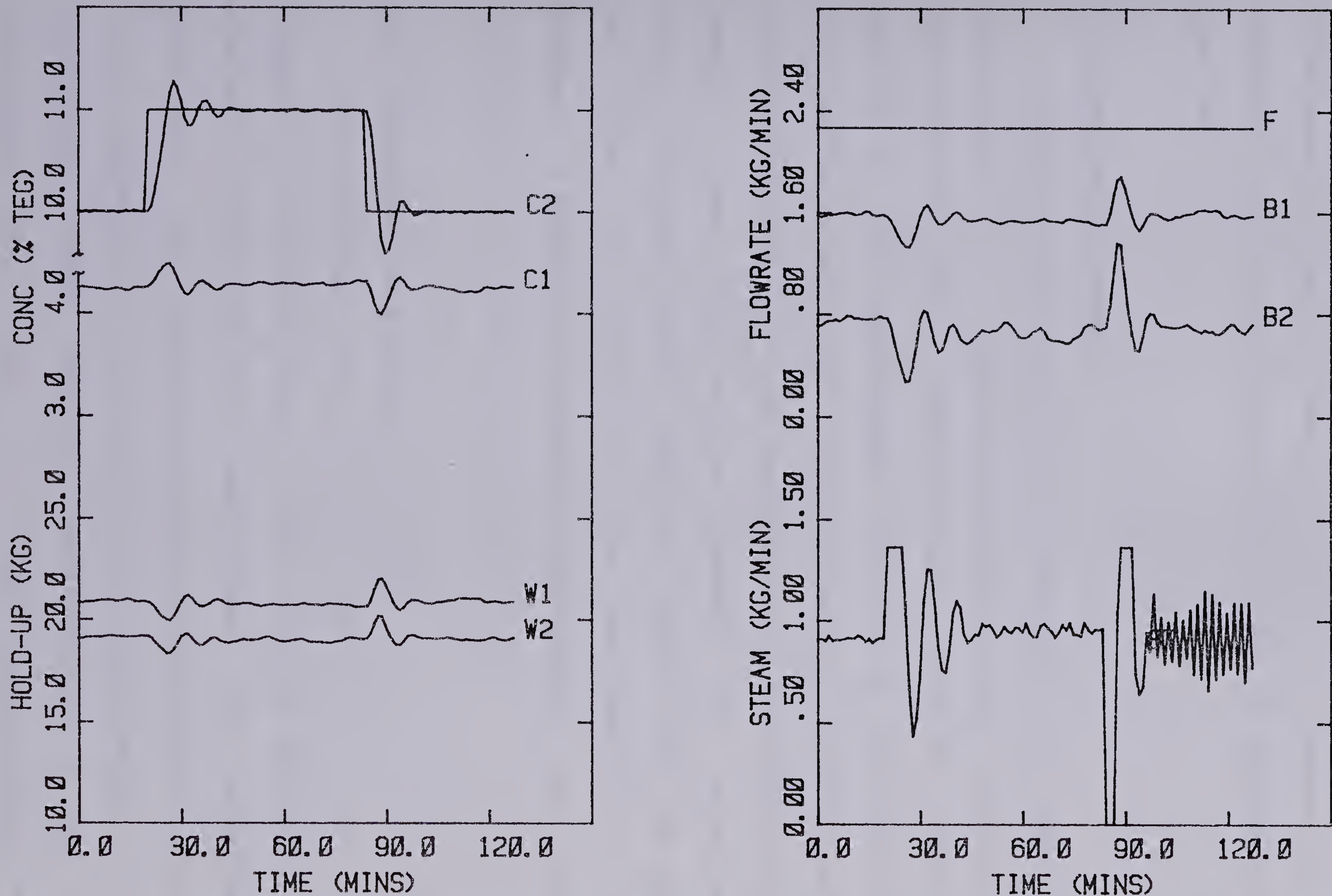


FIGURE 4.19 Simulated Evaporator Response Using STR to Setpoint Changes (STC/ST2012/ITDM/T64/M2/C1/F1/P1/Q0/ 10%SP/ SERVO CONTROL cf. FIG 4.18)

case is more severe than with $P(z^{-1})$ weighting. As a result of these simulation runs, it was concluded that Q-weighting is necessary to get the desired control performance on the evaporator and PI/PID Q-weighting is one of possible design functions.

4.6 Experimental Study

In the previous section the properties of STR/C were investigated in a series of simulation runs. Based on the experience obtained in these simulation runs the STR/C was applied to the experimental control of the pilot scale double effect evaporator. The experimental procedure as well as the computer-controlled evaporator system is described in chapter three. This section presents the experimental results obtained using STR/C in detail and the general conclusions are included in the last section of this chapter. A summary of the STR/C experimental runs conducted is presented in tabular form in Table 4.2.

4.6.1 Experimental Evaluation of STR

The experimental evaluation of STR on the pilot plant evaporator verified some of the results observed in the simulation study but in most cases the experimental performance was significantly worse. Application of the STR to the evaporator in the presence of a step change in feed disturbance caused excessive manipulation of the steam flowrate and as a result the closed loop system became

Table 4.2 List of Experimental Runs Using STR/C

Figure No.	Run No.	Intial $0(0)$	Ts (sec)	Model order	cov matrix	for. factor	P wt	Q wt	Comments
4.21	RT2001	0_4	64	2	0.01	1	1	0	STR b_0=.1 oscillation
4.22	RT2002	0_4	64	2	0.1	1	1	0	STR b_0=.2 stable offset
4.23	RT2003	0_4	64	2	0.1	1	1	$.8-.8z^{-1}$	integral Q-wt with TSM
4.24	RT2004	0_4	64	2	0.1	1	1	$.2-.2z^{-1}$	integral Q-wt unstable
7.2	RT2005	0_4	180	2	0.1	1	1	$.8-.8z^{-1}$	longer sampling time cf. RT2003
	RT2006	0_4	64	2	0.1	1	1	$.8-.8z^{-1}$	integral Q-wt b_0=.1
	RT2007	0_4	64	1	0.1	1	$(.2+.1z^{-1})$	0	P-wt b_0=.0272
	RT2008	0_4	64	1	0.1	1	$(1+.5z^{-1})$	0	P-wt b_0=.2 offset
	RT2009	0_4	64	1	0.1	1	$(1-.5z^{-1})$	0	P-wt b_0=.2 offset
	RT2010	0_4	64	1	0.1	1	$(1-.5z^{-1})$	0	P-wt b_0=.1 unstable
	RT2011	0_4	64	3	0.1	1	1	0	third order model test
	RT2012	0_4	64	3	0.1	1	1	PID	PID-wt unstable oscillation
	RT2013	0_4	64	2	0.1	1	1	PI_1	tuning PI type Q-wt Ti=9.61
	RT2014	0_4	64	2	0.1	1	1	PI_2	tuning PI type Q-wt Ti=5.00
4.30	RT2015	0_4	64	2	0.1	1	1	PI	tuning Q-wt Kp=3.0 Ti=15
	RT2016	0_4	64	2	0.1	1	1	PI	PI type Q-wt, smooth cf.RT2015
	RT2017	0_4	64	3	0.05	1	1	PI	small covariance cf. RT2015
	RT2018	0_4	64	3	0.1	1	1	0	third order model cf. RT2002
	RT2019	0_4	64	2	0.1	1	1	0	limit on steam unstable
	RT2020	0_4	64	2	0.1	1	1	0	limit on steam and setpt.
	RT2021	0_6	64	2	0.1	1	1	0	1st order model plus delay
4.20	RT2022	0_0	64	2	0.1	1	1	0	second order model eq (3.4)
7.1	RT2023	0_0	180	2	0.1	1	1	0	longer sampling time cf. RT2023
4.29	RT2024	0_0	64	2	0.1	1	1	PI	second order PI type Q-wt
4.28	RT2025	0_6	64	2	0.1	1	1	PI	1st order model plus PI Qwt
4.27	RT2026	0_6	64	2	1	1	1	PI	covariance cf. RT2025
4.26	RT2027	0_0+0.0	64	3	1	1	1	PI	third order model cf. RT2024
4.32	RT2028	0_0	64	2	1	1	1	PI	setpt change PI type Q-wt
4.25	RT2029	0_0	64	2	1	1	1	PI	TDM, effect of covariance
4.31	RT2030	0_0	64	2	1	.98	1	PI	forgetting factor cf. RT2029

Note: 0_0 =[.9775 -.000 .0664 .00027]
0_4 =[1.70 -.702 .0272 .01639]
0_6 =[0.9655 .039 .076 .0764 .037]

$PI = (1-z^{-1}) / (3.10-2.90z^{-1})$
$PID = (1-z^{-1}) / (10.88-15.48z^{-1}+5.42z^{-2})$
$PI_1 = (1-z^{-1}) / (4.46-4.18z^{-1})$
$PI_2 = (1-z^{-1}) / (4.88-3.94z^{-1})$

oscillatory and then unstable. One such result is illustrated in Figure 4.20. In an attempt to resolve this situation various experimental runs were conducted: control of the evaporator with longer sampling time (128sec, 180sec); with different model order; with identified initial parameters; and with several different covariance matrices. The choice of process model order was varied from first to third order and the initial model parameters, which are the most significant variables in determining the performance of the STR adaptive controller, were based on the following models.

1) Time domain models (equations (3.2), (3.3) and (3.4))

2) Time series model (equation (3.5))

3) Fifth order state space model (equation (3.6))

4) Model obtained by background identification with RLS

All these approaches resulted in unsatisfactory response similar to the one shown in Figure 4.20. Chang observed similar evaporator responses and was unable to demonsrate satisfactory control of the evaporator using the STR algorithm. One reason for the unstable oscillation is that the leading coefficient, e_0, of polynomial $E(z^{-1})$ in equation (4.18) (Note if there is no timedelay $e_0=b_0$) is small signifying a high gain or a rather sensitive system. Secondly the highly interacting nature of the evaporator output with respect to other input and output intermediary variables is another cause of control difficulties. This may be explained physically as follows: in Figure 4.20 the

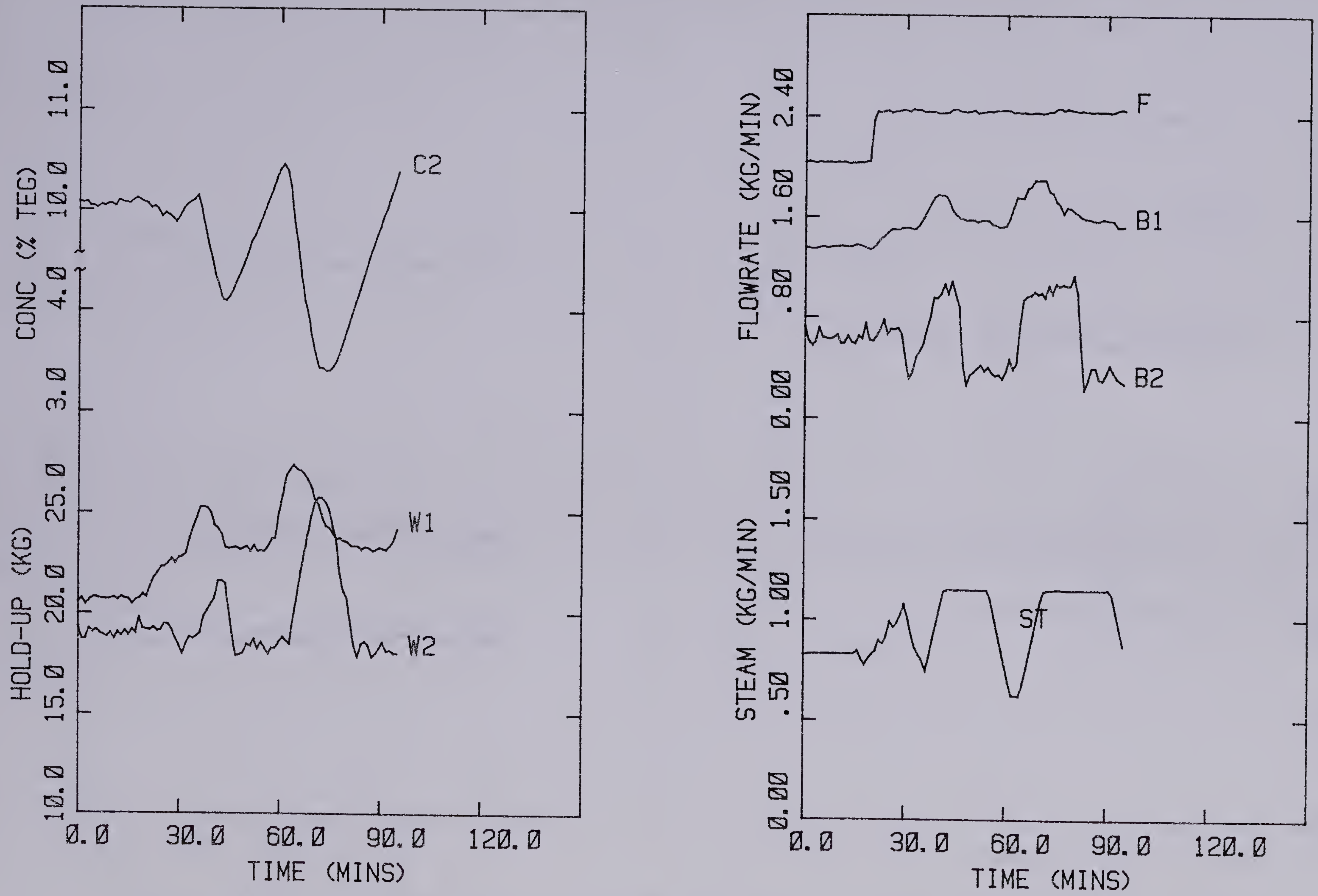


FIGURE 4.20 Evaporator Response using STR with Second Order Curve-Fitted Model
(STC/RT2022/ITDM/T64/M2/C.1/F1/P1/Q0/ 20%FD/ 2ND ORDER IDENTIFIED MODEL)

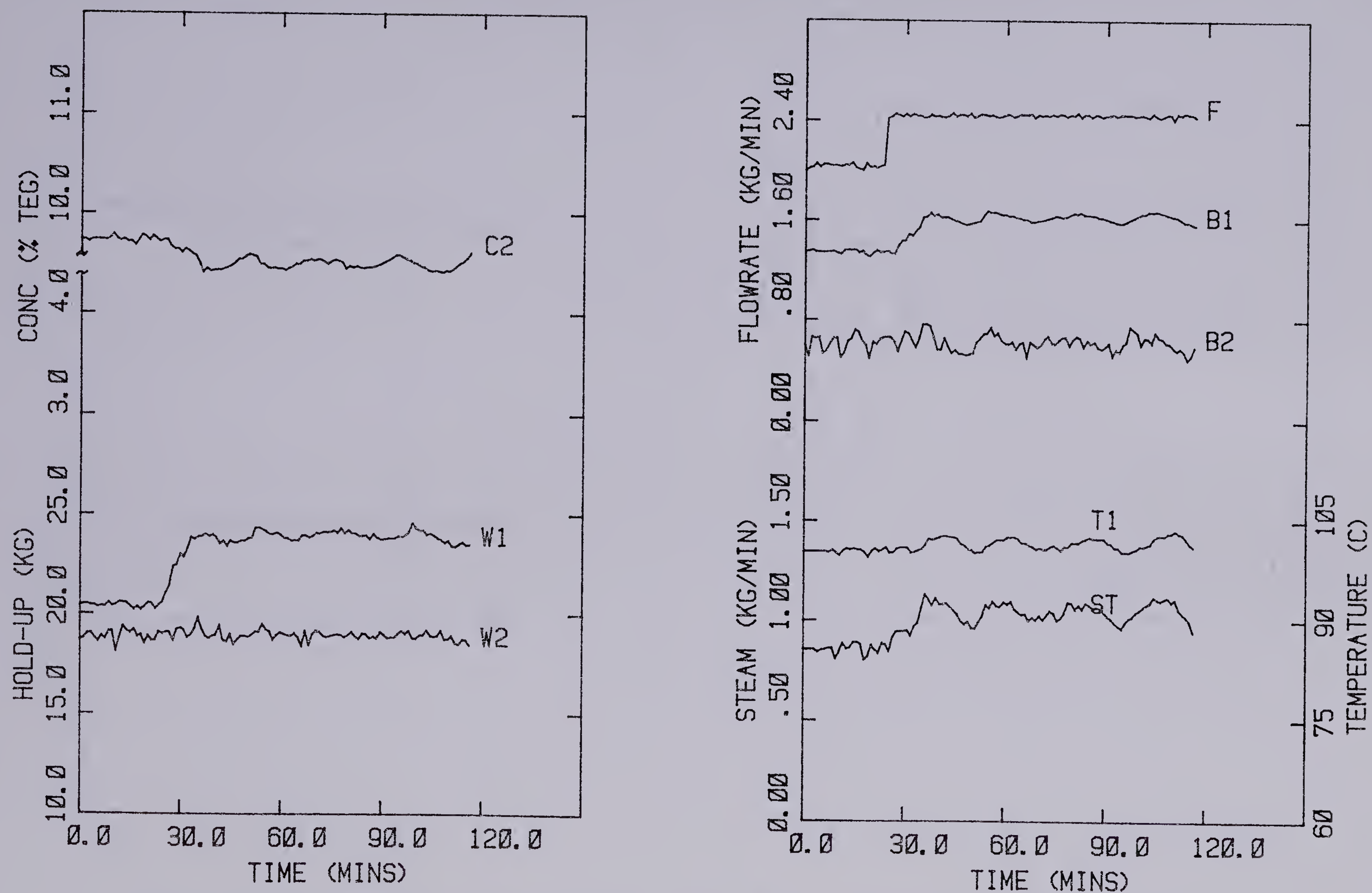


FIGURE 4.21 Evaporator Response Using STR with Time Series Model (e.=0.1)
(STC/RT2001/ITSM/T64/M2/C.1/F1/P1/Q0/ 20%FD/ INCREASED LEADING COEFFICIENT)

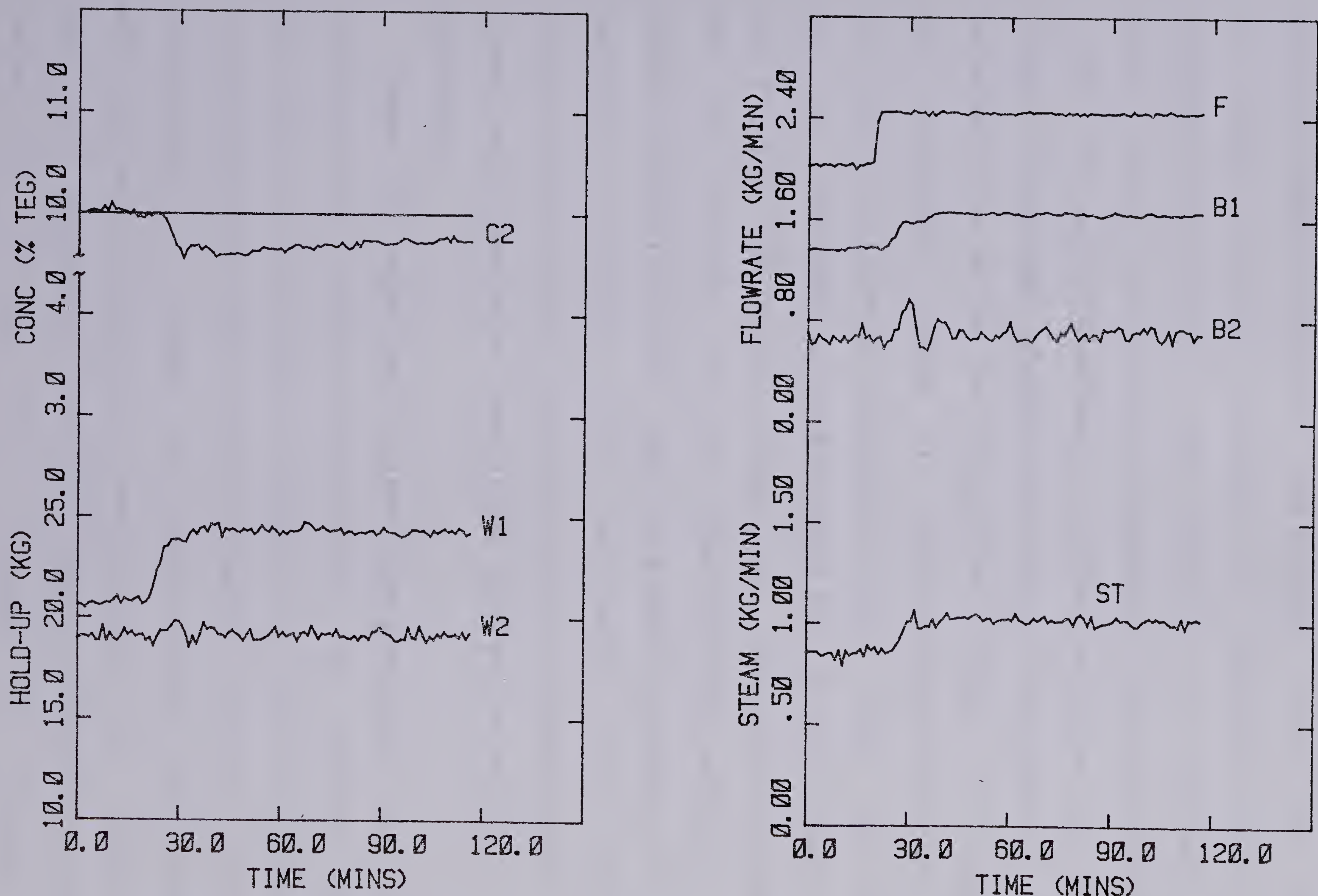


FIGURE 4.22 Evaporator Response Using STR with Time Series Model (e.=0.2)
(STC/RT2002/ITSM/T64/M2/C.1/F1/P1/Q0/ 20%FD/ LARGE LEADING COEFFICIENT)

product concentration C2 drops because of the positive change in feed flowrate, which also causes the increase in the first effect holdup, W1, and bottoms flowrate, B1. Now, the steam flowrate increases to compensate the feed disturbance according to STR control law. However, the control action is too drastic because of the small value of e_o. Once the output overshoots the desired value the steam flowrate decreases sharply again due to the small e_o. When the steam rate is low the first effect holdup is increased due to reduced boiling of glycol solution and hence the first effect bottoms B1 increases as well. When the steam flow is increased, B1 is decreased for the same reason. In this way the first effect bottoms, B1, oscillates or changes 180° out of phase with the steam behaviour affecting the product concentration C2. Thus, if steam fluctuates, all the evaporator variables start oscillating. This physical interpretation together with the grahpical demonstration in Figure 4.20 explains the highly interactive nature of the evaporator.

From the experience gained from the above experiments the importance of the controller gain, $1/e_o$, became clear. The effect of e_o is shown in Figures 4.21 and 4.22 where e_o is increased from .0272 based on equation (3.5) to .1 and .2 respectively. In Figure 4.21 after one and half hours e_o again decreased (through RLS identification) to .0267 and hence the variables once again began to oscillate towards the end of the run. As e_o increases the response is

stabilized but has a bigger offset (Figure 4.22). Note that increasing e_0 is the same as introducing a constant Q-weighting in STC.

The overall conclusion from these series of experimental runs is that STR control of the evaporator resulted in unsatisfactory control. It would seem that some form of smoothing on the manipulated variable (for example through appropriate control weighting) would result in improved control. This is the subject of discussion in the following section.

4.6.2 Experimental Evaluation of STC

A series of runs were conducted using the more general version of Clarke and Gawthrop's STC (1977) to evaluate the performance of this controller on the pilot scale evaporator. The objectives were to verify the simulation results and experimentally demonstrate the influence of the various design parameters on the controller performance. In the remaining part of this section the following items will be discussed followed by a set of conclusions.

1) The choice and effect of weighting functions $Q(z^{-1})$ and $P(z^{-1})$ on the performance of the STC
2) The choice and effect of model order and initial model parameters on the performance of the STC
3) The choice of design parameters for the RLS estimation law: covariance matrix and the forgetting factor

1) Q and P weighting Functions: First of all, to eliminate the offset and also reduce the excessive control effort, integral Q-weighting was introduced, i.e. $Q(z^{-1})=\lambda(1-z^{-1})$. In Figure 4.23 $Q(z^{-1})=.8(1-z^{-1})$ and $e_0=.2$ were used. The offset is gradually reduced but the response is oscillatory. In figure 4.24, where $Q(z^{-1})$ was $.2(1-z^{-1})$ and e_0 equal to .1, offset is reduced however the response is eventually even more oscillatory. The reason for the closed loop system instability can be explained as follows: the closed loop stability of the STC is dependent upon the roots of equation (4.36) in section 4.4.3. From the time series evaporator model on which the initial parameters of the above experiments were based, the locations of closed loop pole are calculated as a function of λ and shown in Table 4.3.

Table 4.3 Closed Loop Poles of an Evaporator Model

λ	closed loop pole locations (3 poles)	
	r_1	r_2, r_3 (complex conjugate pair)
0.1	.4714	.7612 ± .7690j
1.0	.6184	.9974 ± .3325j
5.0	.6758	1.0030 ± .1632j
10.0	.6889	1.0010 ± .1187j

As λ increases the closed loop complex poles migrate toward the unit circle and evently outside it which leads to instability. This analysis suggests a smaller λ at the expense of a excessive control action (Figure 4.24).

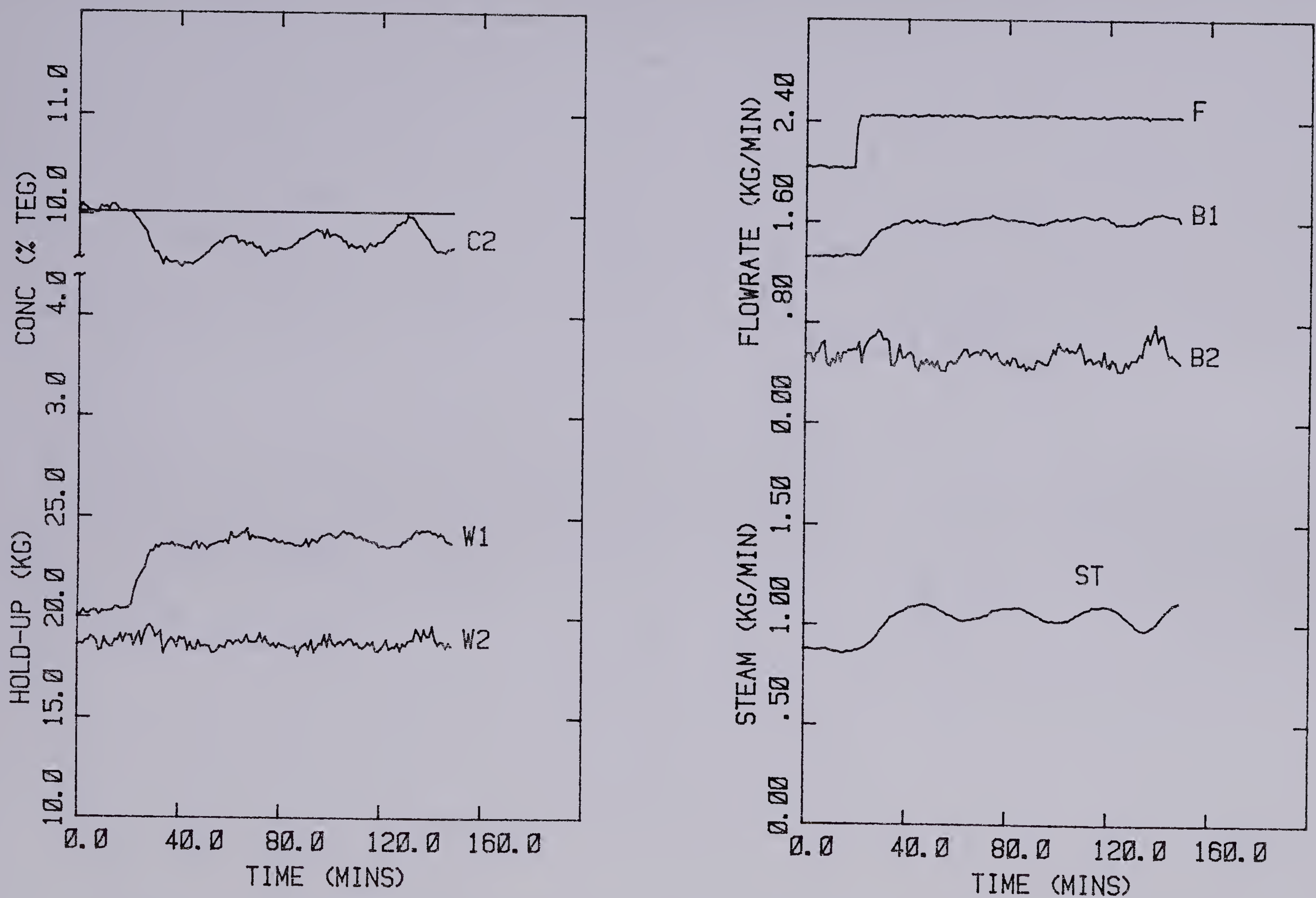


FIGURE 4.23 Evaporator Response Using STC with Integral Q-Weighting (e. =0.2)
(STC/RT2003/ITSM/T64/M2/C.1/F1/P1/Q(.8-.8z)/ 20%FD/ INTEGRAL Q-WT e. =0.2)

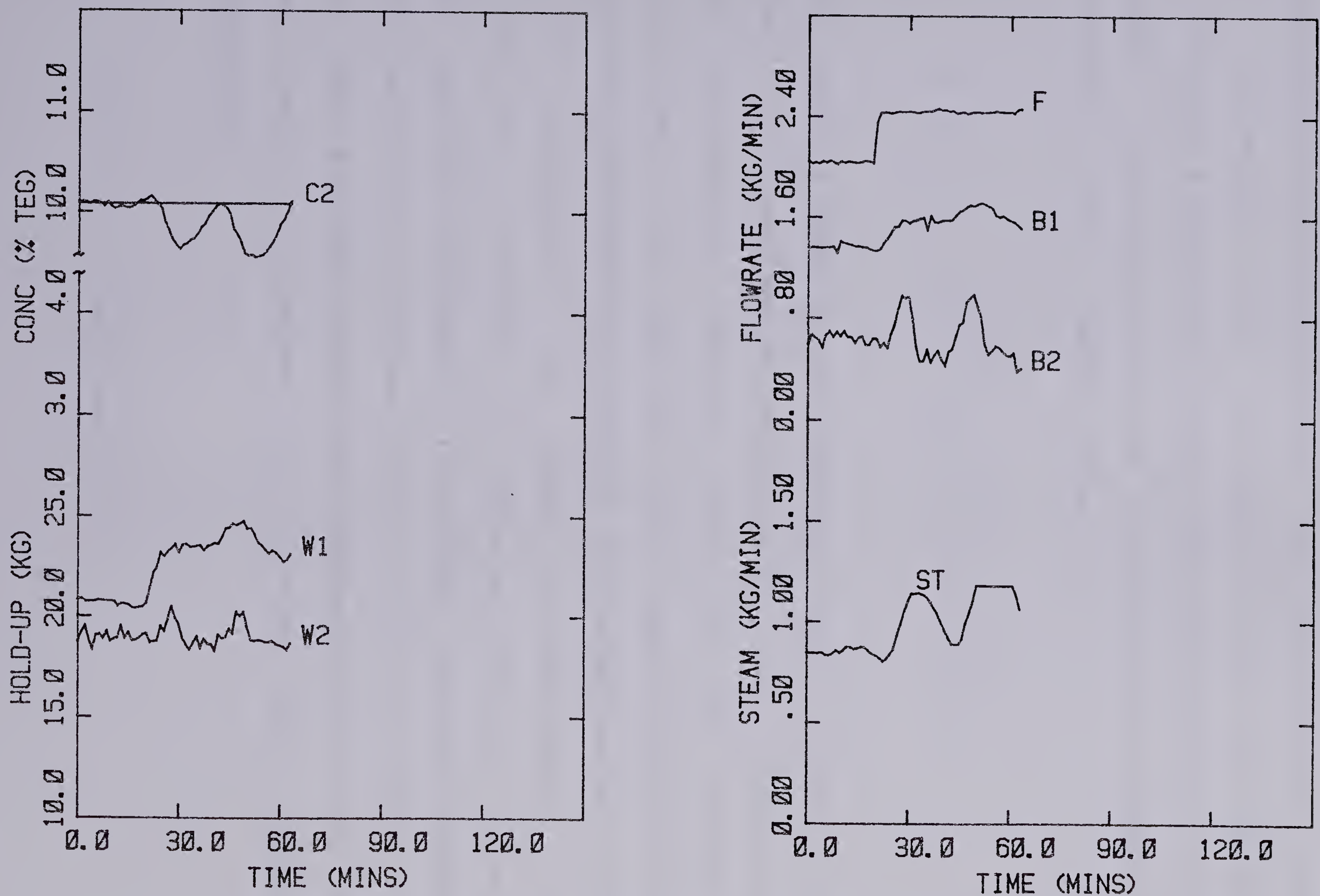


FIGURE 4.24 Evaporator Response Using STC with Integral Q-Weighting (e.=0.1)
(STC/RT2004/ITSM/T64/M2/C.1/F1/P1/Q.2(1-z)/ 20%FD/ INTEGRAL Q-WT e.=0.1)

Since the integral type of Q-weighting could not eliminate the offset and stabilize the response satisfactorily, PID type and PI type Q-weighting as referred to in section 4.4.3 (cf. equation (4.38)) was introduced with P and R being set to unity. The coefficients of the PID type Q polynomial were set to values of the discrete PID constants given in chapter three, i.e.

$$Q^{-1}(z^{-1}) = \frac{(10.88 - 15.48z^{-1} + 5.42z^{-2})}{(1 - z^{-1})}$$

The response based on this weighting function showed large oscillations even though the simulation result (Figure 4.18) was satisfactory. Therefore tuning of the Q-weighting coefficients was required. The PID type Q-weighting needs one more prediction error than PI type weighting and proved very sensitive to the change of coefficient in the Q-polynomial. Furthermore its performance did not prove to be entirely satisfactory. In contrast, PI type Q-weighting gave satisfactory response and its tuning (runs RT2013, 14, 15 (not plotted)) by suitable choices of proportional gain and integral time yielded the following 'best' weighting functions:

$$Q^{-1}(z^{-1}) = \frac{(3.1 - 2.9z^{-1})}{(1 - z^{-1})}$$

With this Q-weighting function the effect of model order, initial model parameters and the RLS design factors were investigated individually.

2) **Model order and initial model parameters:** In Figure 4.25 a second order model was used with the above Q-weighting factor and Figure 4.26 represents the corresponding results obtained using a third order model. These results confirm the simulation results that a second order model performs better than a third order model.

Since the initial model parameters are important to the control performance the effect of the initial model parameters was examined with the PI form of Q-weighting. In Figure 4.27 the initial parameters were calculated based on the first order time delay model, equation (3.3) and in Figure 4.25 the corresponding initial parameters were obtained from the second order curve-fitted model, equation (3.4). The first order model resulted in a bigger deviation in the product concentration. Note that the number of parameters to be estimated is five and four for the first order and the second order model respectively.

These experimental runs show that a second order prediction model performs better than a first order or third order mode for a short term regulation of the evaporator, which verifies the simulation result.

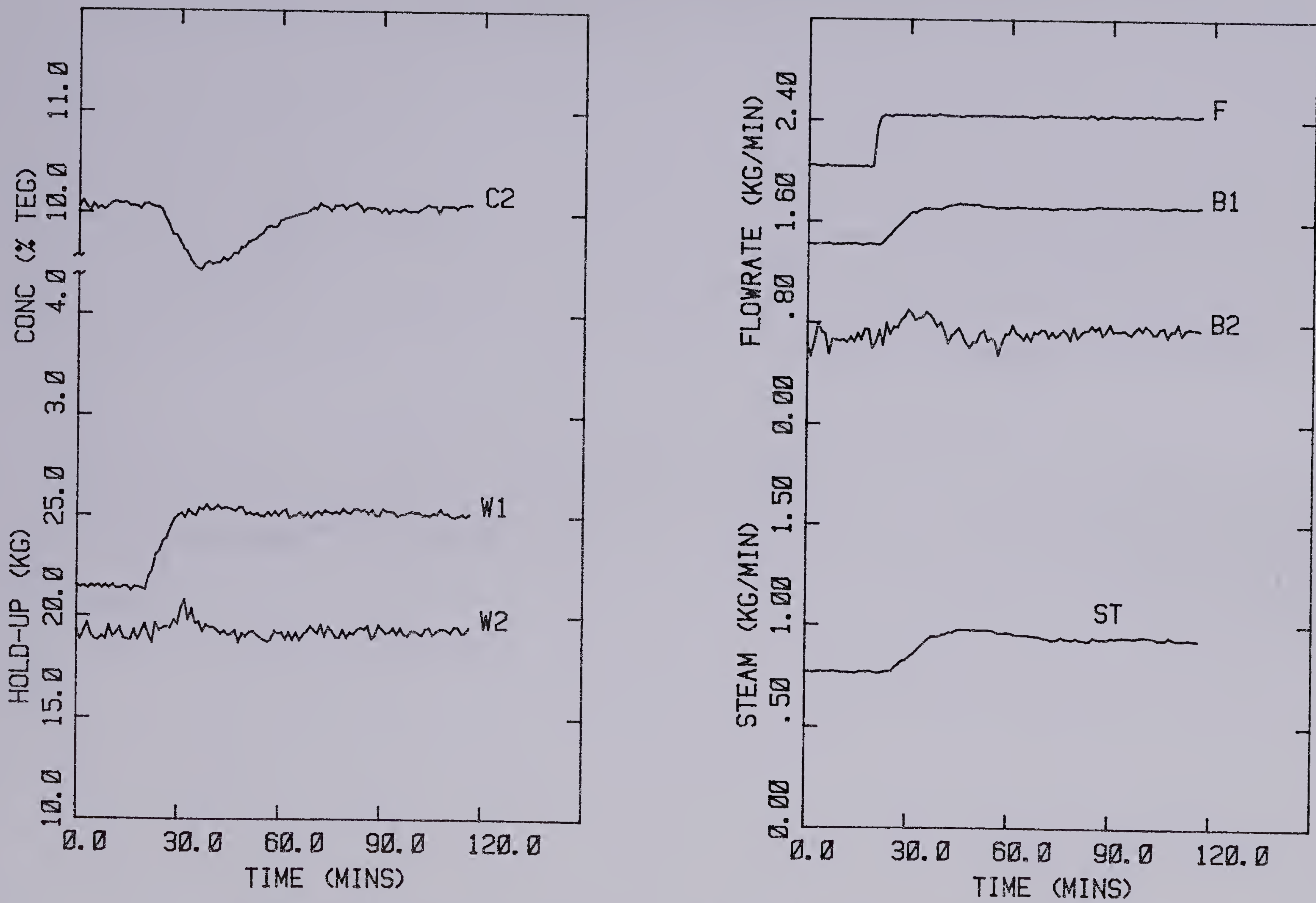
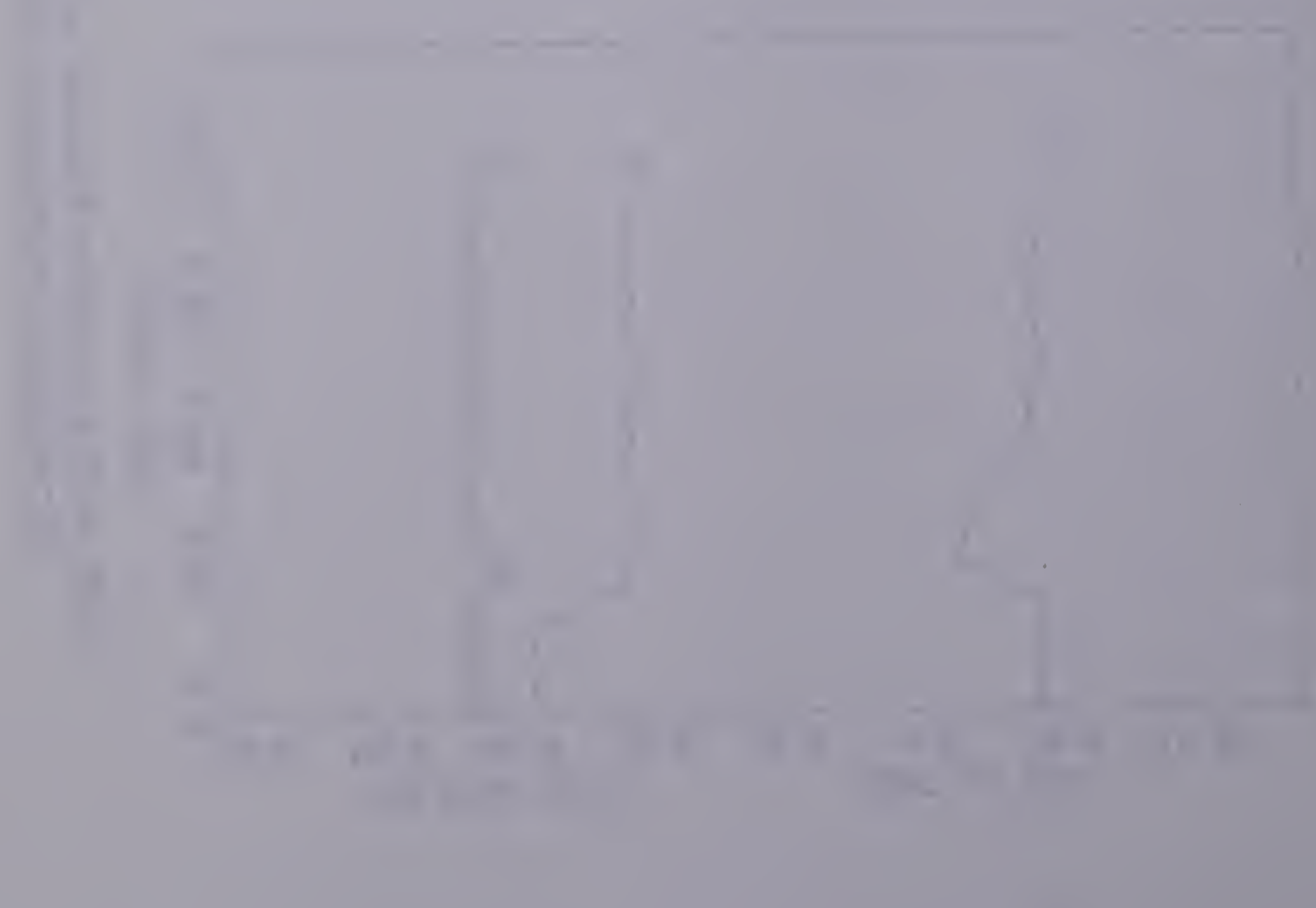


FIGURE 4.25 Evaporator Response by STC with PI Q-wt and Second Order Model
(STC/RT2029/ITDM/T64/M2/C1/F1/P1/Q PI/ 20%FD/ of RT2024 RT2027)



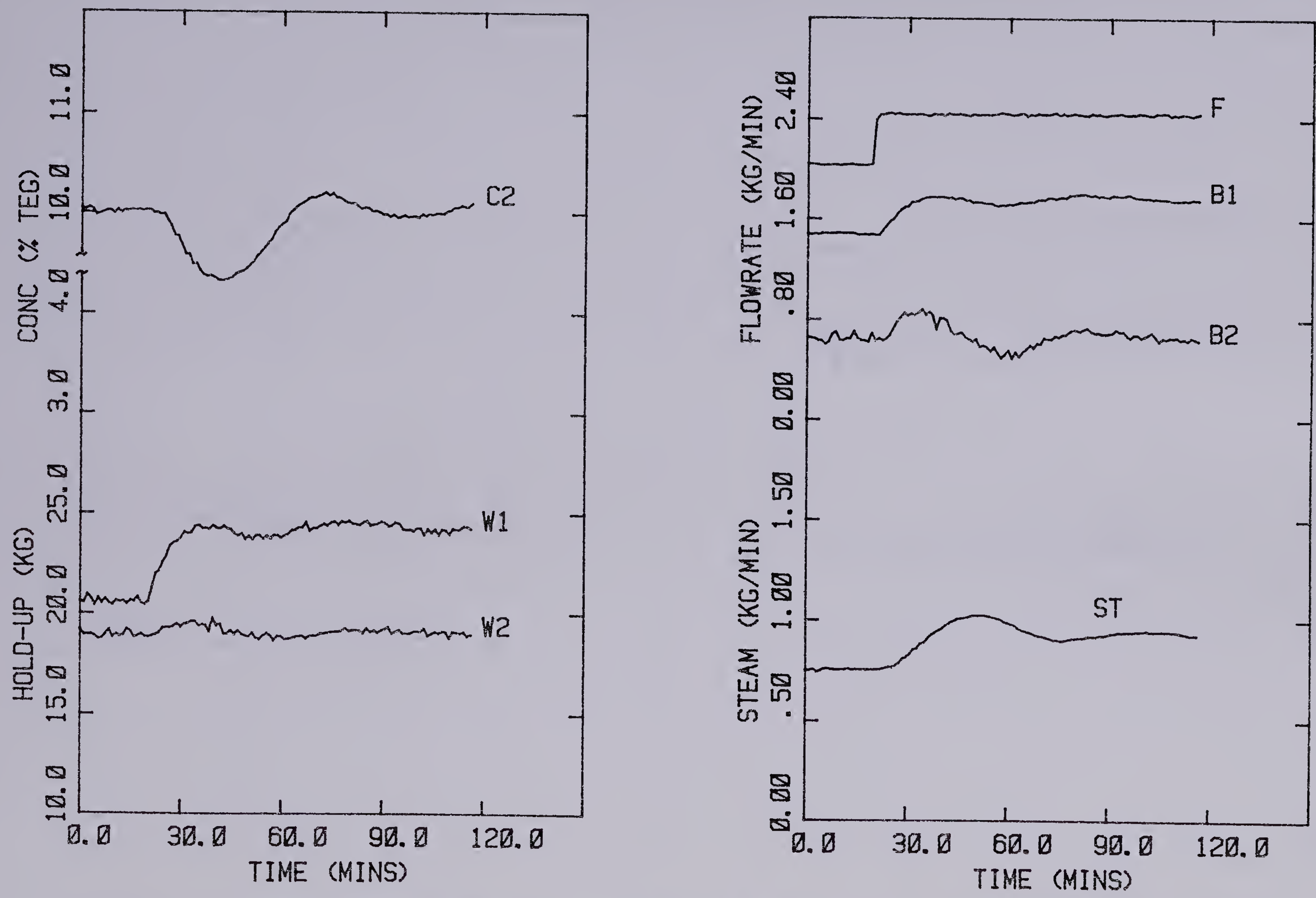


FIGURE 4.26 Evaporator Response by STC with Third Order Model
(STC/RT2027/ITDM/T64/M3/C1/F1/P1/Q PI/ 20%FD/ of RT2024(M=2))

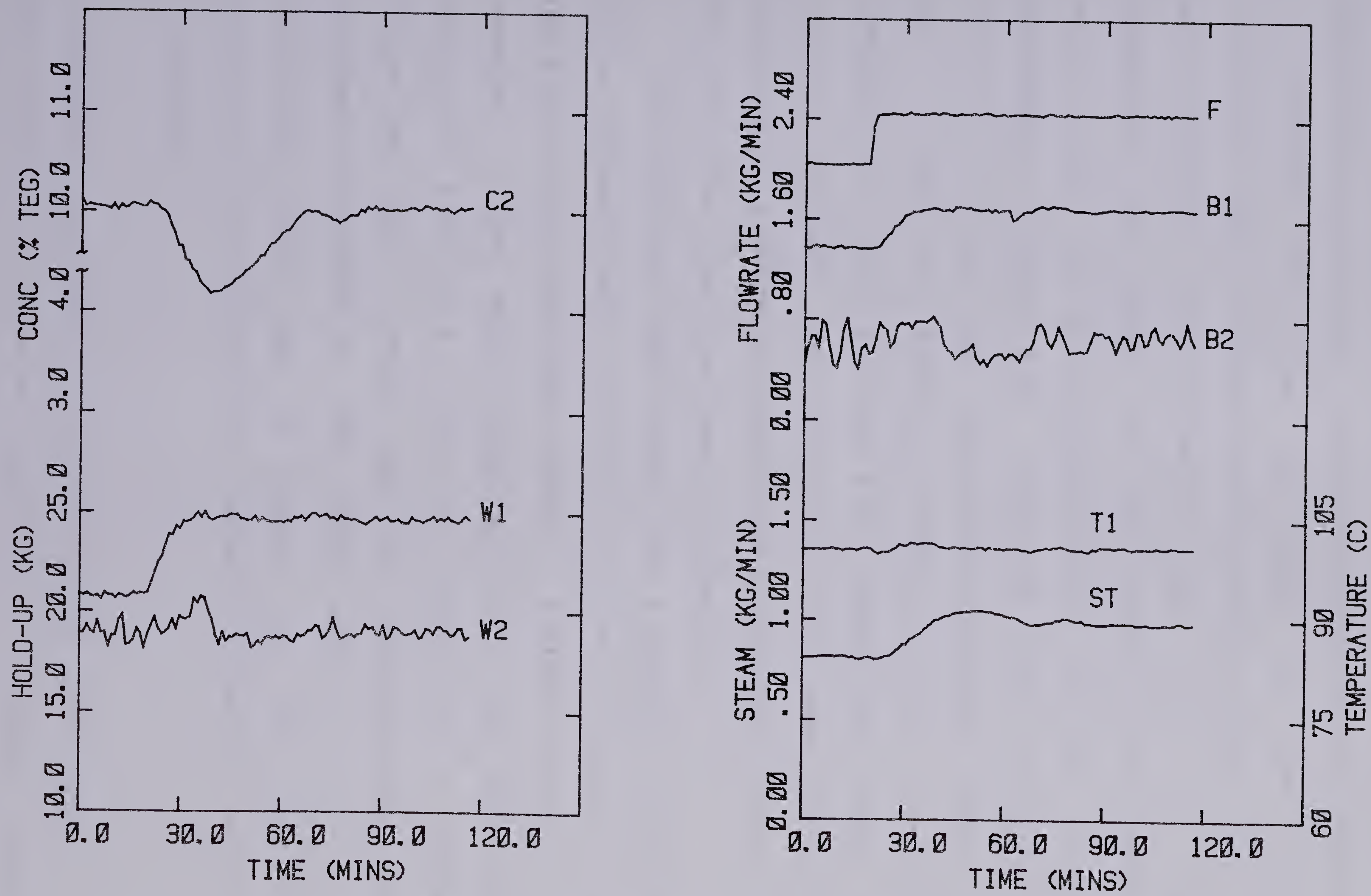


FIGURE 4.27 Evaporator Response by STC with PI Q-wt and First Order Delay Model (STC/RT2026/ITDM/T64/M(1+d)/C1/F1/P1/Q PI/ 20%FD/ of RT2025(COV=.1))

3) Initial covariance matrix and a forgetting factor of RLS estimation law: Because of the sensitivity of the evaporator to parameter variations, the effect of the initial covariance matrix was restricted to small values, i.e. 0.1I and I indicating good confidence in the choice of initial parameters. The effect of large values of the covariance matrix could not be evaluated due to poor control performance. Figure 4.28, 4.29 and 4.30 illustrate the control performance with the initial covariance matrix set to 0.1I. These runs, especially Figures 4.28 and 4.29 can be compared with Figure 4.27 and 4.25 respectively where the initial covariance is set to I. In both cases the control performance is worse with the initial covariance set at 0.1I resulting from slower parameter adaptation.

The simulation study showed that a constant forgetting factor generated more oscillatory I/O variations due to the inflation of the covariance matrix. Similar effects were observed in the actual application to the pilot plant evaporator. Figure 4.31 shows the effect of a constant forgetting factor (ρ=.95) and the resulting oscillatory response as compared to the result with no (or unity) forgetting factor case in Figure 4.25. For long term regulation of the evaporator a variable forgetting factor is recommended.

4) Setpoint tracking: The Q-weighting was also applied in the PI form to examine robustness to setpoint changes.

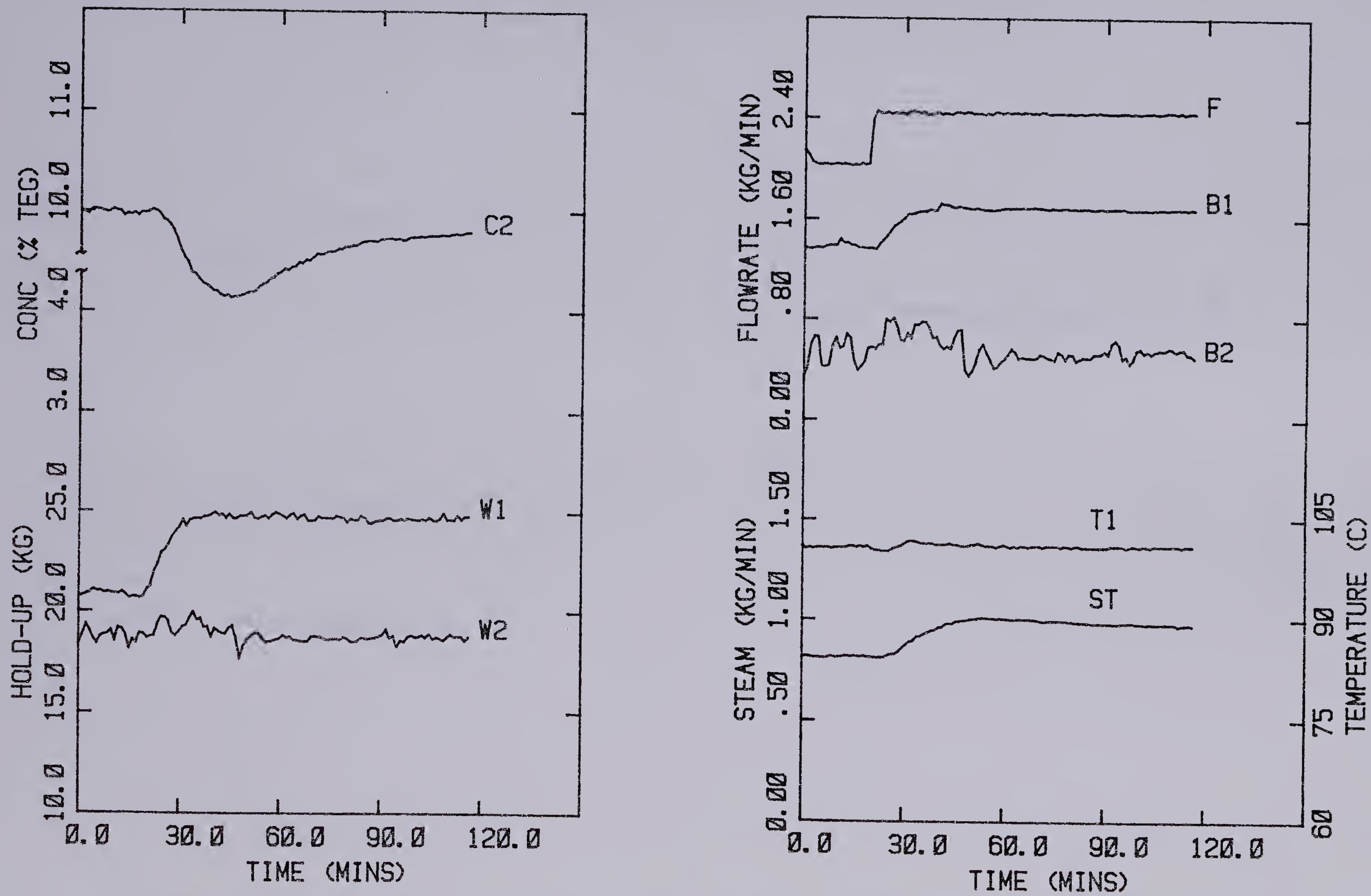


FIGURE 4.28 Evaporator Response by STC with PI Q-wt and Small Covariance Matrix
(STC/RT2025/ITDM/T64/M(1+d)/C1/F1/P1/Q PI/ 20%FD/ COVARIANCE EFFECT of.RT2025)

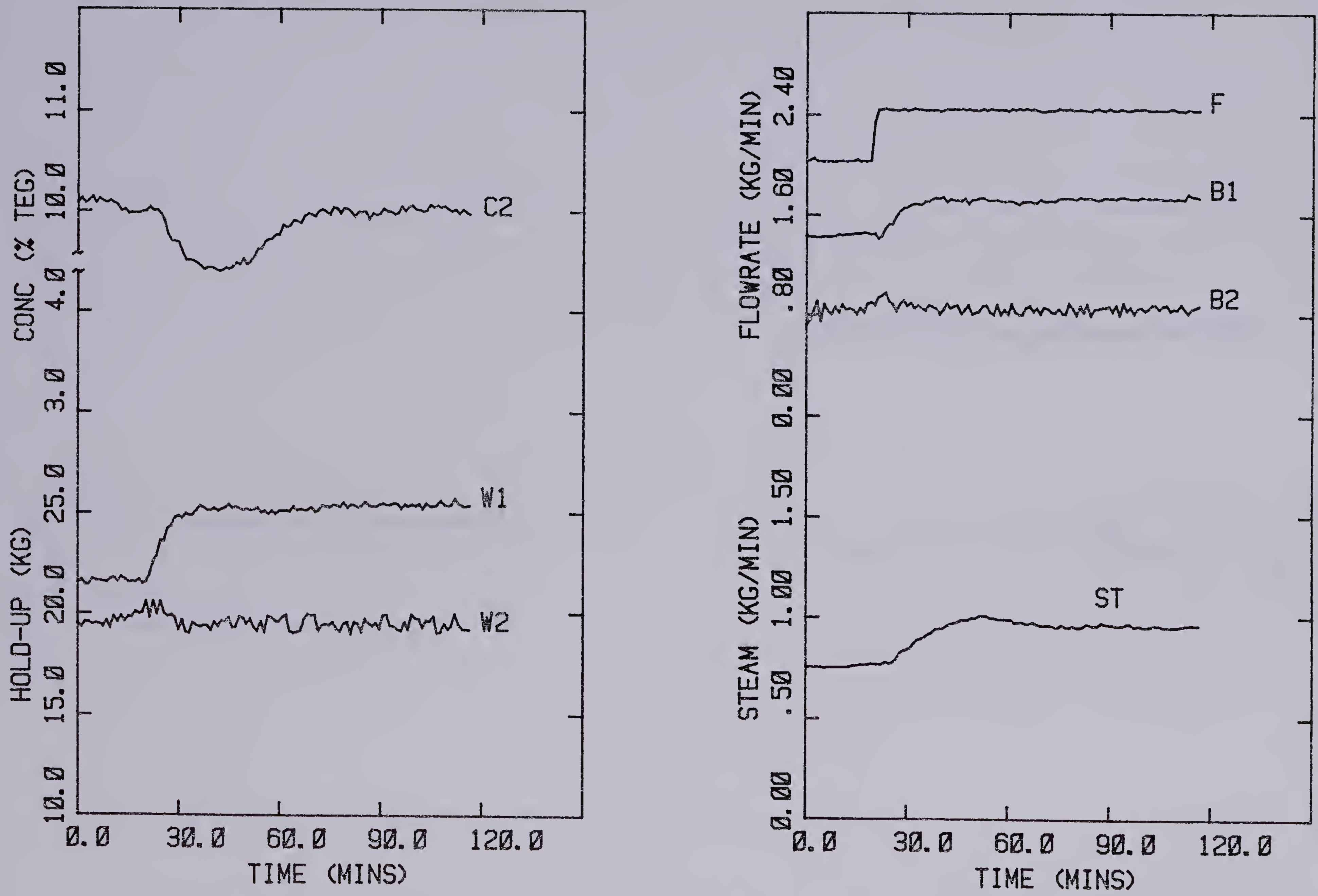


FIGURE 4.29 Evaporator Response by STC with PI Q-wt and Second Order Model
(STC/RT2024/ITDM/T64/M2/C.1/F1/P1/Q PI/ 20%FD/ PI Q-WEIGHTING cf.RP2007)

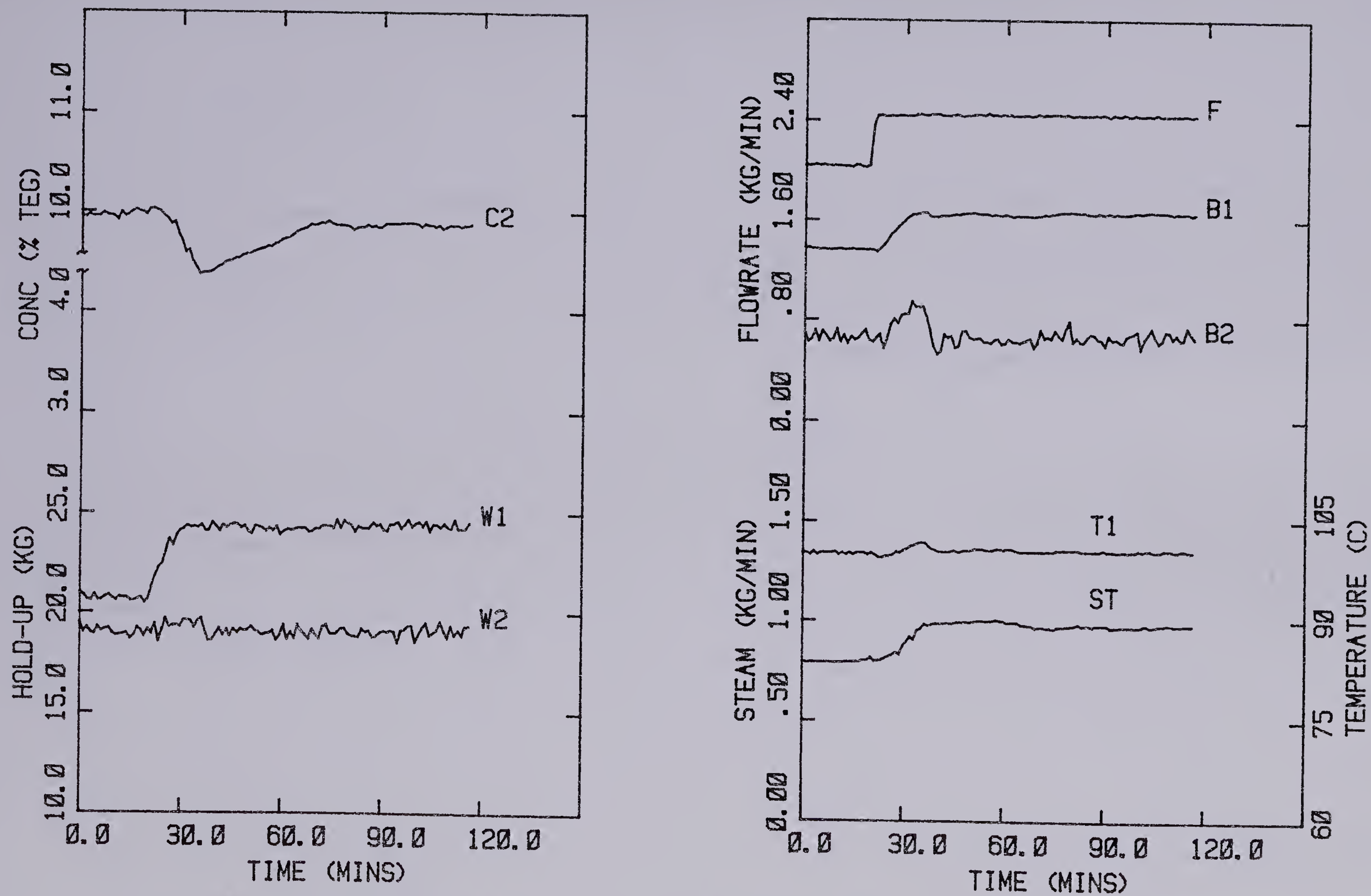
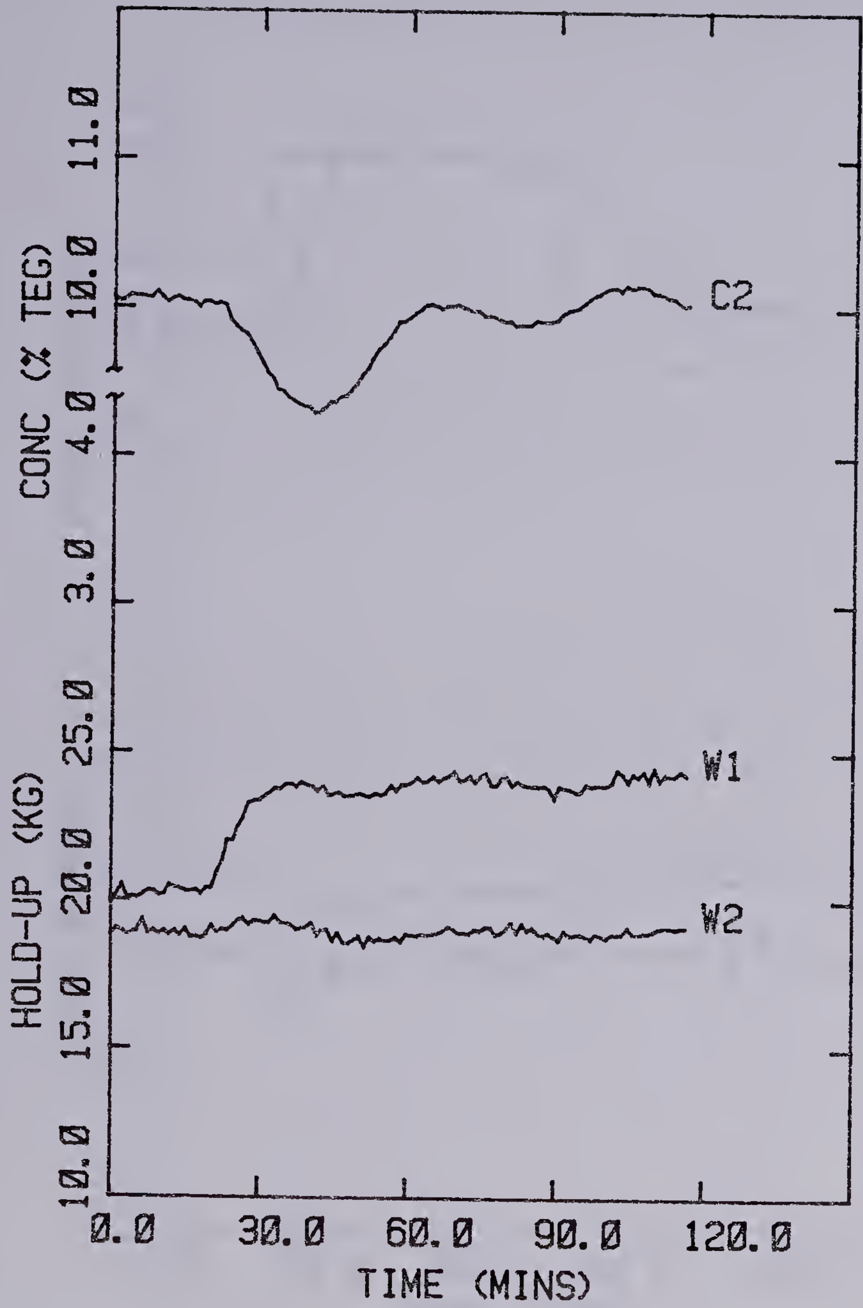


FIGURE 4.30 Evaporator Response by STC with PI Q-wt and Time Series Model
(STC/RT2015/ITSM/T64/M2/C.1/F1/P1/Q PI/ 20%FD/ PI Q-WEIGHTING)



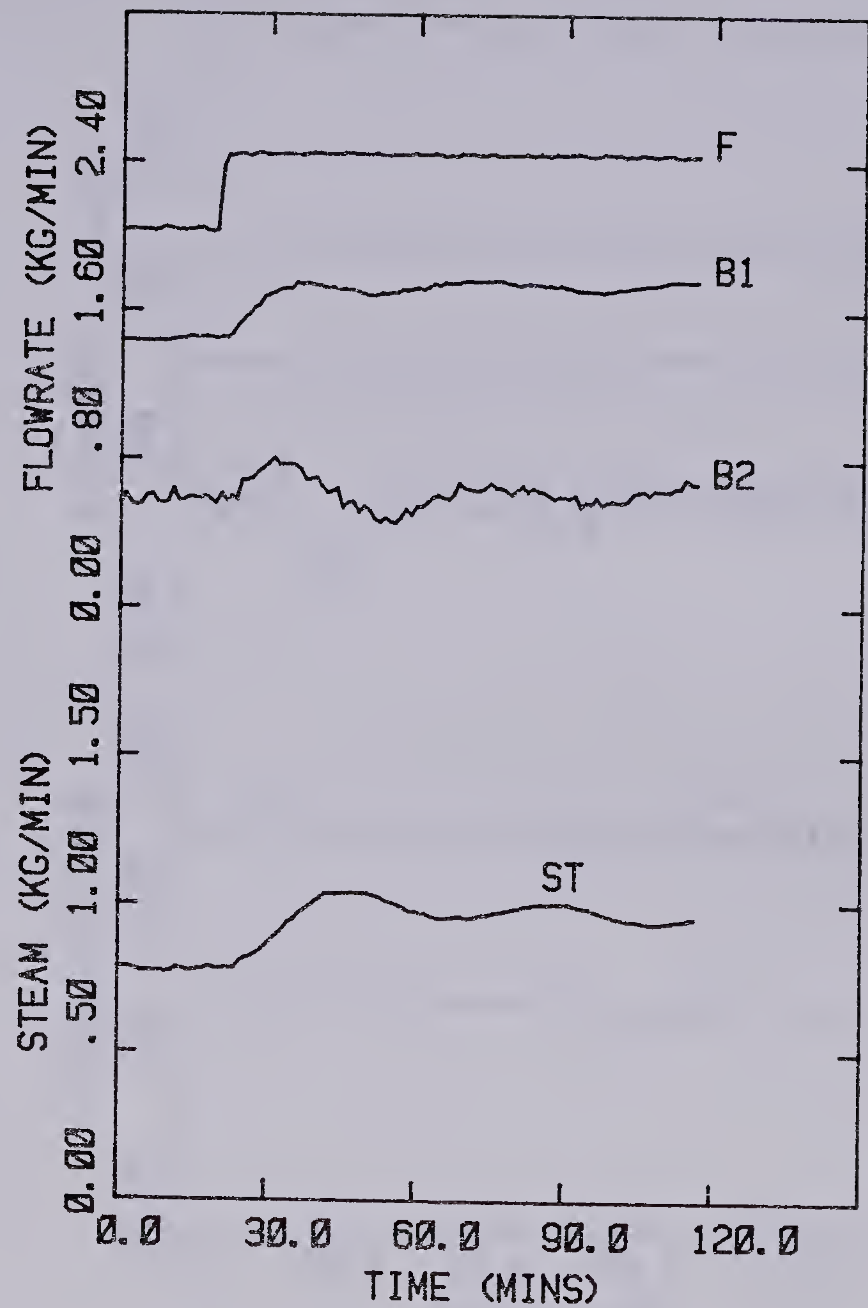


FIGURE 4.31 Evaporator Response Using STC with Constant Forgetting Factor
(STC/RT2030/ITDM/T64/M2/C1/F.95/P1/Q PI/ 20%FD/ of RT2029 RT2024)

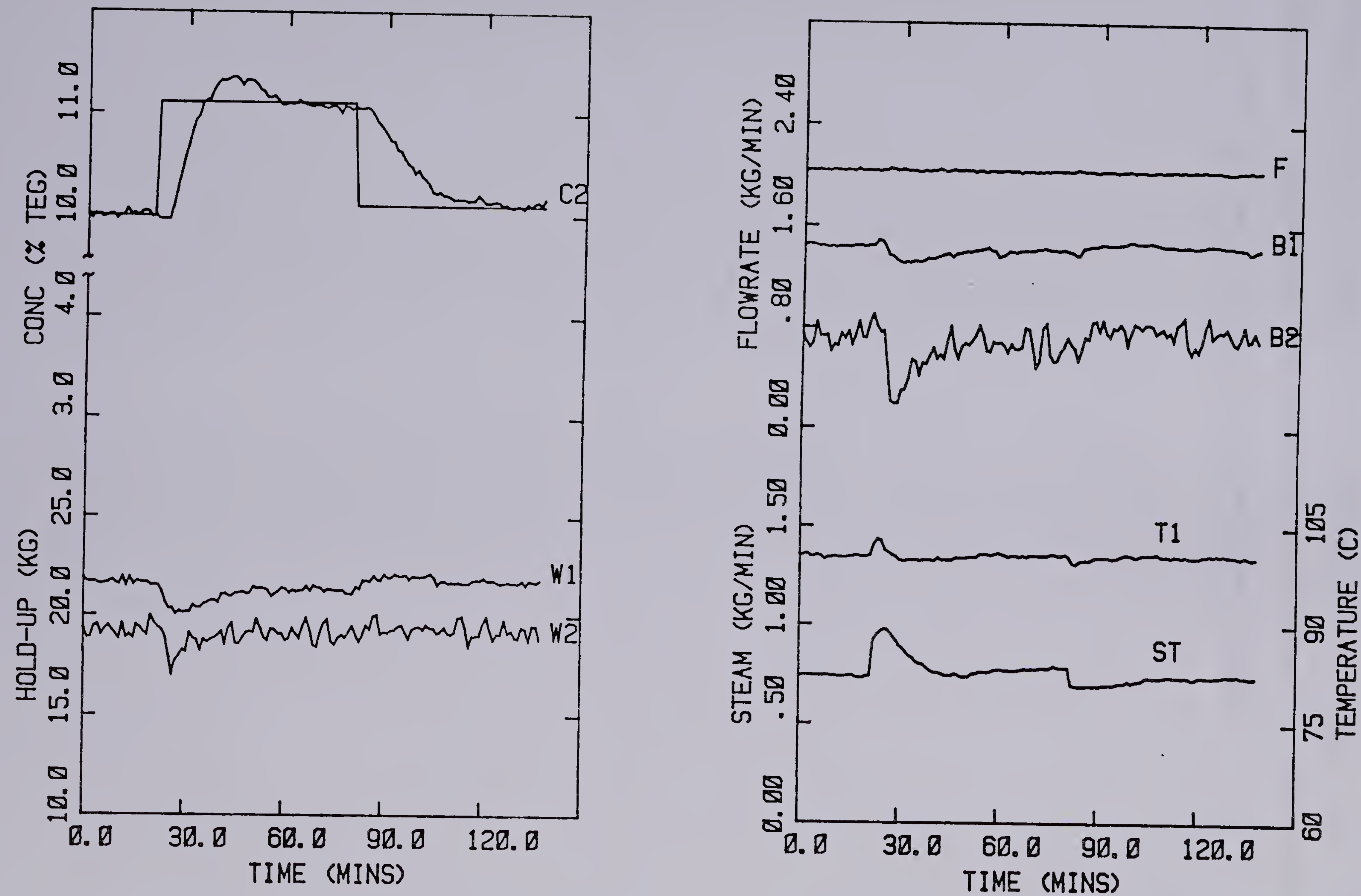


FIGURE 4.32 Evaporator Response Using STC with PI Q-wt to Setpoint Changes (STC/RT2028/ITDM/T64/M2/C1./F1/P1/Q PI/ 10%SP/ SETPOINT CHANGE Q-WEIGHTING)

Figure 4.32 is a response to ±10% setpoint changes and the desired performance was achieved. Note that the STC without Q-weighting can not handle any setpoint changes.

4.7 Conclusions

1. In the simulation study, STR gave satisfactory output performance even with poor initial parameters and conditions. However, this was at the expense of excessive control variance which would not be acceptable in any practical application.

2. In the experimental study the large control variance under STR control could not be eliminated by changing the sampling time, model order, initial model parameters or the design constants of RLS.

3. The control variance was reduced to a desired level by imposing weighting constraints in the performance index of the self-tuning controller. Specifically, the PI type Q-weighting resulted in the desired performance and was robust with respect to the choice of initial parameters and external disturbances. This facility to weight the control or tracking error and the control variable is a very important one from a practical point of view.

4. The choice of a second order model structure proved to be most satisfactory for a short term control of the evaporator. This could have been due to a combination of a better model fit as well as fewer parameters for identification as compared to a third order model.

5. Simulation by itself, is not a satisfactory means of evaluating STR/C control of the evaporator. The overall evaporator response and the value of the design parameters are significantly different in the simulation versus experimental studies.

5. Adaptive Predictive Control System (APCS)

5.1 Introduction

The APCS is an adaptive controller designed to control linear time-invariant but unknown parameter systems subjected to 'bounded' stochastic noise and unmeasurable disturbances. However, this scheme can also be applied to a more general class of control problems including slowly time-variant and nonlinear processes. The adaptive mechanism which is the key feature to this adaptive controller is extremely simple to implement. Therefore, from the practical point of view, APCS is an attractive, realistic controller. In the field of adaptive control one of the long standing questions has been, "Do parameter adaptive controllers which yield asymptotically stable closed loop systems actually exist for linear time-invariant systems?" The real significance of APCS is in rendering one of the first affirmative answers to this question. In other words, APCS guarantees that the outputs of the process in question asymptotically follow the 'desired' output sequence for all initial states, and achieve the control objectives with bounded input sequences.

The design of APCS is based on the following three principles proposed by Martin-Sanchez(1974, 1976a, 1976b)

1. The control vector is chosen at each step so that the predicted output is equal to the desired output vector.

2. The estimated parameters are updated in order to solve the prediction problem, i.e. minimization of the prediction error. Therefore, they are not, in general, required to converge to the actual process parameters.

3. The desired output vector is chosen at each step by a 'driver block' to belong to a desired process output trajectory that satisfies a specified performance criterion.

The schematic diagram of APCS is shown in Figure 5.1 where these principles are conceptually presented as a block diagram. The control block performs exactly as described in the first principle. If the desired output is constant, i.e. regulatory control, the control efforts are calculated such that the effects of disturbances are offset by control action. In this case the calculations are similar to the STR. The estimator block estimates the parameters of the adaptive predictive model based on the process I/O data in such a way that the prediction error, i.e. the error between the actual process output and the model output, is minimized. The prediction model is generally a linear, vector difference equation but the order of the model need not to be the same as that of the real process. This implies that the identification problem of optimal control theory is replaced by an estimation problem. Note also that the choice of the adaptive predictive model affects the steady state control performance as well as the transient response when the basic APCS algorithm is used, e.g. a poor model could result in offset in the controlled variable. The third

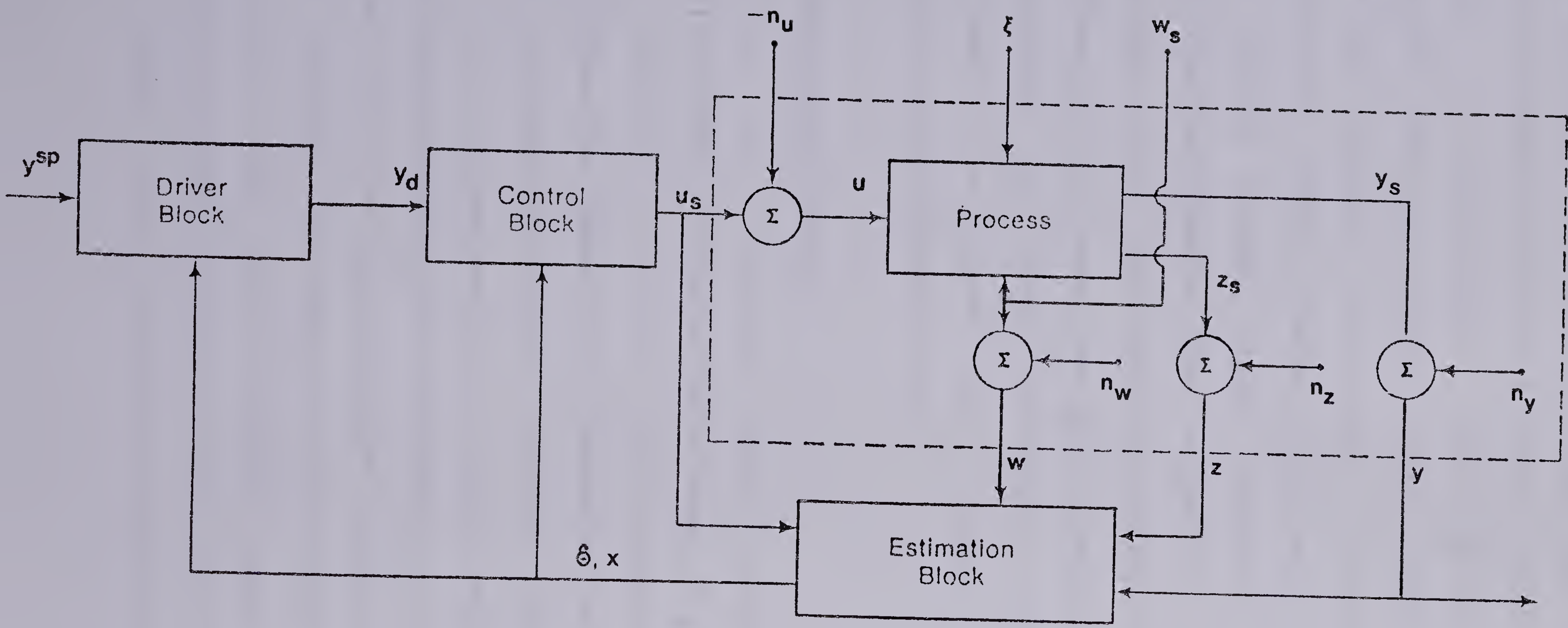


FIGURE 5.1 Schematic Diagram of the Adaptive Predictive Control System (APCS)

briefly states the 'driver block' which can be interpreted as an extension to the traditional concept of the 'reference model' or P-filtering of STC for servo control. At each sampling time the driver block, based on an operator specified setpoint vector for a future sampling time, generates a desired process output vector which belongs to the optimum process output trajectory that satisfies a specified performance index. In addition, an appropriately designed driver block can also provide a basis for handling problems such as nonminimum phase systems. In this study the evaluation of designs including the driver block is excluded. It is assumed that it gives a bounded desired output at each sampling time.

5.2 Literature Survey

In his doctoral thesis published in 1974, Martin-Sanchez introduced the concept of predictive control and combined it with a rather simple parameter adaptive algorithm. The thesis is in Spanish but an overview of APCS was published in English by Martin-Sanchez (1976a). An extension to these results to handle MIMO processes with time-delays together with the general principles of the APCS method, was filed as an US patent by Martin-Sanchez in 1976. The algorithm was designed based on Popov's hyperstability criterion (1963) and a Lyapunov based gradient, error correcting method [Nagumo and Noda, 1967; Mendel, 1973]. In 1978 Goodwin et al. presented an adaptive control algorithm

using predictive control for discrete, MIMO, deterministic systems and included a mathematical proof of global stability and convergence. It is interesting to note that for the case of delay-free discrete systems, the 'projection algorithm I' proposed by Goodwin et al. is identical to the APCS scheme suggested formerly by Martin-Sanchez (1981).

Extensions to the original doctoral work have also been made to include the driver block concept [Martin-Sanchez, 1977]. Basically the driver block transforms the externally specified setpoint value into an internal, physically realizable value in such a way that a specified performance index is minimized. However, the main theoretical extension was published in papers by Martin-Sanchez, Shah and Fisher (1981c) and Martin-Sanchez (1982). In the early work on APCS, Martin-Sanchez (1974, 1976a) showed the convergence of the tracking error of the APCS algorithm using the hyperstability theory and the passivity condition. However, stability, in the sense that convergence is achieved with bounded I/O sequences, was not rigorously proved. In 1981 Martin-Sanchez, Shah and Fisher published the mathematical proof of convergence and stability under reasonable assumptions on the process and its unmeasurable disturbances. This was accomplished through the modification of the original adaptive mechanism to include an adaptation on-off criterion. This result has been further extended to cover general time-delay systems [Martin-Sanchez, 1982].

Recently, Goodwin et al. (1981) also presented a globally convergent adaptive control scheme for discrete, MIMO, stochastic processes. However, from the practical point of view, APCS appears to be more flexible in the sense that the disturbance condition is more moderate and, further, the APCS adaptive scheme turns on and off when necessary, i.e. depending on whether the control error is within or outside a specified bound, whereas the scheme of Goodwin et al. reduces its estimator gain continuously so that it eventually stops adaptation after a certain period. This will be discussed in detail in the discussion section.

There have also been successful applications of APCS to: the control of the highly nonlinear F-8 aircraft [Martin-Sanchez, 1978]; a distillation column which is nonlinear and has relatively long time-delays [Martin-Sanchez et al., 1983]; a mechanical blood pressure control system; simulation of several chemical processes [Martin-Sanchez et al., 1981b]. Some of these applications are discussed in the paper by Martin-Sanchez et al. (1983).

5.3 Theory

This theoretical overview of APCS theory is based on the paper by Martin-Sanchez(1982) which describes a 'basic APCS', i.e. one without a driver block, but includes the stability and convergence proof.

5.3.1 Derivation of APCS

Let the actual process shown in Figure 5.1 be described by a discrete, multivariable, ARMA representation.

$$y_s(k) = \Theta_{io}\Phi_s(k-d) + \Theta_1 u_s(k-d) + \xi(k) \tag{5.1}$$

where

$$\Phi_s(k-d) = [y_s^t(k-d) \cdots, u_s^t(k-d-1) \cdots, z_s^t(k-d) \cdots, w_s^t(k-d) \cdots] \tag{5.2}$$

i.e. a vector of past values of the actual process output vector, y_s; control input, u_s; for generality, other process variables, z_s; and external variables in the vector, w_s. The input and output vectors are assumed to be of dimension n. The dimension of Φ_s depends on the assumed order of the process representation. Integer d denotes the the process time-delay including sample delay. $\xi(k)$ represents the effect of unmeasured disturbances on the process output. Θ_{io} and Θ_1 are the process parameter matrices to be estimated.

The available measured process variables differ from the actual values due to measurement errors plus noise, etc., i.e.

$$y(k) = y_s(k) + ny(k)$$
$$u(k) = u_s(k) + nu(k) \tag{5.3}$$

$$z(k) = z_s(k) + nz(k)$$

$$w(k) = w_s(k) + nw(k)$$

and the corresponding measured Φ now becomes;

$$\Phi(k) = \Phi_s(k) + N\Phi(k) \tag{5.4}$$

$$= [y^t(k) \cdots,\ u^t(k-1) \cdots,\ z^t(k) \cdots, w^t(k) \cdots]$$

where $N\Phi(k)$ is the noise component of $\Phi(k)$. Substitution of (5.3) and (5.4) into the model equation (5.1) gives;

$$y(k) = \Theta\Psi(k-d) + \Delta(k) \tag{5.5}$$

where

$\Theta = [\Theta_{io},\Theta_1]$ = process parameter matrix

$\Psi^t(k-d) = [\Phi^t(k-d),\ u^t(k-d)]$ = process I/O vector

$N\Psi^t(k-d) = [N\Phi^t(k-d),\ nu^t(k-d)]$ = noise vector

$$\Delta(k) = ny(k) - \Theta\cdot N\Psi(k-d) + \xi(k) \tag{5.6}$$

$\Delta(k)$ is refered to as the perturbation vector.

The description in (5.5) can be used to represent a general class of stable-inverse processes. The unknown parameter matrix, Θ, is adaptively estimated by the APCS estimation algorithm described below. First, let the a priori estimation, $\hat{y}(k|k-1)$, of the process output $y(k)$, based on the estimated parameters, $\hat{\theta}(k-1)$, at time k-1, be defined as;

$$\hat{y}(k|k-1) = \theta(k-1)\Psi(k-d) \tag{5.7}$$

Then, the corresponding a priori estimation error is given by;

$$\begin{aligned} e(k|k-1) &= y(k) - \hat{y}(k|k-1) \\ &= y(k) - \theta(k-1)\Psi(k-d) \end{aligned} \tag{5.8}$$

where the parameter estimates are updated by the following recursive relationship.

$$\theta_i(k) = \theta_i(k-1) + \frac{a_i(k)e_i(k|k-1)\Psi(k-d)}{1 + a_i(k)\Psi^t(k-d)\Psi(k-d)}, \quad (i=1,n) \tag{5.9}$$

where $\theta_i{}^t(k)$ is the ith row of the process parameter matrix $\theta(k)$, and $e_i(k|k-1)$ is the ith component of $e(k|k-1)$. The nonnegative scalar constants $a_i(k)$ $(i=1,n)$, as defined below, provide the means for stopping or continuing parameter adaptation which is essential for the proof of stability.

i) $a_i(k) = 0$ if and only if

$$|e_i(k|k-1)| \leq \Delta'_{id}(a_{io},\Delta_{id},k) \leq 2\Delta_{id} < \infty \tag{5.10}$$

where the function Δ'_{id} is defined as:

$$\Delta'_{id}(g(k),\Delta_{id},k) = \frac{2 + 2g(k)\Psi^t(k-d)\Psi(k-d)}{2 + g(k)\Psi^t(k-d)\Psi(k-d)}\Delta_{id} \tag{5.11}$$

with $0 < a_{io} < \infty$, and $\Delta_{id} \geq \Delta_{im} = \max\limits_{0<k\leq\infty} |\Delta_i(k)|$ (5.12)

In the above equations $\Delta_i(k)$, Δ_{id} and Δ_{im} are respectively the ith components of the vectors $\Delta(k)$, Δ_d and Δ_m; where Δ_d is an estimate of a constant, upper bound on the absolute value of the perturbation vector for all k, and Δ_m is the minimum value of this upper bound.

ii) $a_{io} < a(k) \leq a_{id}(k) \leq a_{i1} < \infty$ if and only if

$$|e_i(k|k-1)| > \Delta_{id}^{!}(a_{io},\Delta_{id},k) \geq \Delta_{id} \tag{5.13}$$

where $a_{id}(k)$ is defined as follows;

(1) $a_{id}(k) = a_{i1}$ (5.14)

if $|e_i(k|k-1)| > \Delta_{id}^{!}(a_{i1},\Delta_{id},k)$

where function $\Delta_{id}^{!}(a_{i1},\Delta_{id},k)$ is given by (5.11).

$$(2)\quad a_{id}(k) = \frac{2[|e_i(k|k-1)| - \Delta_{id}]}{[2\Delta_{id}-|e_i(k|k-1)|]\Psi^t(k-d)\Psi(k-d)} \tag{5.15}$$

if $\Delta_{id}^{!}(a_{io},\Delta_{id},k)<|e_i(k|k-1)|<\Delta_{id}^{!}(a_{i1},\Delta_{id},k)$

Then, for all nonzero $a_i(k)$ the following inequality is followed [Martin-Sanchez et al., 1981c; Martin-Sanchez, 1982].

$$|e_i(k|k-1)| \geq \Delta_{id}^{!}(a(k),\Delta_{id},k) \tag{5.16}$$

Consequently, along the solution of the adaptive algorithm defined by equations (5.7) to (5.15), the adaptation of $\theta_i(k)$ will be stopped at time k, i.e. $\theta_i(k)$ will be equal to $\theta_i(k-1)$, if the absolute value of the ith component of the a

priori estimation error, $|e_i(k|k-1)|$, is less than or equal to $\Delta'_{id}(a_{io},\Delta_{id},k)$. If the adaptation is not stopped the error correcting factor $a_i(k)$ can be chosen in an interval greater than a selected value a_{io} and less than or equal to $a_{id}(k)$, which have been defined in such a way that condition (5.16) is satisfied. The reason for this definition is clarified in the stability proof by Martin-Sanchez (1982).

The prediction $\hat{y}(k+d|k)$, at time k, of the process output at time k+d, is given by;

$$\begin{aligned}\hat{y}(k+d|k) &= \theta(k)\Psi(k) \\ &= \theta_0(k)\Psi(k) + \theta_1(k)u(k) \qquad (5.17)\end{aligned}$$

where $\theta_0(k)$ and $\theta_1(k)$ are estimates of the actual process matrices Θ_0 and Θ_1, and

$$\theta(k) = [\theta_0(k),\ \theta_1(k)]$$

The corresponding prediction error is

$$e(k+d|k) = y(k+d) - \hat{y}(k+d|k)$$

The control vector u(k) can be calculated to make the predicted output $\hat{y}(k+d|k)$ equal to the desired output, $y_d(k+d)$, which is prescribed by the operator or by the output of the driver block.

$$u(k) = \theta_1^{-1}(k)\,[y_d(k+d) - \theta_0(k)\Psi(k)] \qquad (5.18)$$

In this control law calculation it is assumed that the number of outputs is equal to the number of inputs and $\theta_1(k)$ is assumed to be nonsingular. It has been shown by Martin-Sanchez et al. (1981c) that $\theta_1(k)$ can always be made nonsingular by selecting an appropriate set of $a_i(k)$ $(i=1,n)$.

The control or tracking error, $\epsilon(k)$, which is equal to the prediction error, $e(k|k-d)$, is defined as;

$$\epsilon(k) = y(k) - y_d(k) \qquad (5.19)$$

Equations (5.7) to (5.18) describe the basic APCS algorithm. The important properties of APCS including stability and convergence will be discussed in the following section.

5.3.2 Stability and Convergence Analysis

The stability and convergence of APCS have been established under the following conditions;

i) An upper bound on the dimension of the process parameter matrix and the process time-delay, d, are known.

ii) The perturbation vector $\Delta(k)$ in equation (5.6) is bounded. Two cases are considered:

(1) The general stochastic case where a constant

upper bound, Δ_d, on the absolute value of $\Delta(k)$ for all k is known and

$$\Delta_{id}-\Delta_{im} = \delta_i \text{ where } \delta_i>0 \text{ for } i=1,n \text{ and}$$

$$\Delta_m = \max_{0<k\leq\infty} |\Delta(k)| \qquad (5.20)$$

(2) The deterministic case where

$$\Delta(k) = \Delta_d = \Delta_m = \delta = 0 \qquad (5.21)$$

iii) The desired process output at time (k+d) is known at time k and bounded, i.e.

$$||y_d(k+d)|| \leq \lambda^2 < \infty, \quad \forall\ k$$

iv) The sequence $\{||\Psi(k)||\}$ is unbounded if and only if there is a subsequence $\{k_s\}$ such that

(1) $\lim_{k_s\to\infty} ||\Psi(k_s-d)|| = \infty$ and

(2) $||y(k_s)|| > \alpha_1||\Psi(k_s-d)|| - \alpha_2, \quad \forall\ k_s$

where α_1 and α_2 are finite scalar constants. This is a standard result for MIMO, ARMA, stable-inverse processes of the form equation (5.5), where the I/O vector does not include vector z and w, matrix Θ_1 is nonsingular and $\{\Delta(k)\}$ is bounded. If vector z and w are bounded for all k, their inclusion in the I/O vector does not violate the result.

The global stability and convergence of the APCS are summarized in theorem 5.1 for a process exposed to unmeasurable bounded disturbances and/or to stochastic noise. Theorem 5.2 is simply a special case of theorem 5.1 which is applicable to deterministic processes, i.e. those

with no unmeasured disturbances or noise.

__Theorem__ __5.1__: Subject to the conditions i), (1) of ii), iii) and iv) stated above, the following properties are true if APCS algorithms (5.7) to (5.19) are applied to a process described by (5.1) to (5.3).

a) $||\Psi(k)|| < \infty, \qquad \forall\ k$

b) There exists a finite integer, k_0 such that

$\theta(k)=\theta(k-1), \qquad \forall\ k>k_0$

c) $|\epsilon_i(k)| \leq \Delta'_{id}(a_{io},\Delta_{id},k) \leq 2\Delta_{id}$ $(i=1,n)$, $\forall\ k>k_0+d-1$

__Theorem__ __5.2__: Subject to the conditions i), (2) of ii), iii) and iv), the following properties are true if APCS algorithms (5.7) to (5.19) are applied to a process described by (5.1) to (5.3).

a) $||\Psi(k)|| < \infty, \qquad \forall\ k$

b) $\lim_{k\to\infty} [\theta(k) - \theta(k-1)] = 0$

c) $\lim_{k\to\infty} \epsilon_i(k) = 0 \qquad (i=1,n)$

__Proof__: Proofs for these theorems are omitted for the sake of brevity. The complete proofs are in Martin-Sanchez(1982).

5.3.3 Weighted APCS

The basic form of APCS, i.e. without a driver block, is analogous to a discrete time, dead beat controller. In other words the basic APCS results in minimal settling time plus zero steady state error. However, the basic APCS also has some of the same shortcomings as dead beat controllers, e.g. excessive control signals may be generated which in some cases cause severe closed-loop oscillations. In this case a detuned approach such as the pole assignment method [Prager and Wellstead, 1981] or weighting on the manipulating variable similar to STC, is able to moderate the excessive control signals associated with the optimal APCS control law. In this work, to avoid the control problem created by the large excursions of the input variable the following type of performance index was introduced. It is similar to the one used in STC. This approach makes the APCS control algorithm more flexible and practical.

$$J = E\{[P(z^{-1})y(k+d)-R(z^{-1})y_d(k+d)]^2+[Q'(z^{-1})u(k)]^2\} \quad (5.22)$$

Where $P(z^{-1})$, $Q'(z^{-1})$ and $R(z^{-1})$ are user specified, design polynomials in z^{-1}. The design and the effect of these polynomials are discussed in section 4.4.3. The corresponding control law can be obtained by replacing $y(k+d)$ by its predicted value $\hat{y}(k+d|k)$ from equation (5.17) and minimizing the performance index with respect to $u(k)$. The result is:

$$u(k) = Q^{-1}(z^{-1})[P(z^{-1})y(k+d)-R(z^{-1})y_d(k+d)] \tag{5.23}$$

where $Q(z^{-1}) = q'_0 \cdot Q'(z^{-1})/p_0 \cdot \theta_1$. This control law has the same form as STC, equation (4.18). The only difference is that the weighed, predicted output of STC is replaced by the corresponding weighted output obtained by the APCS prediction law. In this way the APCS can be directly compared with the STC.

5.3.4 Discussion of APCS

1. Adaptive Mechanism

The importance of the parameter adaptive algorithm of an adaptive controller has already been discussed in the previous chapter. APCS takes advantage of a rather simple parameter estimation scheme (cf. equation (5.9)). The parameter estimator can be thought of a modified algorithm of the 'learning method' proposed by Nagumo and Noda(1967) which is based on the error correcting training procedure. The differences are that the denominator term, $x^t x$ of the learning method is replaced in the APCS estimation scheme by $(1+a_i(k)x^t x)$ so that the possibility of division by zero is totally eliminated and secondly the error correcting coefficient $a(k)$ is introduced into the APCS estimator to facilitate proof of global stability and convergence.

As can be seen from equation (5.9), both the APCS and the learning methods perform parameter estimation without

using a process I/O information matrix such as the parameter covariance matrix used in the recursive least squares estimation method. Thus, as far as implementation goes, the learning method is extremely simple and requires less computational effort than RLS or its equivalent schemes. It may suffer from slow convergence and does not provide a measure of the accuracy of parameter estimates (Morris et al., 1982) because of absence of a covariance matrix. The APCS estimator, however, does not suffer from 'windup' as does ordinary RLS, nor from 'estimator shrinkage' as does algorithm proposed by Goodwin et al. for stochastic process. The estimator proposed by Goodwin et al. (1981), has the form

$$\theta(k) = \theta(k-1) + \frac{a_0}{\gamma(k-1)} e(k|k-1)\Psi(k-1) \tag{5.24}$$

$$\gamma(k-1) = \gamma(k-2) + \Psi^t(k-1)\Psi(k-1), \quad \gamma(0) = 1. \tag{5.25}$$

where a_0 is a positive scalar constant. The error correcting coefficient $a_0/\gamma(k-1)$ tends to zero as time goes on. This behavior is only acceptable for time-invariant processes where parameter convergence is faster than the rate of estimator gain decreases.

Since APCS has an adaptation stopping criterion supervision of steady state operation to prevent 'windup' is not required. Also APCS does not require an external signal to give persistent excitation to the process.

If the APCS parameter estimation scheme (equation (5.9)) is closely examined, it can be seen that there are two extreme cases depending upon the magnitude of the I/O vector. When the I/O vector is very small in magnitude, as frequently occurs if normalized perturbation variables are used, its norm is negligible, i.e. $\Psi^t(k)\Psi(k)<<1$, and the estimator can be approximated by;

$$\hat{\theta}_i(k) \cong \hat{\theta}_i(k-1) + a_i(k)e_i(k|k-1)\Psi(k-d), \quad (i=1,n) \quad (5.26)$$

In this case the effect of the error correcting factor $a_i(k)$ is very significant and hence it should be chosen carefully. If it is too large the parameter estimates may fluctuate too rapidly, which in turn results in poor performance. On the other hand, when the magnitude of the I/O vector is much larger than unity, as occurs during large process fluctuations and/or with the selection of specific engineering units, the estimator equation can be expressed approximately as;

$$\hat{\theta}_i(k) \cong \hat{\theta}_i(k-1) + \frac{e_i(k|k-1)\Psi(k-d)}{\Psi^t(k-d)\Psi(k-d)}, \quad (i=1,n) \quad (5.27)$$

Note that the error correcting factor does not influence the parameter estimation at all. It is recommended that the representation of the process I/O variables should be in perturbation and/or normalized form in order to make

selection of the error correcting factor $a_i(k)$ easier. When the I/O vector Ψ increases in magnitude, equation (5.27) suggests that the rate of parameter adaptation decreases. However, the error e_i also increases as Ψ increases and it has been observed that the overall effect of large variations in the process I/O variables is to increase the rate of parameter adaptation.

2. Control law

The control law of APCS is more general than the STR of Aström and Wittenmark (1973) in the sense that a setpoint or reference value is introduced as part of the control calculation. However, the APCS control law calculation is also based on the certainty equivalency principle and design is a stochastic analogy to a discrete parameter adaptive 'dead beat' controller. Therefore, the basic APCS may be expected to give unacceptable I/O variations during the initial startup period if initial parameter estimates are poor. One practical way to improve the performance for these cases is to penalize the control effort by introducing a quadratic cost function as discussed previously. The effect and design of the cost function will be discussed in the simulation and experimental sections.

3. Perturbation vector

Since the adaptive predictive model of APCS does not have to have the same structure as the process being

controlled, the perturbation vector $\Delta(k)$ can include 'modelling residual' as well as bounded unmeasured disturbances and noise if is assumed that the residuals are bounded. In actual applications the exact upper bound of the perturbation vector is not usually known for a particular process. However, starting with a relatively large bound for Δ_{id}, ensures that condition (5.20) is satisfied. The APCS estimator with this large initial Δ_{id} will estimate the parameters so that the magnitude of the control error is bounded as defined by (c) of theorem 5.1. However,this large bound may not be satisfactory. If this is the case the absolute bound on the perturbation vector can be reduced gradually. For each new bound, Δ_{id}, estimation will resume and give better parameter estimates in the sense that the norm of the parameter error vector will be smaller. In this manner the upper bound on the absolute value of the perturbation vector can be brought close to the minimum upper bound, Δ_{im}, and control performance will improve. If the user specified bound, Δ_{id} is chosen smaller than the minimum upper bound, the parameter estimator will adjust the parameters unnecessarily, which requires more computer time and produces no improvement in control performance. In practice even though convergence of the parameter estimates to their true values is not guaranteed, it is possible to show that the magnitude of the APCS control error at the steady state, is bounded by the value that would be obtained if the actual parameters were known [Martin-Sanchez, 1982].

4. **Parameter convergence**

It is proven as part of theorem 5.1 that the parameter estimates converge to constant values in a finite number of sampling intervals. As discussed in the introduction section, the adaptive predictive model of APCS in general need not exactly match the actual process. If the predictive model is not exact it will give an approximate description of the process dynamics and the control based on this approximate process model may result in poorer control especially during transient periods. The important features of the predictive model used in adaptive control, are the 'suitability' and 'adaptability' rather than the exactness of the process description, which is almost impossible to obtain in real situations [Ljung, 1978]. The stability analysis of APCS proves parameter convergence in a finite number of sampling intervals. In addition, the APCS estimation algorithm produces a parameter error which is nonincreasing in its norm. In other words, the parameter estimates tend towards values which decrease the prediction error.

5.4 Implementation

5.4.1 Initial Conditions

A basic APCS, as described by equations (5.7) to (5.18), can be easily implemented on a microprocessor or a minicomputer and used to control a variety of actual

processes. However, before execution of the algorithm, APCS like any other controller, requires that some parameters be specified in order to achieve the desired control performance. These include the order and time-delay of the adaptive predictive model, lower and upper values of the error correcting factor and an upper bound on the unmeasurable disturbances (perturbation variable $\Delta(k)$).

First of all, the choice of the initial adaptive predictive model parameters is an important step in the implementation of APCS. It influences not only the control performance during initial transient but also the final values to which the parameters will converge, e.g. to which local optimum the parameters converge. As stated earlier in APCS principle (2) the adaptive predictive model need not be an exact description of the process to be controlled but it should provide a reasonable basis for predicting future values of the process output (prediction problem). The process model, equation (5.5), used by APCS is little different from that of STC, equation (4.1). If an equation (4.1) type process description is available it should be transformed into the form of equation (5.5) in order to find the dimension of the process model and the corresponding coefficients. This can be done by successive substitution of $y(k-i)$, $(i=1,\cdots d-1)$ or using the Diophantine equation (cf. equation (4.4)) and letting the stochastic noise term be the perturbation variable, $\Delta(k)$.

For completely unknown processes the 'suitability' and 'adaptability' of the chosen model should be very carefully considered. In fact, there is strong motivation to consider low order adaptive controllers. Several papers have achieved excellent low order designs [Goodwin and Sin, 1979b; Goodwin and Ramage, 1979; Hsia, 1970]. Higher order models in general take more time to converge and more computational effort. If the order of the model is higher than that of the actual process, the performance of the adaptive controller should be optimal, and the corresponding converged parameters will contain a common factor unless one of them converges its true value. If one of them converges to the true value the extra coefficients of the model will tend towards zero.

The time delay is perhaps the most crucial parameter to choose in the application of discrete controllers. It is usually represented in terms of an integer multiple of the control interval. However, in real applications, it is impractical to always choose the sampling time such that the system time delay can be accurately represented by an integer multiple since the sampling time must be selected to reflect the process dynamics as well as the process time delay. In general, discretization of a continuous model leaves a fractional part of the pure time delay, which introduces an extra system zero. This zero will migrate outside the unit circle in the z-plane as the fractional part increases from zero to unity and thereby give

nonminimum phase behavior even though the process is minimum phase. From a practical point of view, the time delay should be chosen to be equal to or slightly larger than the actual time delay (modified Z-transforms should be used for systems that have a fractional delay). If the time delay, d, is chosen to be less than its true value, d_s, the adaptive controller tries to make the predicted d-step-ahead output of the process equal to the desired output using the current control action, u(k). However, u(k) can only affect the actual process output at d_s sample intervals in the future. Therefore, the cross correlation between the current u(k) and the d-step-ahead output of the process y(k+d) will be close to zero which may result in a large control action and a highly oscillatory response. In the case where the time delay is significantly greater than the actual delay the effect will be similar for analogous reasons.

The effect of initial model parameters has already been discussed in the section on the implementation of STC. The choice of the upper limiting values that determines the error correcting factor, a_{i1}, depends upon the accuracy of the initial parameter estimates. Larger a_{i1} should be used with poor initial estimates in order to achieve faster convergence. However,it has been found that even if the initial parameters values are zero the value of a_{i1} should not be as large as the covariance matrix of RLS for the same case. This may be explained from the estimation equation. In equation (5.9) the error correcting factor, $a_i(k)$, weights

each element of the I/O vector, $\Psi(k-d)$, by the same amount. Thus each parameter estimate changes by $a_i(k)\Psi_i(k-d)$ multiplied by scalar term, $[e(k|k-1)/(1+a_i(k)\Psi^t(k-d)\Psi(k-d)]$. A large $a_i(k)$ can make the parameter change too large and thereby destroy a well balanced set of converged parameters. On the other hand, the lower limits on $a_i(k)$ should be as small as possible. If a_{io} is chosen to be zero and the disturbance bound Δ_{id} to be equal to Δ_{im}, then APCS produces minimum output variance. Note that, as discussed in the previous section, the magnitude of the dead zone in which APCS parameter estimation is turned off, i.e. where $a(k)=0$, is directly proportional to the magnitude of Δ_{id}. Therefore, Δ_{id} should be set as small as practical (ideally $\Delta_{id}\to\Delta_{im}$). The method of choosing Δ_{id} has already been discussed in the previous section.

APCS was evaluated via a number of simulated and experimental runs on the double effect evaporator.

5.5 Simulation study

The properties of APCS, such as those discussed in the preceeding sections, were investigated in a series of simulated applications to the double effect evaporator. Wherever possible, the simulation conditions were chosen to facilitate comparison of APCS with the PID, STC and SFC runs described in other chapters.

As expected, based on experience gained with STC, the choice of initial parameters and design constants was

critical. Several runs were made to illustrate the effect of:

1. model order
2. choice of initial model parameters
3. APCS adaptive mechanism
4. bound on unmeasured disturbances

Unfortunately, these factors interact and it is impossible to evaluate them individually. Moreover, it also became apparent after several weeks of effort that for the evaporator application the performance using APCS was not practical or robust enough without the addition of P and Q weighting on the output and control variables respectively. Therefore, several additional runs were completed using weighting functions comparable to those that proved particularly effective in the STC runs.

A summary of the APCS simulation runs is given in Table 5.1 and can be comparable directly with the STC runs in Table 4.1 and the SFC runs in Table 6.1. Note that the APCS runs described in this chapter were done primarily for comparison with SFC and not intended as an independent and/or complete evaluation of APCS. As explained previously, the adaptive law of APCS can be turned on/off at any time. The straight line just above the time axis in the figures (cf. Figure 5.2) indicates whether adaptation is on (dots) or off (blank space). The next section discusses the runs individually. The general conclusions based on both the simulated and experimental APCS results are included in the

Table 5.1 List of Simulation Runs Using APCS

Figure No.	Run No.	Intial $\Theta(0)$	Ts (sec)	Model order	a(k) (upper)	Noise Bound	P wt	Q wt	Comments
5.9	SP2001	Θ_1	64	2	1	0.0	1	0	zero initial parameters cf. ST2001
5.5 5.10	SP2002	Θ_2	128	2	1000	0.0	1	0	effect of a(k) and b_0
	SP2003	Θ_3	64	2	1	0.0	1	0	effect of a(k) and b_0
	SP2004	Θ_2	64	2	1000	.005	1	0	zero initial cf.SP2002
	SP2005	Θ_3	64	2	1	.005	1	0	effect of a(k) and noise bound
5.6	SP2006	Θ_4	64	2	1	.005	1	0	initial parameter from eq (3.5)
5.2	SP2007	Θ_5	64	1	1	.005	1	1	first order model eq (3.2)
5.7	SP2008	Θ_6	64	1	1	.005	1	0	first order model plus delay
5.3	SP2009	Θ_0	64	2	1	.005	1	0	second order model eq (3.4)
5.17	SP2010	Θ_0	64	2	1	.005	1	0	setpoint change cf. ST2012
5.16	SP2011	Θ_0	64	2	1	.005	$(1-.8z^{-1})$	0	setpoint change, P-wt cf.ST2013
	SP2012	Θ_0	64	2	1	.005	$(1-.5z^{-1})$	0	setpoint change, P-wt cf.ST2013
5.12	SP2013	Θ_0	64	2	1	.005	1	PI	PI type Q-wt stable and smooth
5.13	SP2014	Θ_0	64	2	1	.005	1	PID	PID type Q-wt cf. SP2013
	SP2015	Θ_0	64	2	1	.005	$(1+.5z^{-1})$	PI	setpoint and feed change
5.14	SP2016	Θ_2	64	2	1000	.005	1	PI	zero initial parameters and Q-wt
	SP2017	Θ_3	64	2	10	.005	1	0	effect of a(k) cf. SP2005
5.8	SP2018	Θ_1	64	2	1000	0.0	1	0	effect of a(k) and disturbance
	SP3001	Θ_3+0.0	64	3	1	.005	1	0	third order model cf. SP2003
5.4	SP3002	Θ_0+0.0	64	3	1	.005	1	0	third order cf. SP2009
	SP3003	Θ_0+0.0	64	3	1	.005	$(1-.5z^{-1})$	0	P-wt, feed change cf. SP2012
	SP3004	Θ_6+0.0	64	3	1	.005	$(1-.8z^{-1})$	0	P-wt, setpoint, cf. SP2011
5.15	SP3005	Θ_0+0.0	64	3	1	.005	1	PI	PI type Q-wt, cf. SP2013
	SP3006	Θ_0+0.0	64	3	1	.005	1	PID	PID type Q-wt, cf. SP2014
	SP3007	Θ_0+0.0	64	3	1	.005	$(1-.5z^{-1})$	PI	feed and setpt, smooth

Note (i):
Θ_0 =[.9775 -.000 .0664 .00027]
Θ_1 =[.00 .00 .05 .00]
Θ_2 =[.00 .00 1.00 .00]
Θ_3 =[.00 .00 .10 .00]
Θ_4 =[1.70 -.702 .0272 .01639]
Θ_5 =[0.9775 .0667]
Θ_6 =[0.9655 .00 .039 .076 .0764 .037]

$$PI = (1-z^{-1}) / (4.64-4.18z^{-1})$$
$$PID = (1-z^{-1}) / (10.88-15.48z^{-1}+5.42z^{-2})$$

Note (ii): Measurement noise is added to all runs.

last section of this chapter.

5.5.1 Model Order

The order of the adaptive predictive model of APCS determines the number of parameters to be estimated and the controller dynamics. The following simulation runs demonstrate the effect of model order on the closed loop evaporator dynamics and the control performance. Basically three different discrete models, first order, second order and third order, were tested. When the prediction model is assumed to be first order with no time delay the basic APCS control law is equivalent to a variable gain proportional feedback controller. Figure 5.2 is the evaporator response when the first order model, equation (3.2) was used. The input and output variables fluctuate unacceptably as time goes on mainly because the controller gain, $\theta_0(k)/\theta_1(k)$, is increasd as adaptation proceeds. Figure 5.3 shows the corresponding response when the second order model, equation (3.3), was used in which case the output and the input are stabilized compared to those of the first order case. In figure 5.4 a third order model was used to predict the evaporator dynamics and the initial parameters were chosen based on equation (3.3). When the model order was increased the output was not improved at all and manipulated variables were oscillatory due to the difficulty of higher order parameter estimtion. In this particular application second order appears to be the best predictive model for the

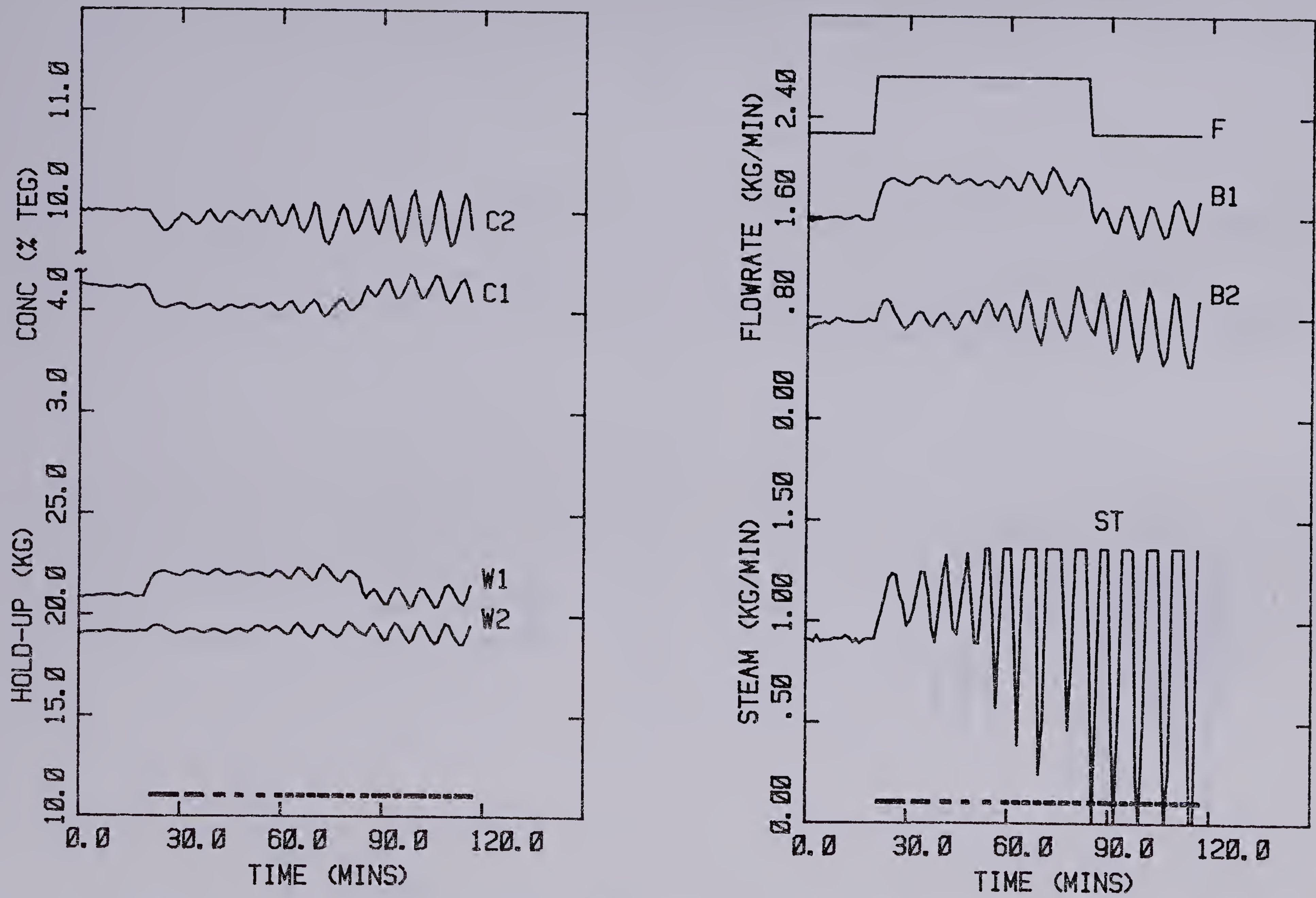


FIGURE 5.2 Simulated Evaporator Response Using APCS with Time Domain ID model (APCS/SP2007/ITDM/T64/M1/C1/D.005/P1/Q0/ 20%FD/ FIRST ORDER PREDICTIVE MODEL)

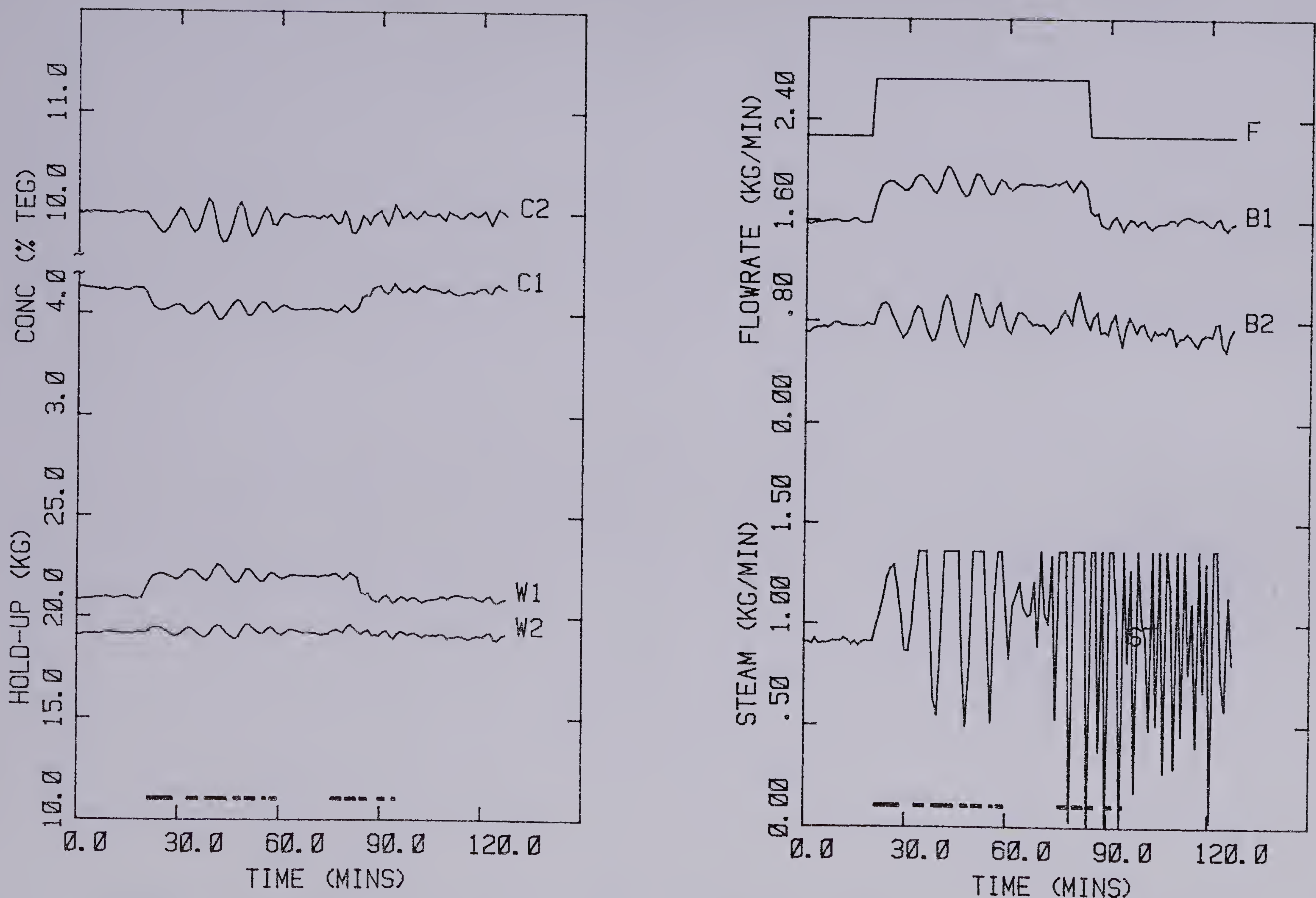


FIGURE 5.3 Simulated Evaporator Response Using APCS with Second Order Model
(APCS/SP2009/ITDM/T64/M2/C1/D.005/P1/Q0/ 20%FD/ SECOND ORDER PREDICTIVE MODEL)

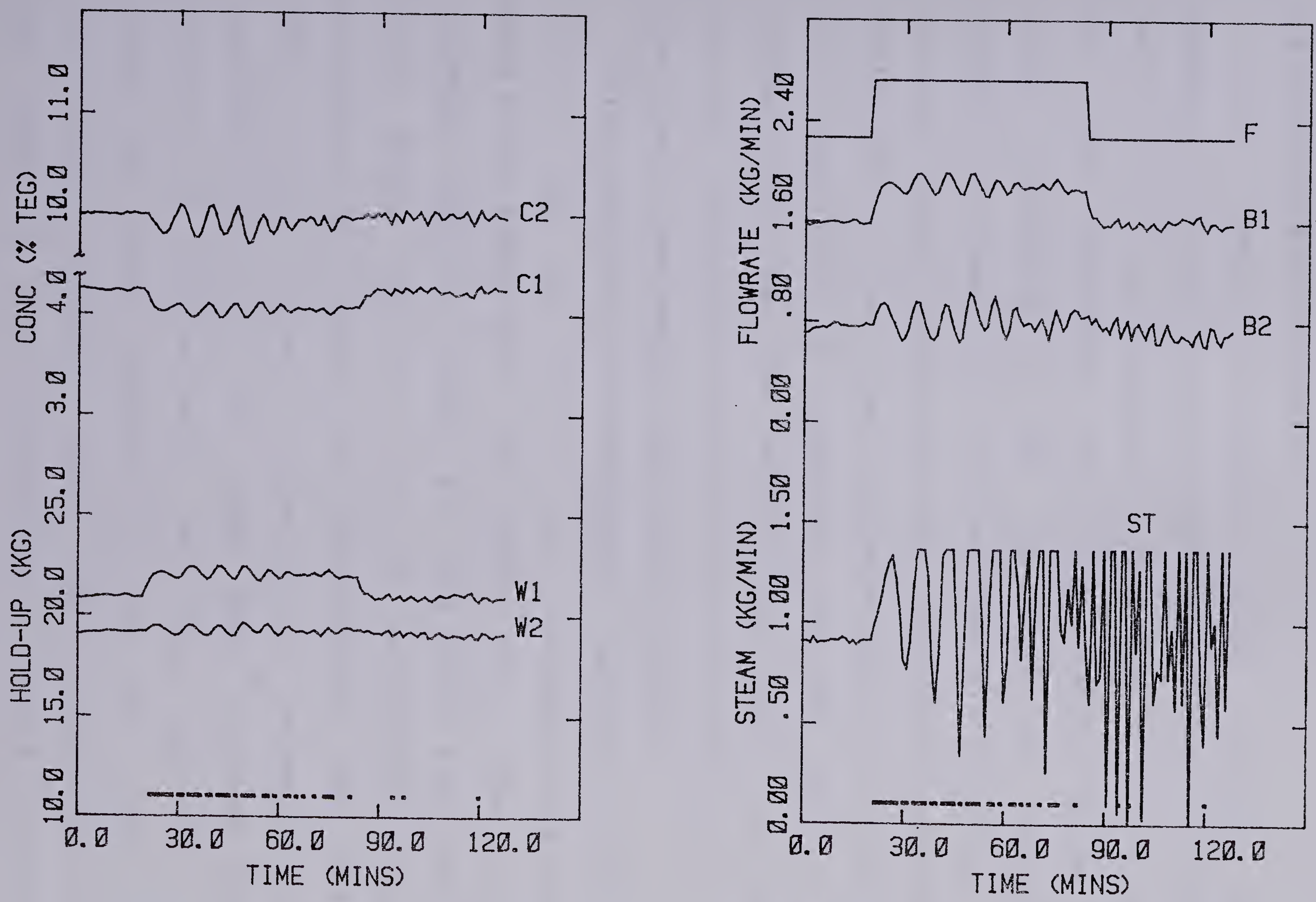


FIGURE 5.4 Simulated Evaporator Response Using APCS with Third Order Model (APCS/SP3ØØ2/ITDM/T64/M3/C1/D.ØØ5/P1/QØ/ 2Ø%FD/ THIRD ORDER PREDICTIVE MODEL)

evaporator control.

5.5.2 Initial Model Parameters

For a completely unknown process, initial model parameters for APCS are often set to zero except the leading coefficient of the polynomial corresponding to the input. The control performance may be unsatisfactory because the control calculation of APCS is based on uncertain parameters. Figure 5.5 is one of those examples where the input and output variables are very oscillatory. The leading coefficient of the input polynomial acts as a controller gain in the control law calculation of APCS and it was observed that when the coefficient got smaller, the input and output variables became more oscillatory. Note that the true value of the leading coefficient is 0.014.

Since the zero initial parameters resulted in excessive I/O variation, the initial model parameters were chosen based on the models given in chapter three. The initial parameters used in Figure 5.6 were chosen from the time series model, equation (3.5), and satisfactory output performance was achieved but the controller used too much control effort. In Figure 5.7 the initial parameters were calculated from the first order model with time delay, equation (3.3), and the control was worse than when the second order model, equation (3.4), was used (Figure 5.3). From the above simulation results it may be concluded that a good choice of start-up parameters is helpful in reducing

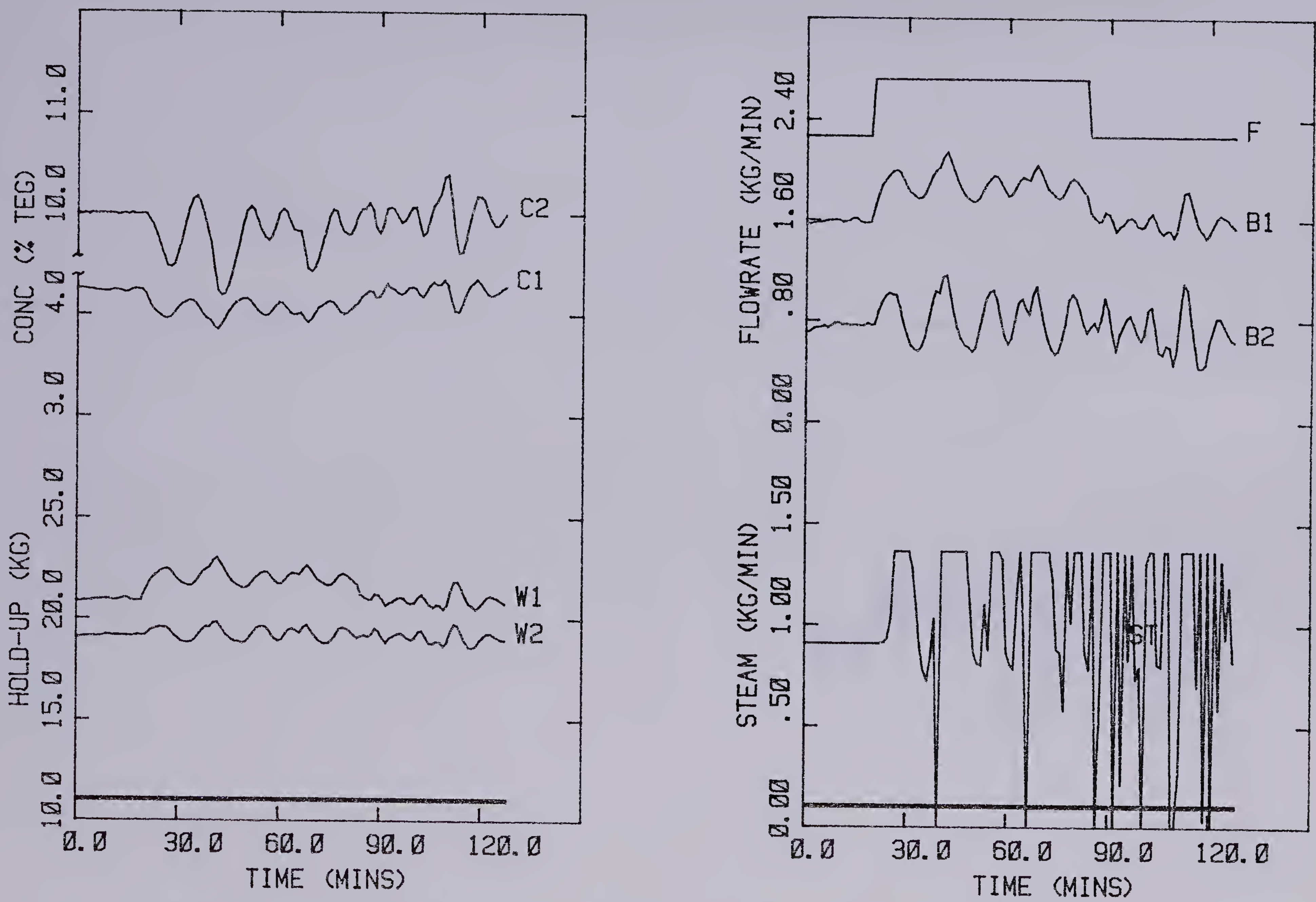


FIGURE 5.5 Simulated Evaporator Response by APCS with Zero Initial Parameters
(APCS/SP2002/I0/T64/M2/C1000/D0/P1/Q0/ 20%FD ZERO INITIAL PARAMETERS)

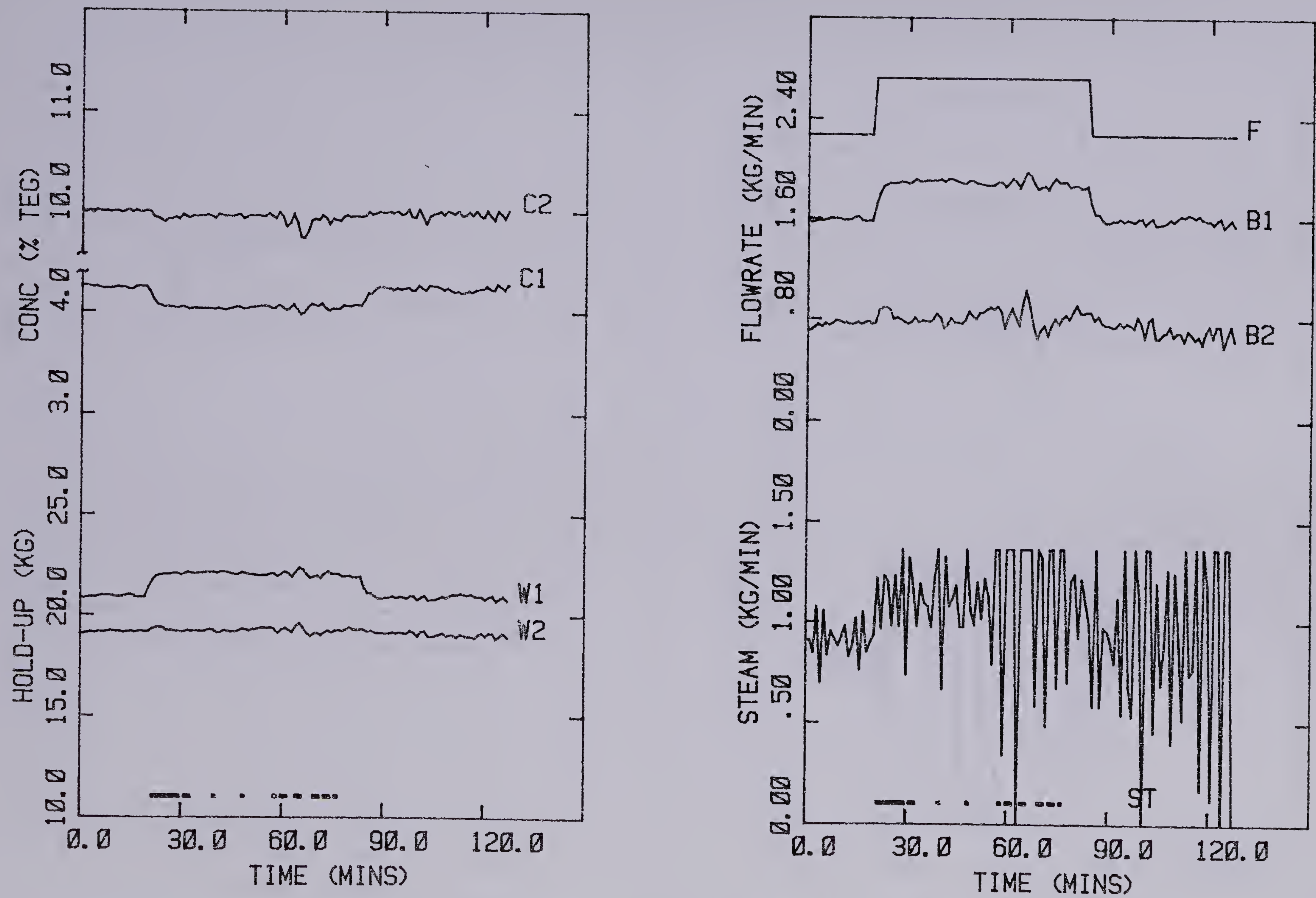


FIGURE 5.6 Simulated Evaporator Response Using APCS with Time Series Model
(APCS/SP2006/ITSM/T64/M2/C1/D.005/P1/Q0/ 20%FD/ TIME SERIES MODEL PARAMETERS)

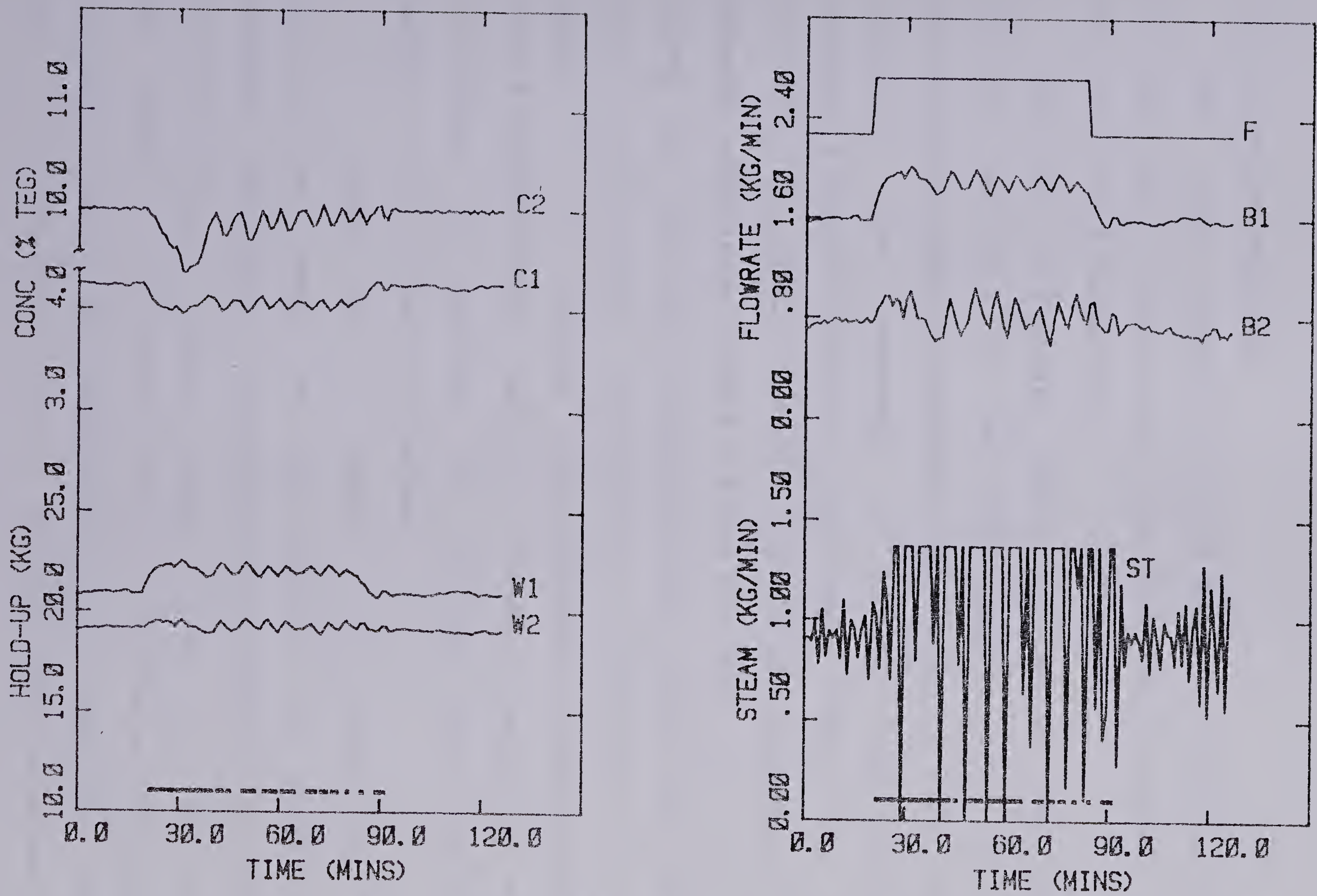


FIGURE 5.7 Simulated Evaporator Response by APCS with 1st Order Timedelay Model
(APCS/SP2008/ITDM/T64/M1+d/C1/D.005/P1/Q0/ 20%FD/INITIAL PARAMETER)

the output deviation of the evaporator but fails to smooth out the manipulated variable.

5.5.3 Adaptive Mechanism

Here, some properties of APCS adaptive mechanism will be illustrated by simulations using different values of the error correcting factor a(k).

The error correcting factor is one of the most important variables in the APCS adaptive mechanism. It determines the speed of paremeter adaptation and also stops adaptation when necessary. In general, when initial parameter estimates are poor a large upper limit, a_{i1}, on the error correcting factor is preferred to produce fast parameter adaptation. Figure 5.8 and 5.9 show two extreme choices of the error correcting factor. These show that a large a_{i1} gives fast convergence of parameters for the output, A's (cf. Figure 5.10). However, it also strongly affects parameters for the input, θ's which results in very oscillatory I/O variables (not plotted) due to the fast changes in the parameter estimates.

As pointed out in section 5.3.4, when the I/O variations are small the APCS adaptive law moves the parameter estimates in the same direction whether the error correcting factor is large or small (Figure 5.8 and 5.9). This results in slow identification of system parameters so the final values of the parameters in this simulation were poor. In Figures 5.10 and 5.11 the APCS adaptive scheme is

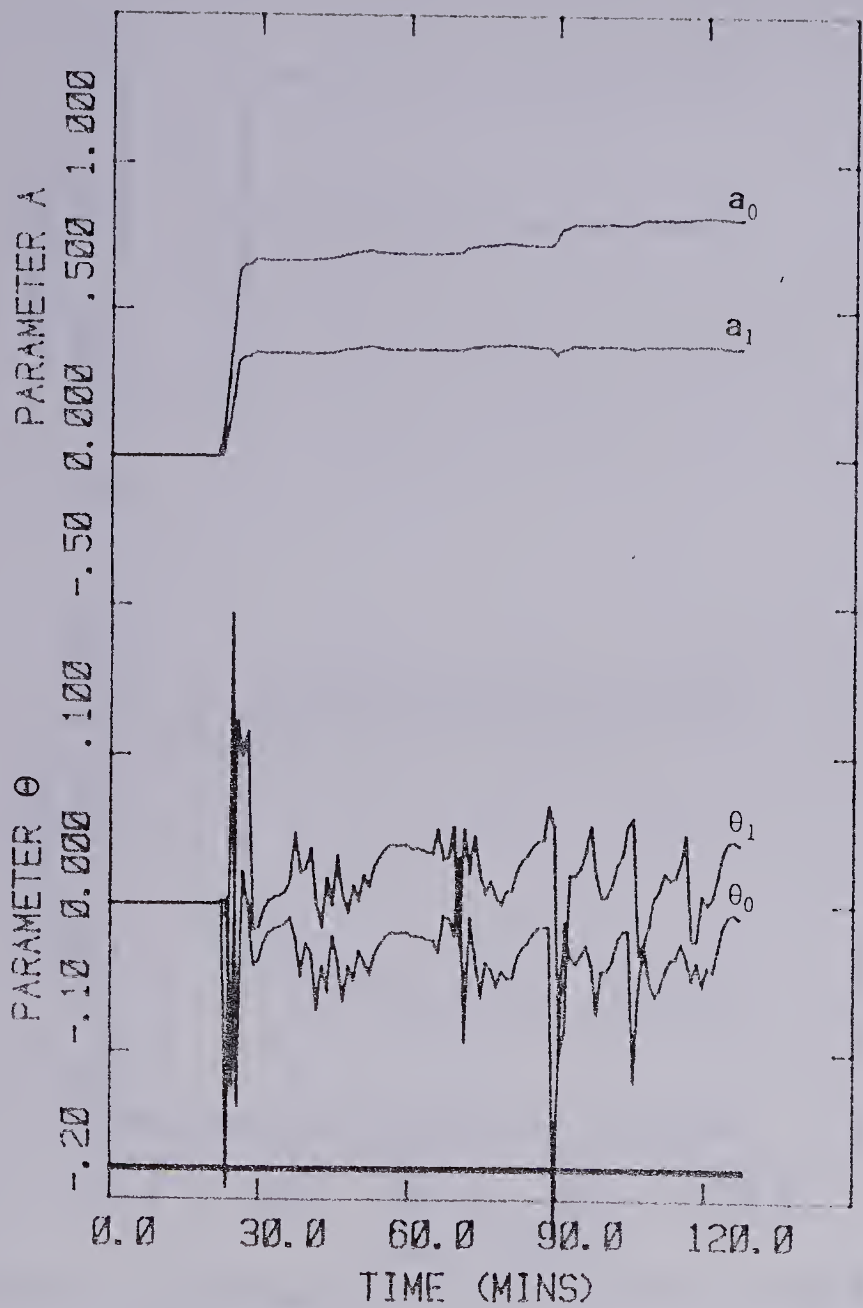


FIGURE 5.8 Parameter Convergence of APCS with a_1=1000 and Zero Initial Parameters (θ_1=.05)

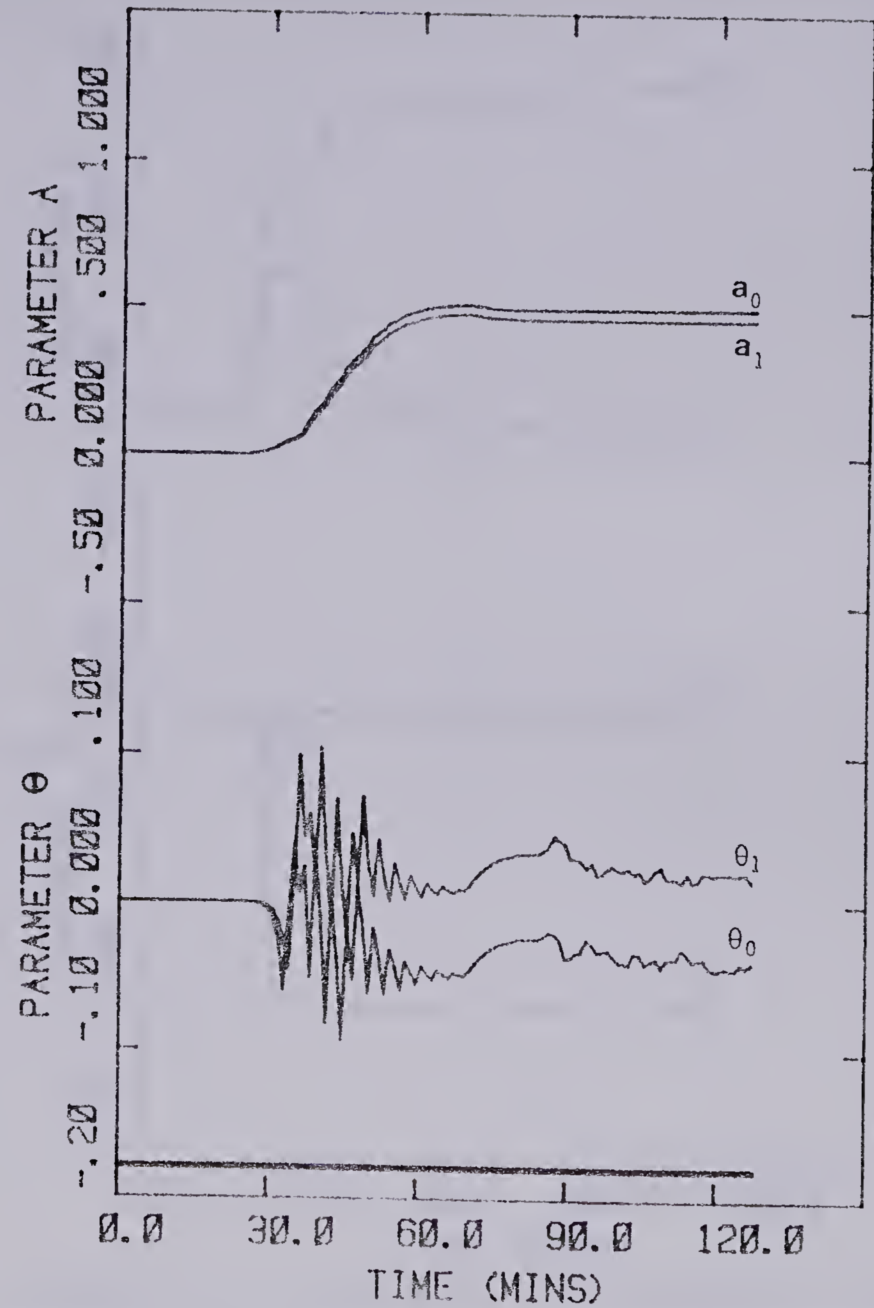


FIGURE 5.9 Parameter Convergence of APCS with a =1 and Zero Initial Parameters (θ_1=.05)

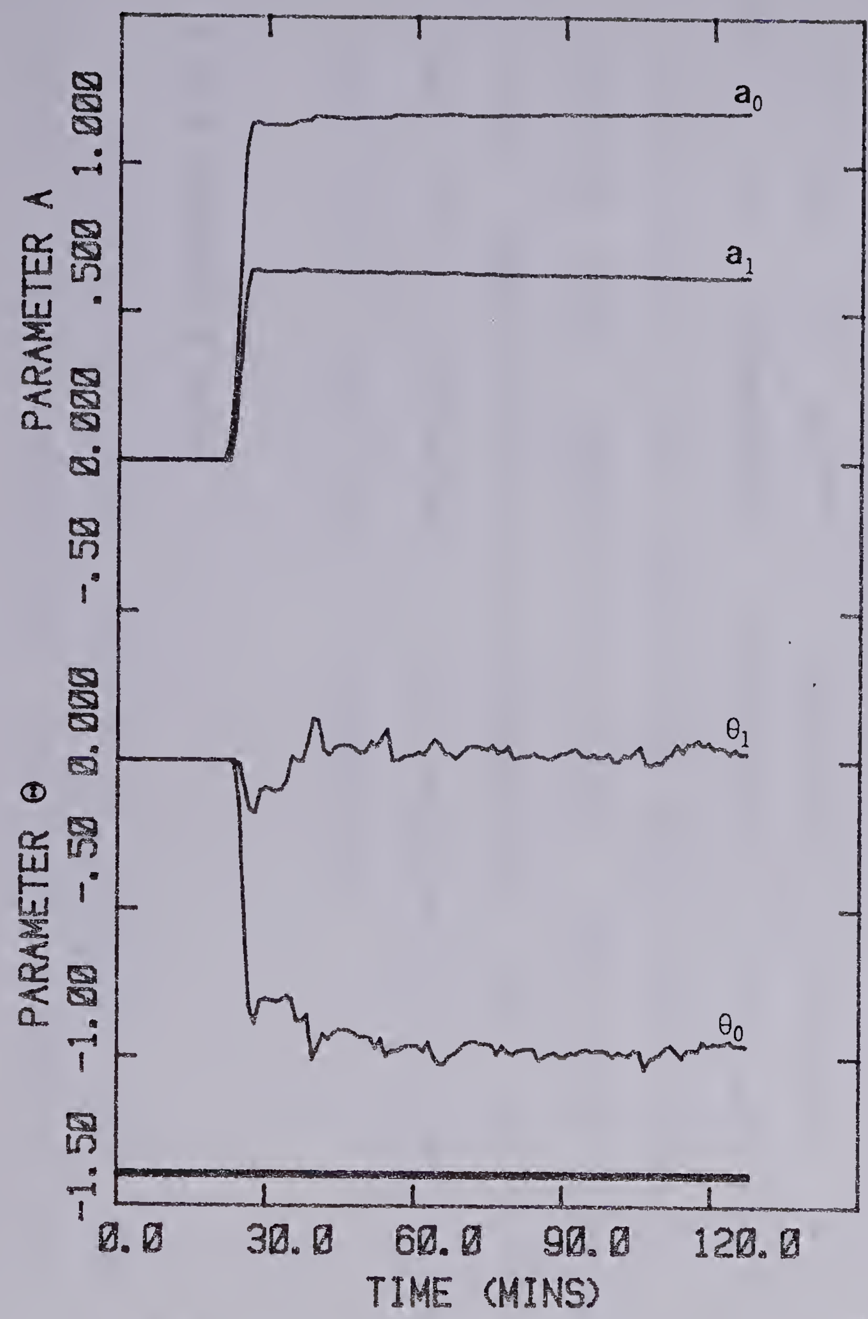


FIGURE 5.10 Parameter Convergence of APCS with a =1000 and Zero Initial Parameters (θ_0=1.0)

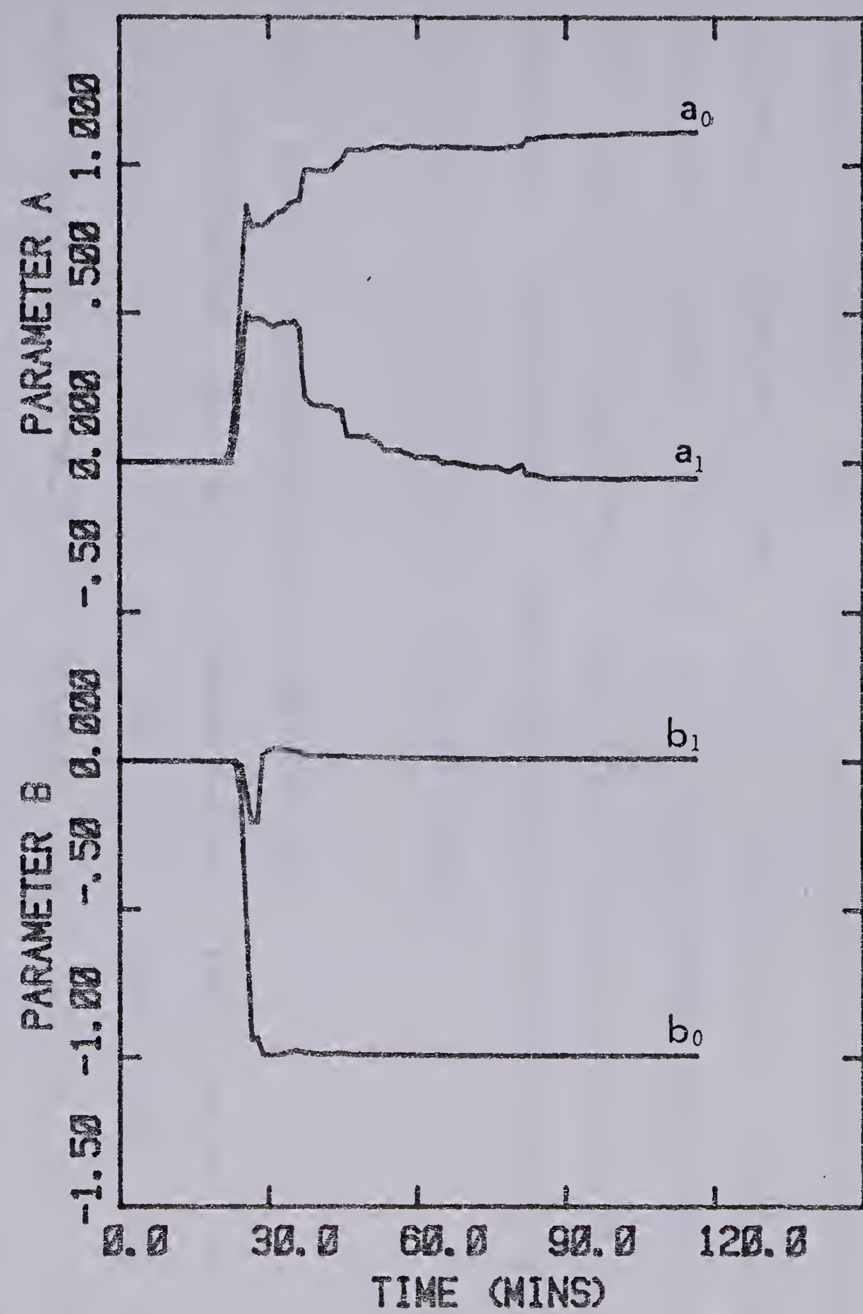


FIGURE 5.11 Parameter Convergence of RLS with cov=1000 and Zero Initial Parameters (b_0=1.0)

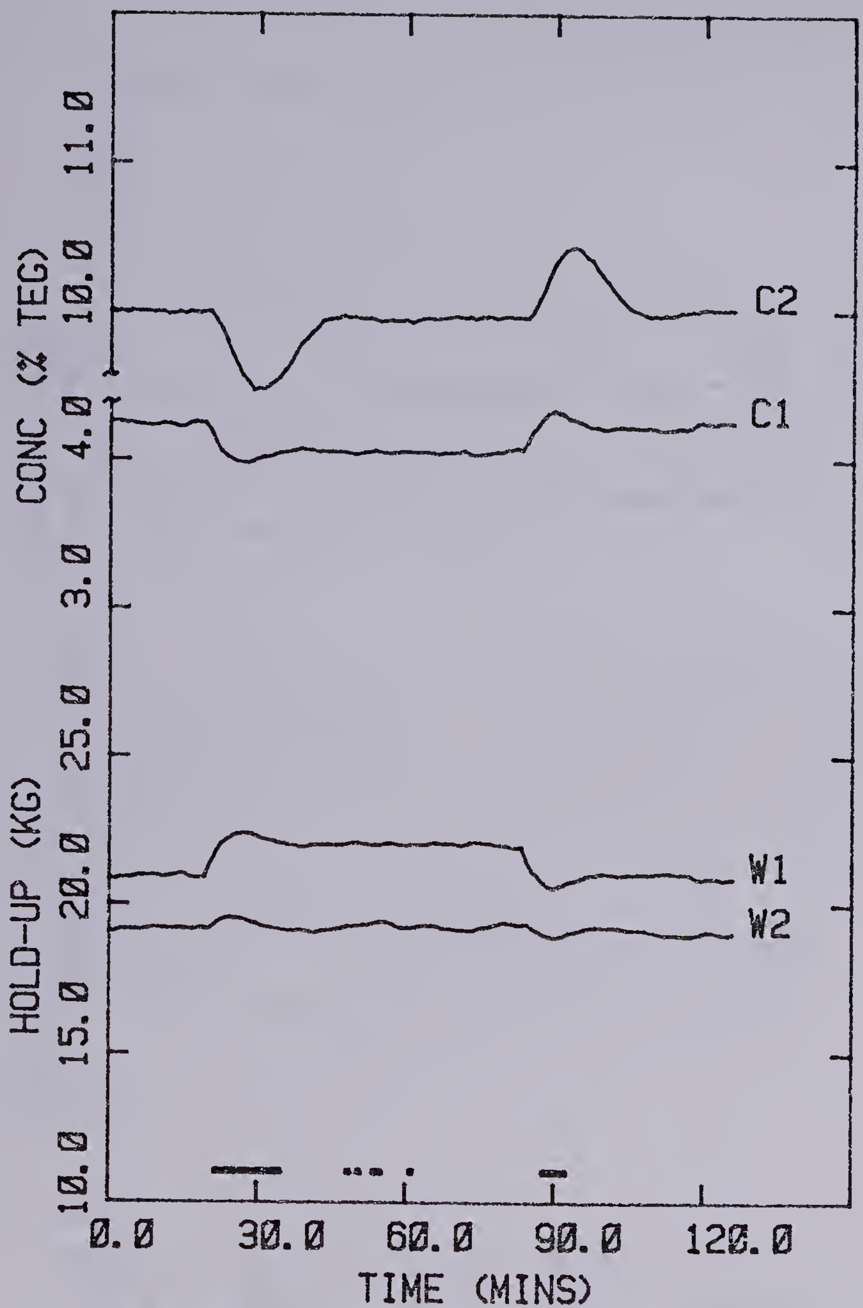
compared with the RLS method. Both methods gave fast convergence of the coefficients corresponding to the input dynamics due to the large variations in the manipulated variable. However, the output polynomial coefficients converged closer to the true values when RLS rather than APCS adaptation was used. Note that the true value of a_1 is negative. Additional comparisons of the APCS adaptive law with RLS are presented in chapter seven.

5.5.4 Bound on Unmeasured Disturbance

The bound, Δ_d, on the unmeasured disturbance variable determines the range of the adaptation dead zone. The ideal value would be the minimal upper bound which is usually unknown. The effect of Δ_d on the control performance was examined under the assumption that it was unknown. When Δ_d is set to zero (cf. Figure 5.9, SP2003, SP2004 (not plotted) and Figure 5.8) parameter adaptation proceeds all the time but the corresponding performance is the same as the case when Δ_d set to 0.005 (SP2004 and SP2005 (not plotted)) which required less computation effort. Note that the actual minimal upper bound on the unmeasured noise is 0.004 and that setting the bound to zero represents a violation of the (sufficient) conditions for stability.

5.5.5 Weighted APCS

All the previous simulation results show that the desired output performance can be achieved but only with



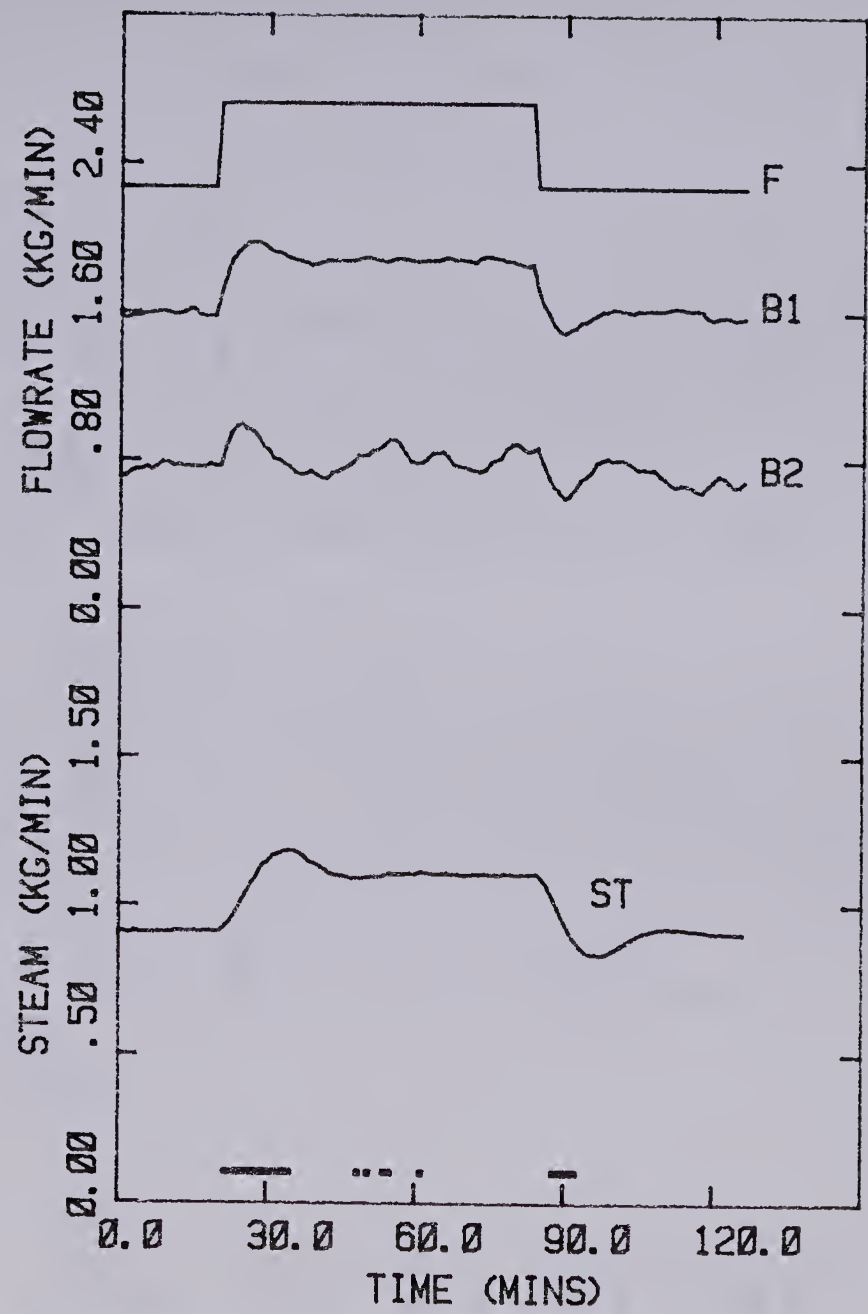


FIGURE 5.12 Simulated Evaporator Response Using APCS with PI type Q-Weighting (APCS/SP2013/ITDM/T64/M2/C1/D.005/P1/Q PI/ 20%FD/ SMOOTH CONTROL EFFORT)

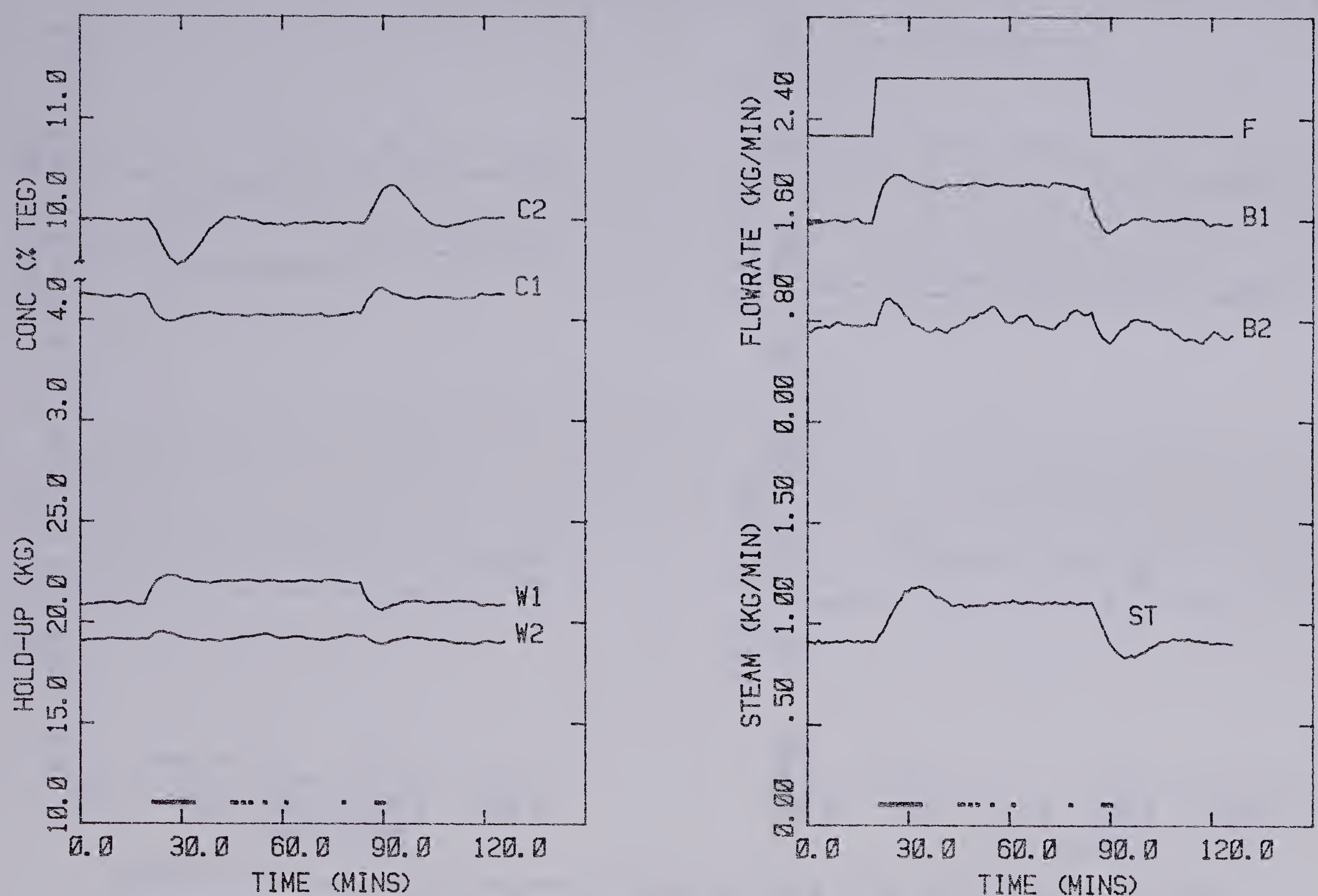


FIGURE 5.13 Simulated Evaporator Response Using APCS with PID Type Q-Weighting (APCS/SP2014/ITDM/T64/M2/C1/D.005/P1/Q PID/ 20%FD/ WEIGHT ON CONTROL ACTION)

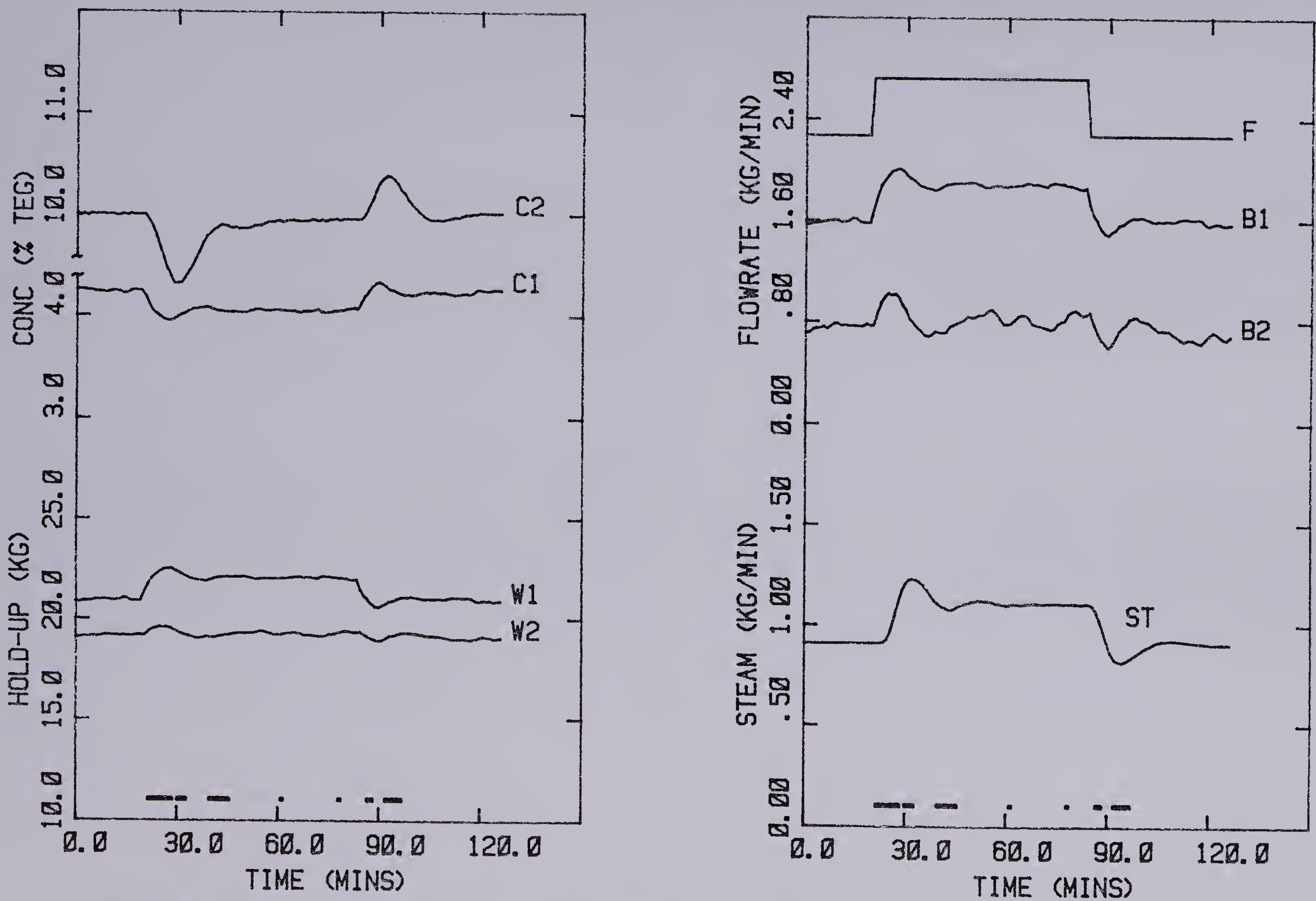


FIGURE 5.14 Simulated Evaporator Response Using APCS with PI Q-Wt (b=1.0)
(APCS/SP2016/I0/T64/M2/C1000/D.005/P1/Q PI/ 20%FD/ Q-WT AND ZERO INITIAL)

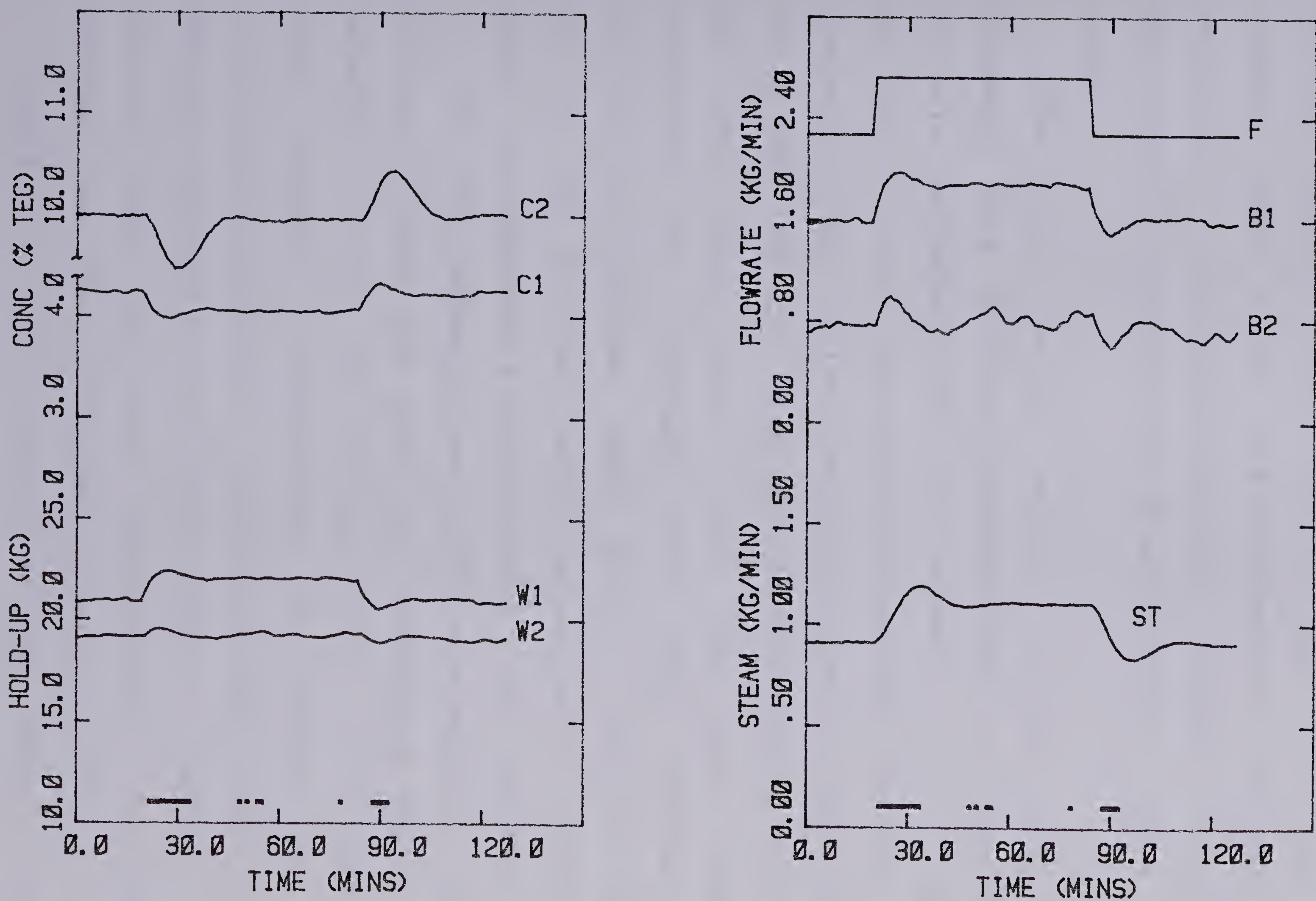


FIGURE 5.15 Simulated Evaporator Response by APCS with PI Q-Wt and Third Order (APCS/SP3005/ITDM/T64/M3/C1/D.005/P1/Q PID/ 20%FD/ Q-WT AND THIRD ORDER MODEL)

control signals which are unacceptably large from an application point of view. Similar results were observed in the STR simulation study, but it was noted that the excessive control action could be successfully eliminated by penalizing the manipulated variable. Therefore the weighed APCS introduced in section 5.3.3 was applied to solve the problem.

Q-weighting : The $Q(z^{-1})$ weighting function can be any polynomial form but here, was restricted to the same cases that were considered in STC, i.e. constant, pure integral form and PI or PID type weighting functions. As in the STC simulation study, constant Q-weighting gave an offset when P and R were set to unity. This follows directly from the performance index equation (5.22). When pure integral Q-weighting was introduced the output response was oscillatory for the same reason given in section 4.6.2. Thus, PI and PID type Q-weightings were mainly considered and for the comparison with the results of STC, the same PI and PID design parameters used in STC were employed. Figure 5.12 and 5.13 show the effect of Q-weighting which are comparable to the corresponding results of STC shown in Figure 4.16 and 4.17 respectively. In contrast with the non-Q-weighting presented in the previous section both cases remarkably smoothed out the control signal in addition to improving control performance. No difference in the control performance was observed whether the weighting function was PI (Figure 5.12) or PID (Figure 5.13) type. Thus, in Figure

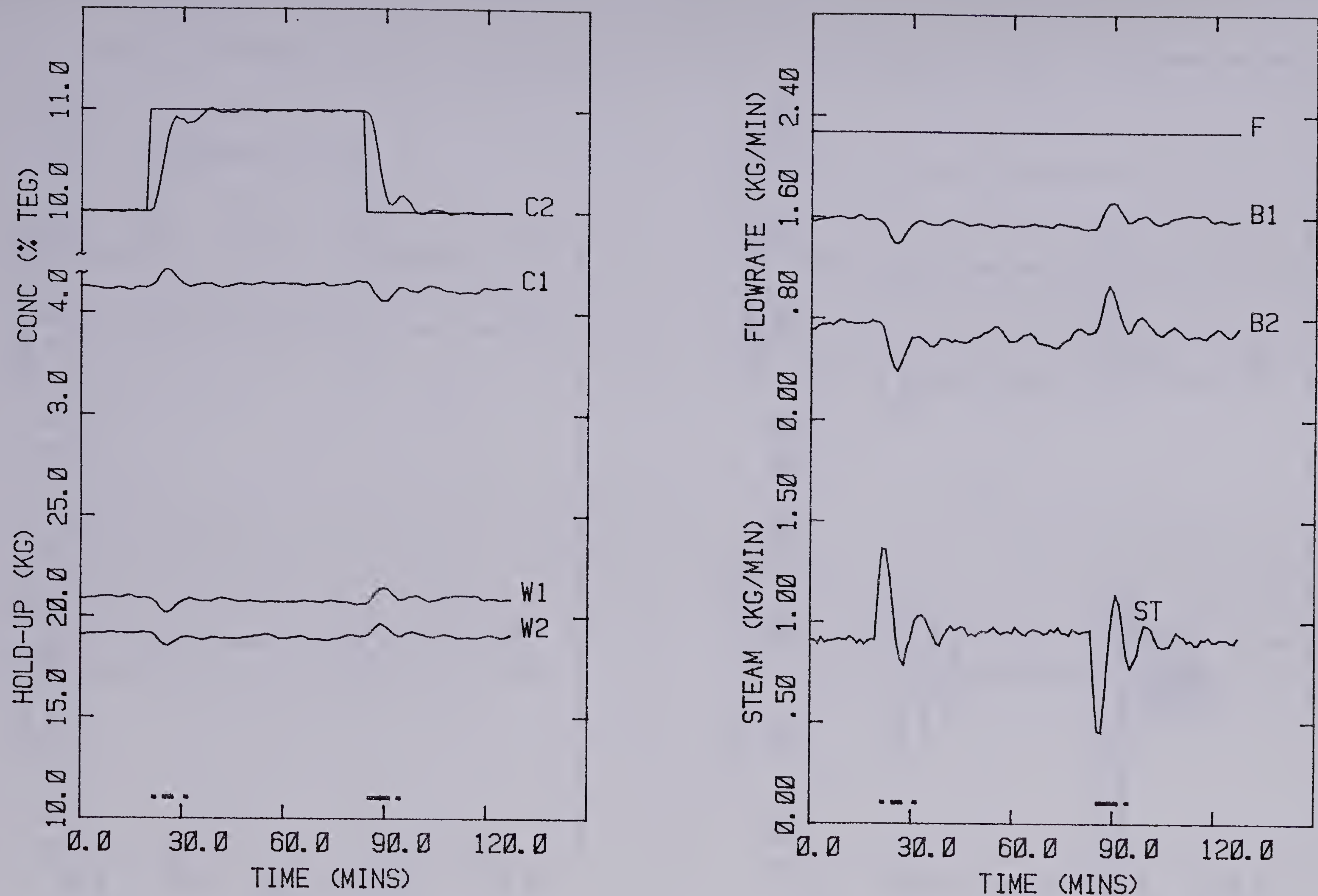


FIGURE 5.16 Simulated Evaporator Response BY APCS With P-Wt to Setpoint changes
(APCS/SP2011/ITDM/T64/M2/C1/D.005/P(1-.8z)/Q0/ 10%SP/ EFFECT OF P-WEIGHTING)

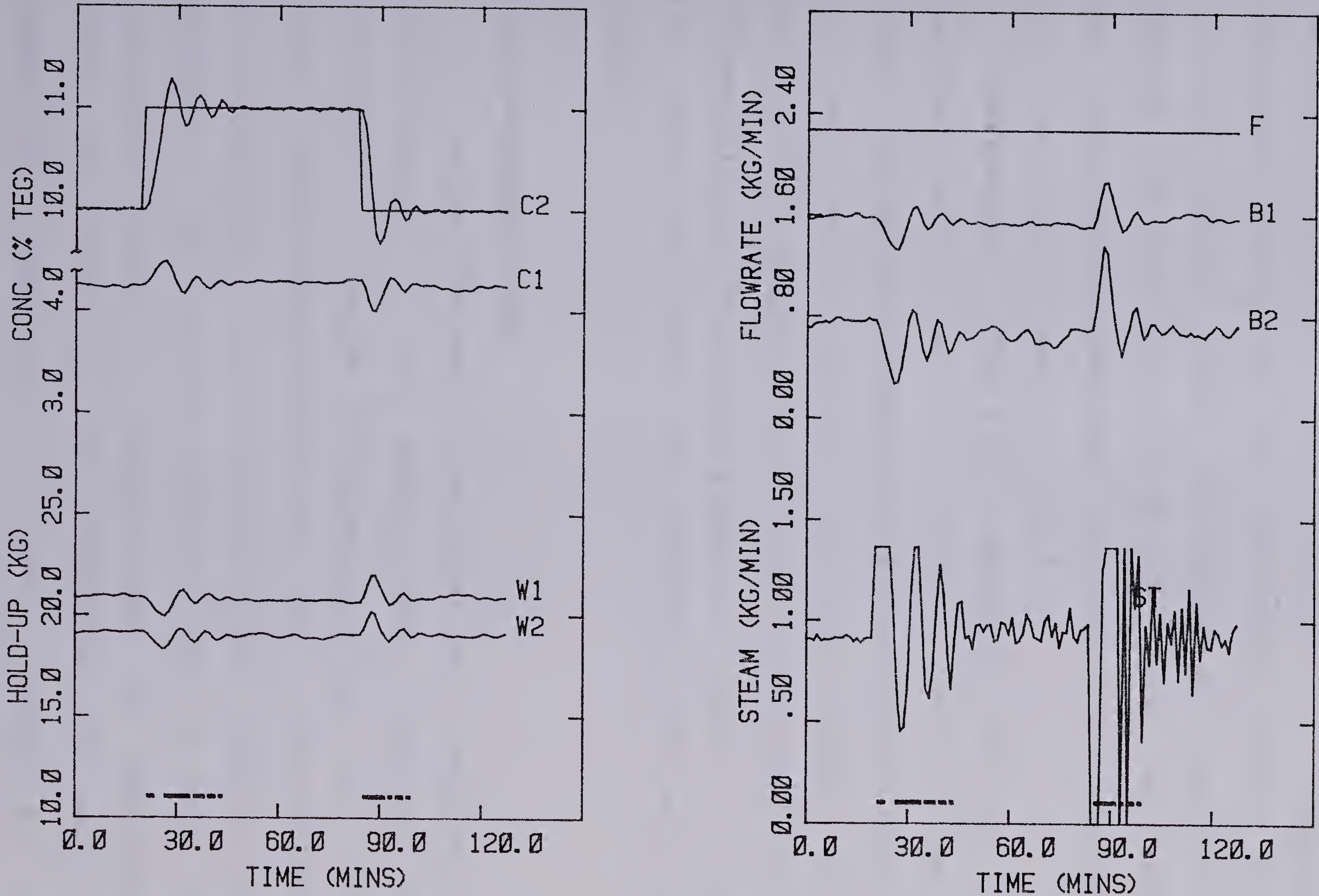


FIGURE 5.17 Simulated Evaporator Response Using APCS to Setpoint Changes (APCS/SP2010/ITDM/T64/M2/C1/D.005/P1/Q0/ 10%SP/ cf. P-WT IN FIGURE 5.16)

5.14 PI type Q-weighting was also applied to the zero initial parameter case represented in Figure 5.5. Although the initial deviation in C2 of Figure 5.14 is larger than the case initialized with well identified parameters (Figure 5.13) the desired output performance was obtained with moderate input variations. Figure 5.15 shows the response when the predictive model was third order and it is similar to the second order case in Figure 5.12.

P-weighting : The $P(z^{-1})$ weighting function, which could be interpreted as a driver block in the sense that it filters the setpoint change given by the operator, was used for a servo control simulation. One simple example, Figure 5.16, clearly demonstrated the effect of P-weighting. The corresponding non-P-weighting example is shown in Figure 5.17. Note particularly the difference in the manipulated variable.

5.6 Experimental study

In the previous section the properties of APCS and the characteristics of the double effect evaporator were investigated in a series of simulation studies. The results obtained from these simulations were used in the design of a set of experimental runs on the pilot plant, double effect evaporator. This section presents some of these APCS experimental results and compares them with experimental results from STR/C and with conventional, PID control. A complete list of experimental runs is in Table 5.2 (cf.

Table 5.2 List of Experimental Runs Using APCS

Figure No.	Run No.	Intial $\theta(0)$	Ts (sec)	Model order	a(k) (upper)	Noise Bound	P wt	Q wt	Comments
5.19	RP2001	θ_4	64	2	0.1	.005	1	0	basic APCS cf. RP2002
7.3	RP2002	θ_4	180	2	0.1	.005	1	0	basic APCS and sampling time
5.18	RP2003	θ_0	64	2	0.1	.005	1	0	initial parameter eq (3.3)
5.20	RP2004	θ_0	64	2	0.1	.005	$(1-z^{-1})$	0	Incremental form of APCS
5.23	RP2005	θ_6	64	1	1	.005	1	PI	1st order model, PI type Q-wt
5.24	RP2006	θ_4	64	2	1	.005	1	PI	TSM and PI type Q-wt cf. RT2015
5.21	RP2007	θ_0	64	2	1	.005	1	PI	TDM and PI type Q-wt cf. RT2024
5.25	RP2008	θ_0	64	2	1	.005	1	PI	setpoint change, TDM model
	RP2009	θ_4	64	2	1	.005	1	0	effect of a(k) cf. RP2001
5.22	RP3001	θ_0+0.0	64	3	1	.005	1	PI	third order model cf. RP2007

Note: θ_0 =[.9775 -.000 .0664 .00027]
θ_4 =[1.70 -.702 .0272 .01639]
θ_6 =[0.9655 .00 .039 .076 .0764 .037]

$PI = (1-z^{-1}) / (3.10-2.90z^{-1})$

Table 4.2 and 6.2).

5.6.1 Basic APCS

As noted in the simulation study, concentration control of the evaporator using the basic APCS algorithm results in severely fluctuating input variations which make the closed loop system very oscillatory because of the interactions between the evaporator variables. It was found over a period of several weeks that the evaporator response was very hard to stabilize by changing values for sampling time, model order, initial model parameters and/or the design parameters of the adaptive law such as the error correcting factor. The initial model parameters were the most important ones to choose properly to eliminate the extra fluctuation due to the uncertainty in the control parameters. Several different initial values were calculated based on the well identified models given in chapter three, i.e.

1) Time domain curve-fitted models (equations 3.3, 3.4)
2) Time series model (equation 3.5)
3) fifth order state space model (equation 3.6)

However, no matter what model was used as a basis for choosing the initial values, the basic APCS scheme resulted in unstable, oscillatory control performance mainly due to the large controller gain. In fact the control performance was very similar to the performance obtained by STR. Figure

5.18 shows one of the examples. This example is comparable with Figure 4.20 in the case of STR.

Since the main reason for this unstable, oscillatory response was the small value of the leading coefficient of the input polynomial, this value was artificially increased from 0.0272 (cf. equation (3.5)) to 0.2. As can be seen from Figure 5.19 all the evaporator variables are stabilized with an offset in the product concentration. The offset decreases as the APCS parameter estimation proceeds. In other words, as the estimation goes on the leading coefficient gets smaller which means the controller gain gets larger and larger and the oscillatoin problem arises again. Therefore, increasing the leading coefficient is not a satisfactory solution.

In many applications the incremental form of APCS is helpful in eliminating offset. An incremental form of APCS which contains integral action was used to eliminate the offset in the final product concentration in Figure 5.20. However, the control result also ended up with unstable oscillations due to high controller gains.

5.6.2 Weighted APCS

The previous experimental applications of the basic APCS on the double effect evaporator showed that the closed loop response could be stabilized if the control action were reduced sufficiently. This conclusion is the same as obtained from the APCS simulation studies and also from the

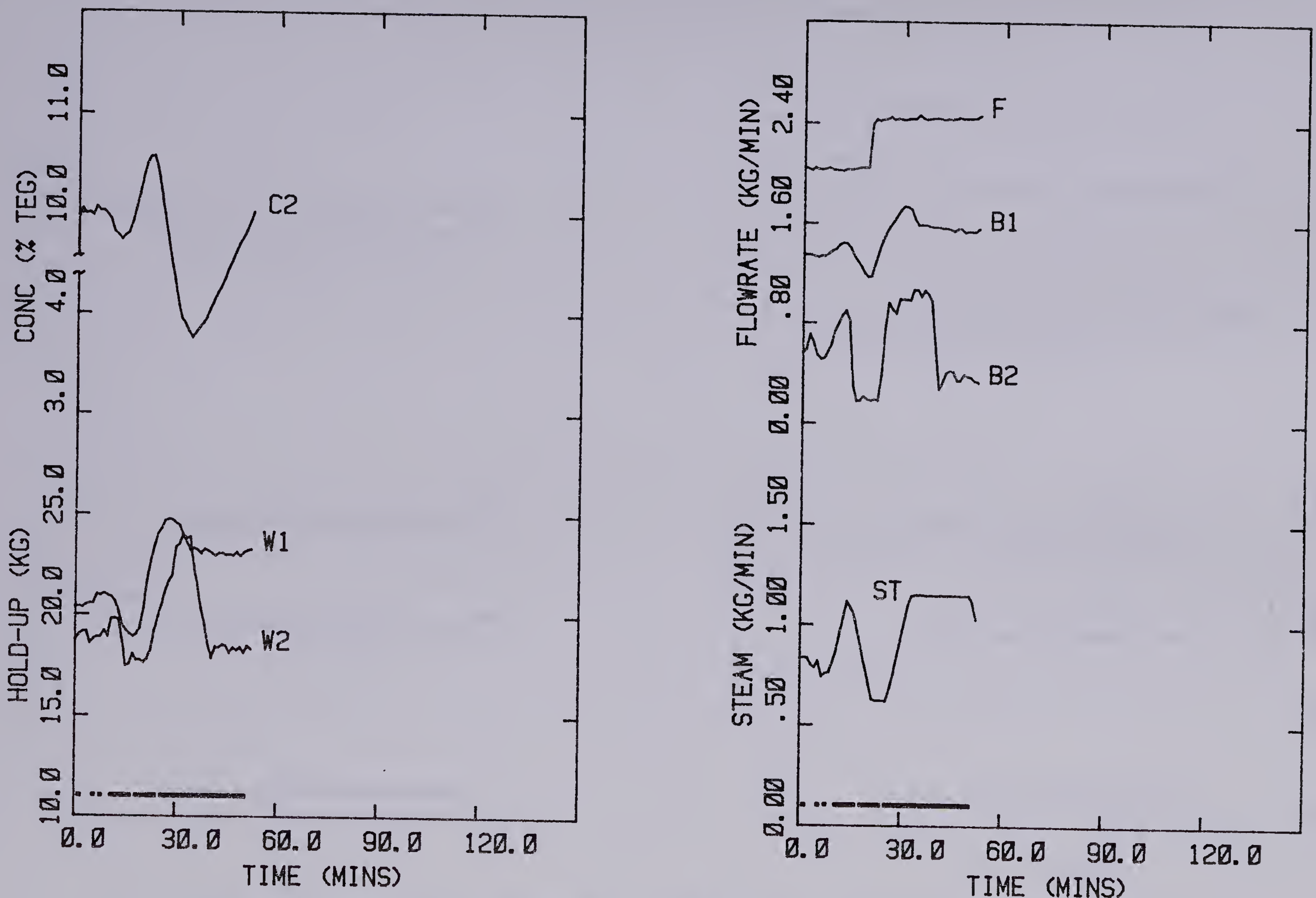


FIGURE 5.18 Evaporator Response Using APCS with Second Identified Model
(APCS/RP2003/ITDM/T64/M2/C.1/D.005/P1/Q0/ 20%FD/ INITIAL PARAMETERS)

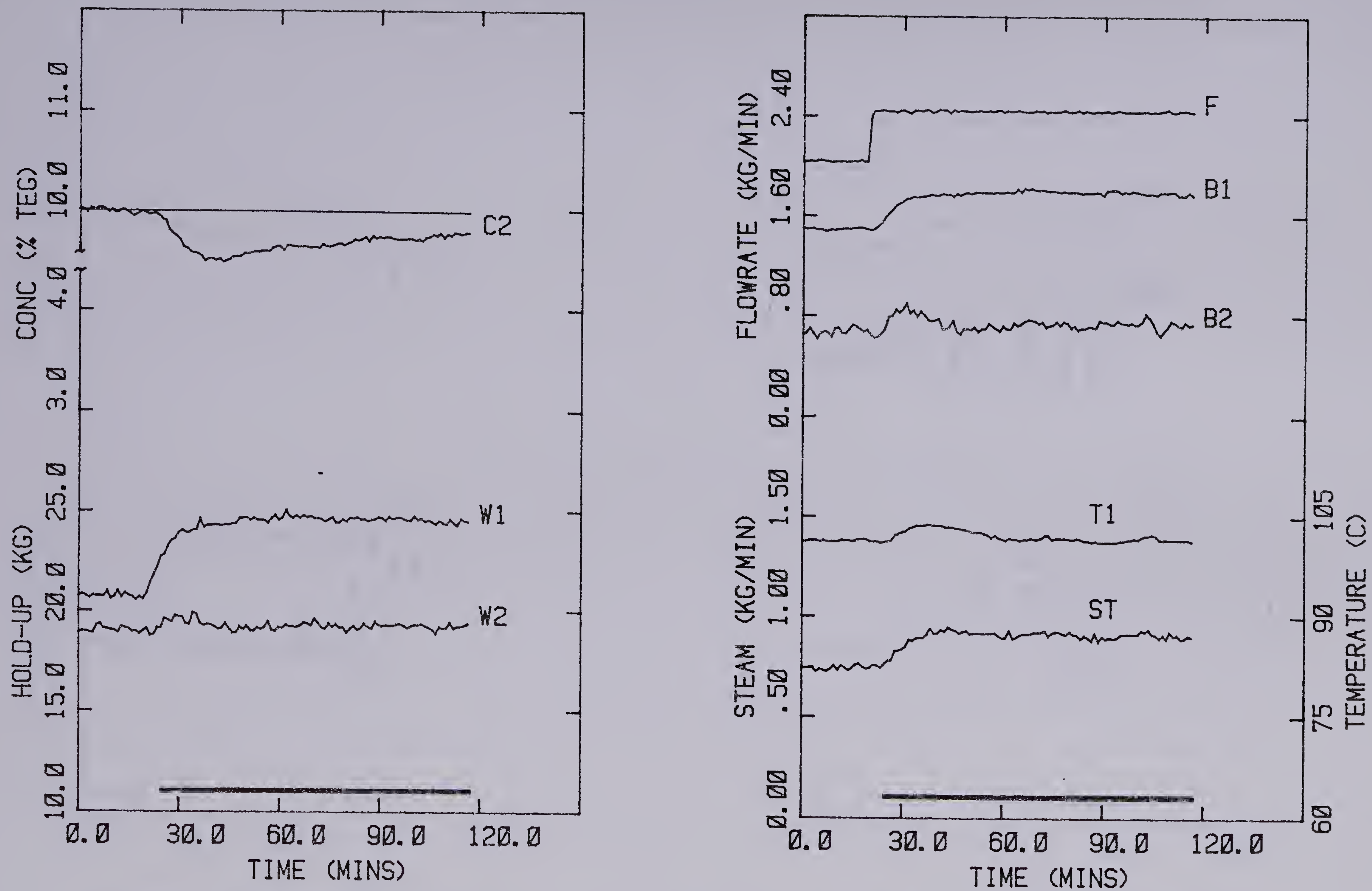


FIGURE 5.19 Evaporator Response by APCS with Time Series Model (bØ=.2)
(APCS/RP2001/ITSM/T64/M2/C.1/D.005/P1/Q0/ 20%FD/ bØ cf. RT2002)

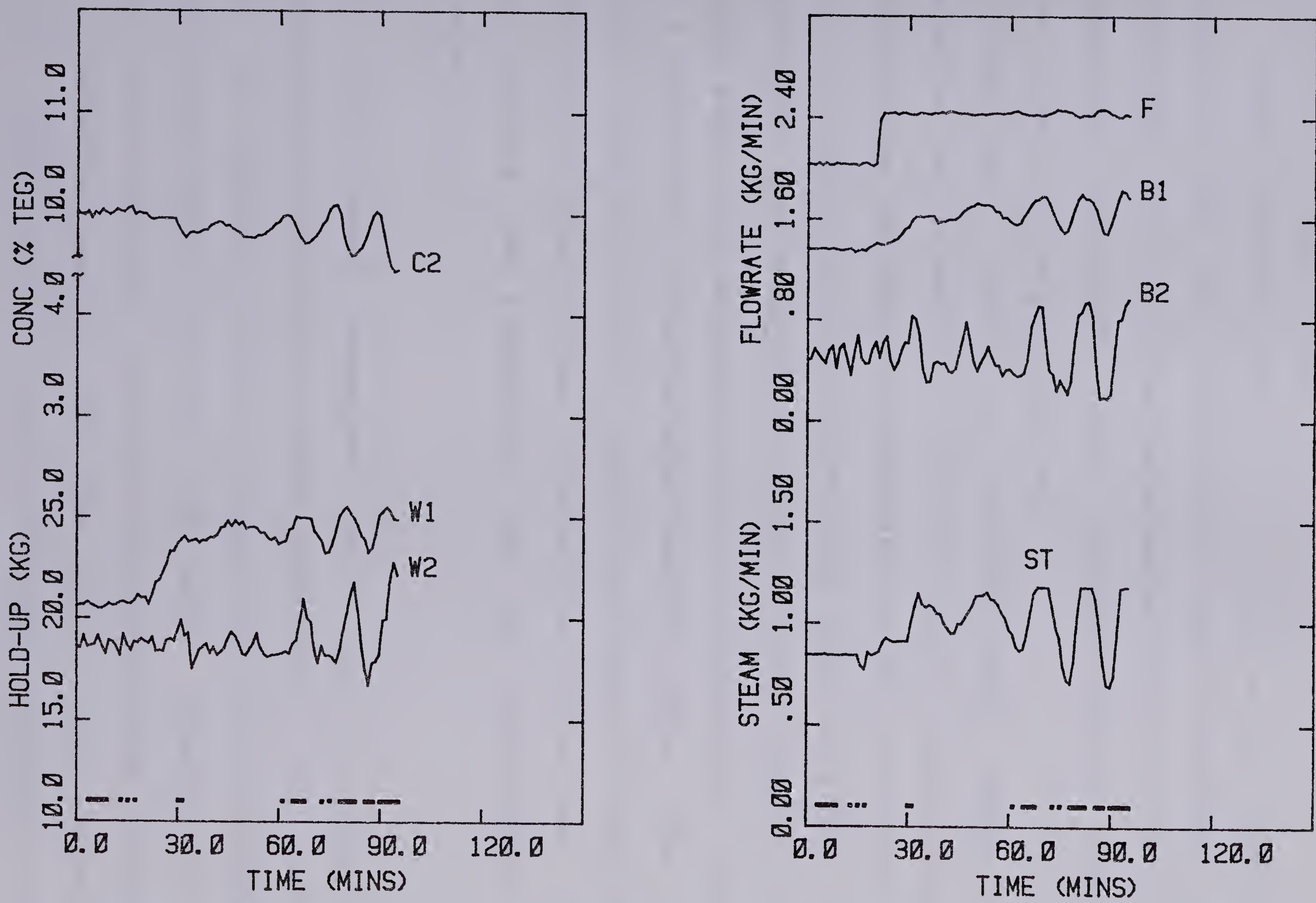


FIGURE 5.20 Evaporator Response Using Incremental APCS with Second Order Model
(APCS/RP2004/ITDM/T64/M2/C.1/D.005/P(1-Z)/Q0/ 20%FD/ INCREMENTAL APCS)

STC experimental runs. Thus, the weighted APCS introduced in section 5.3.3 was used to moderate the excessive control action of the basic APCS.

Since the main concern of this work was the regulatory control of the final concentration only Q-weighting was considered in the actual experimental runs. To facilitate comparison with the STC, the PI type Q-weighting function was chosen for the all APCS runs.

1) **Model order** : First of all, different model orders were examined even though the simulation results showed that the second order was the most preferrable. Figure 5.21 shows the control results using the second order model and Figure 5.22 represents the corresponding results obtained from the third order model. This experiment verifies the simulation result, i.e. shows that the second order model is better than the third order as far as the control performance is concerned. Note that in general higher order models require more time to estimate the parameters. They may perform better in longer runs.

2) **Initial Model Parameters** : The PI type Q-weighting was also applied with different model parameters to the control of the evaporator. Figure 5.23 uses the initial parameters calculated based on the first order model, equation (3.3), and Figure 5.24 shows the control results obtained using the initial values based on the time series model, equation (3.5). These control results were quite comparable to the

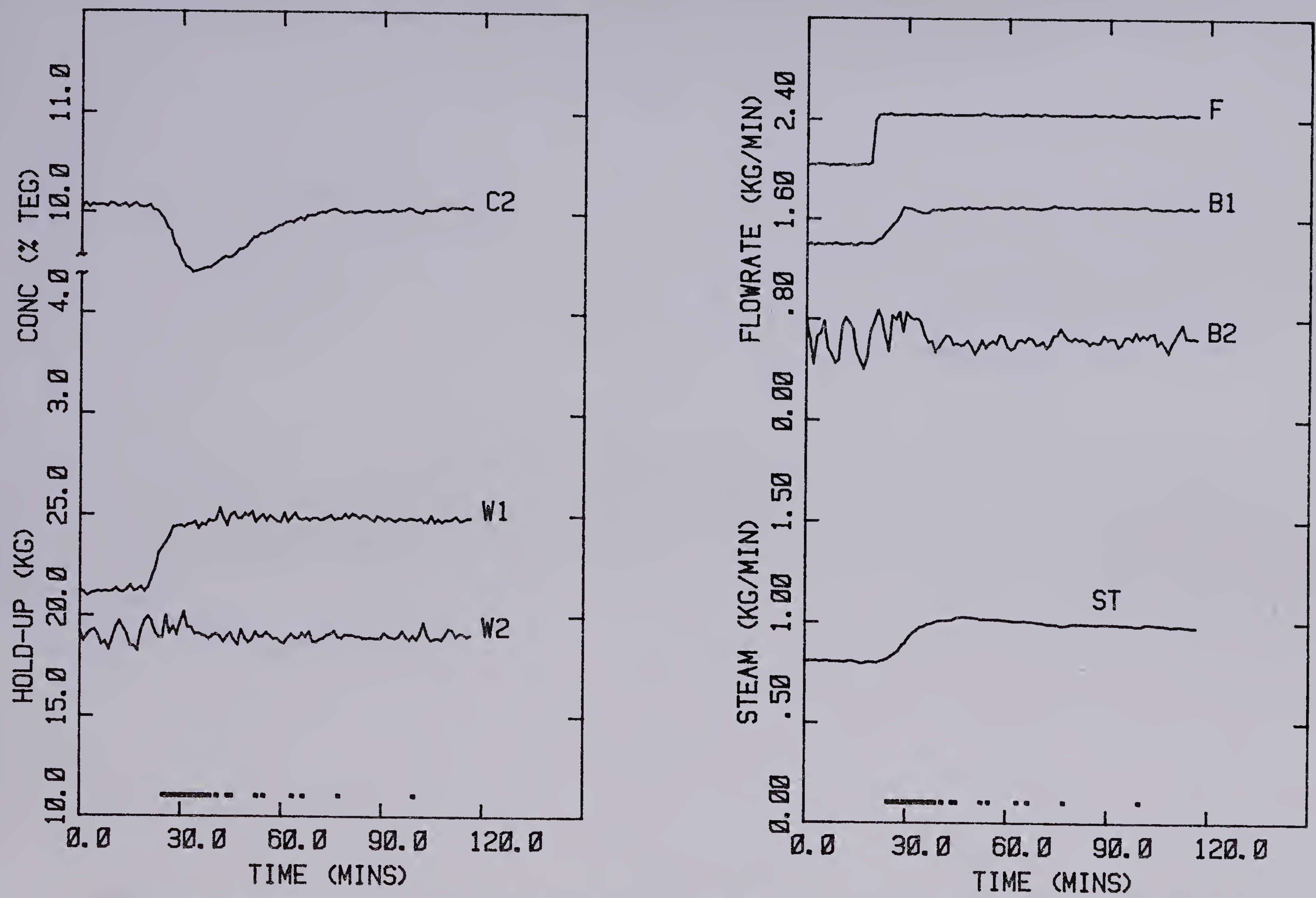


FIGURE 5.21 Evaporator Response Using APCS with PI Q-wt and Second Order Model
(APCS/RP2007/ITDM/T64/M2/C1./D.005/P1/Q PI/ 20%FD/ PI Q-WEIGHTING cf.RT2024)

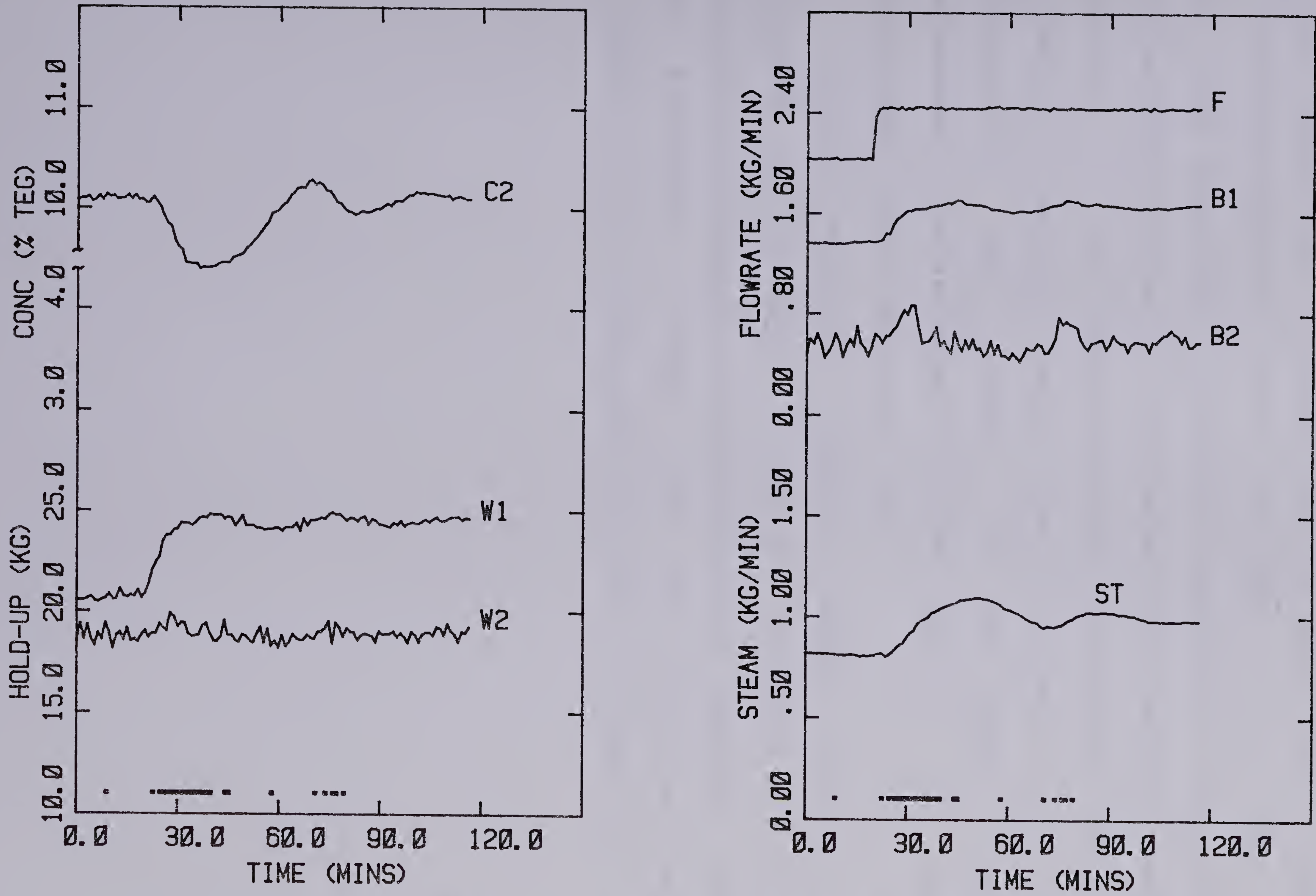


FIGURE 5.22 Evaporator Response Using APCS with PI Q-wt and Third Order Model
(APCS/RP3001/ITDM/T64/M3/C1/D.005/P1/Q PI/ 20%FD/ PI Q-WT AND THIRD ORDER)

corresponding STC results shown in Figure 4.27 and 4.30, respectively. As can be seen from the figures the initial model parameters calculated from the second order, whether it is the time domain curve-fitted model (Figure 5.21) or the time series model, give better results than the initial values obtained from the first order with time delay model, equation 3.3 in Figure 5.23. Note that although the first order plus time delay model gives the best fit to the experimental data (Figure 3.3) the number of model parameters to be estimated is higher than for the second order model due to the delay term.

Without Q-weighting, setpoint changes could not be achieved by the APCS. To show the robustness to external disturbances produced by Q-weighting a 10% setpoint change was introduced in Figure 5.25. This result can also be compared to the corresponding STC result in Figure 4.32.

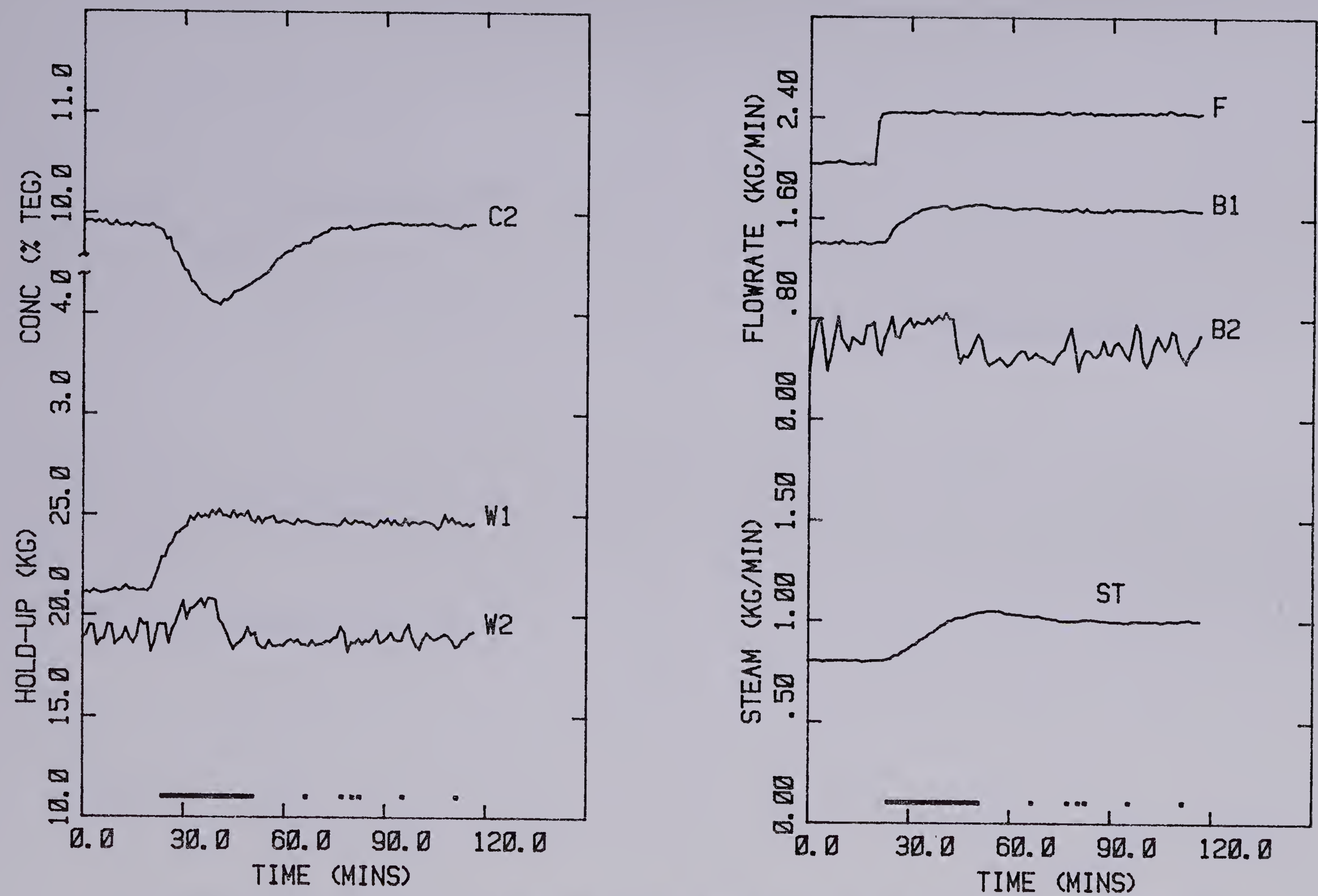


FIGURE 5.23 Evaporator Response Using APCS with Q-wt and 1st Order Model
(APCS/RP2005/ITDM/T64/M1/C1/D.005/P1/Q PI/ 20%FD/ PI Q-WT AND 1ST ORDER)

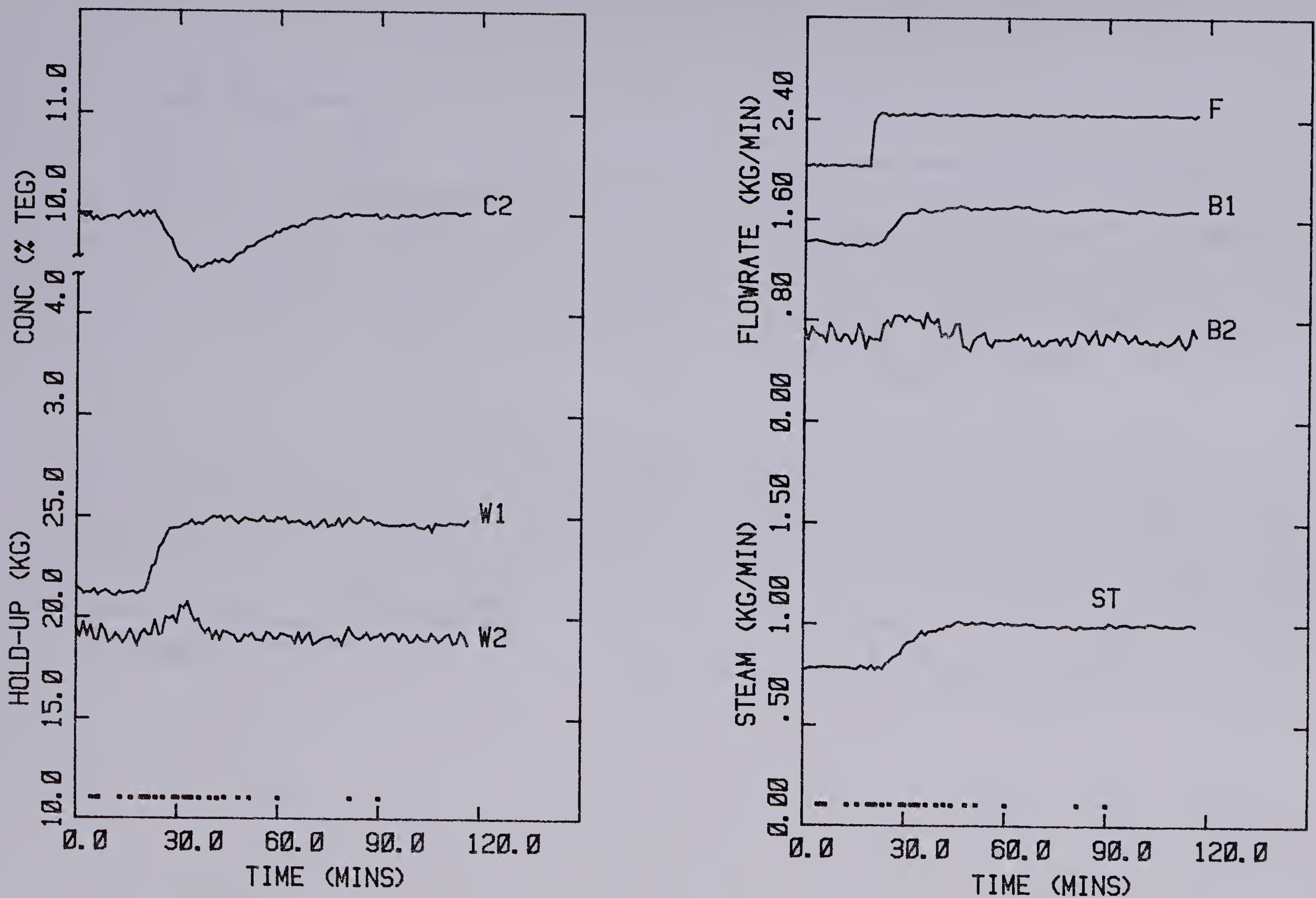



FIGURE 5.24 Evaporator Response Using APCS with Q-wt and Time Series Model
(APCS/RP2006/ITSM/T64/M2/C1/D.005/P1/Q PI/ 20%FD/ PI TYPE Q-WT)

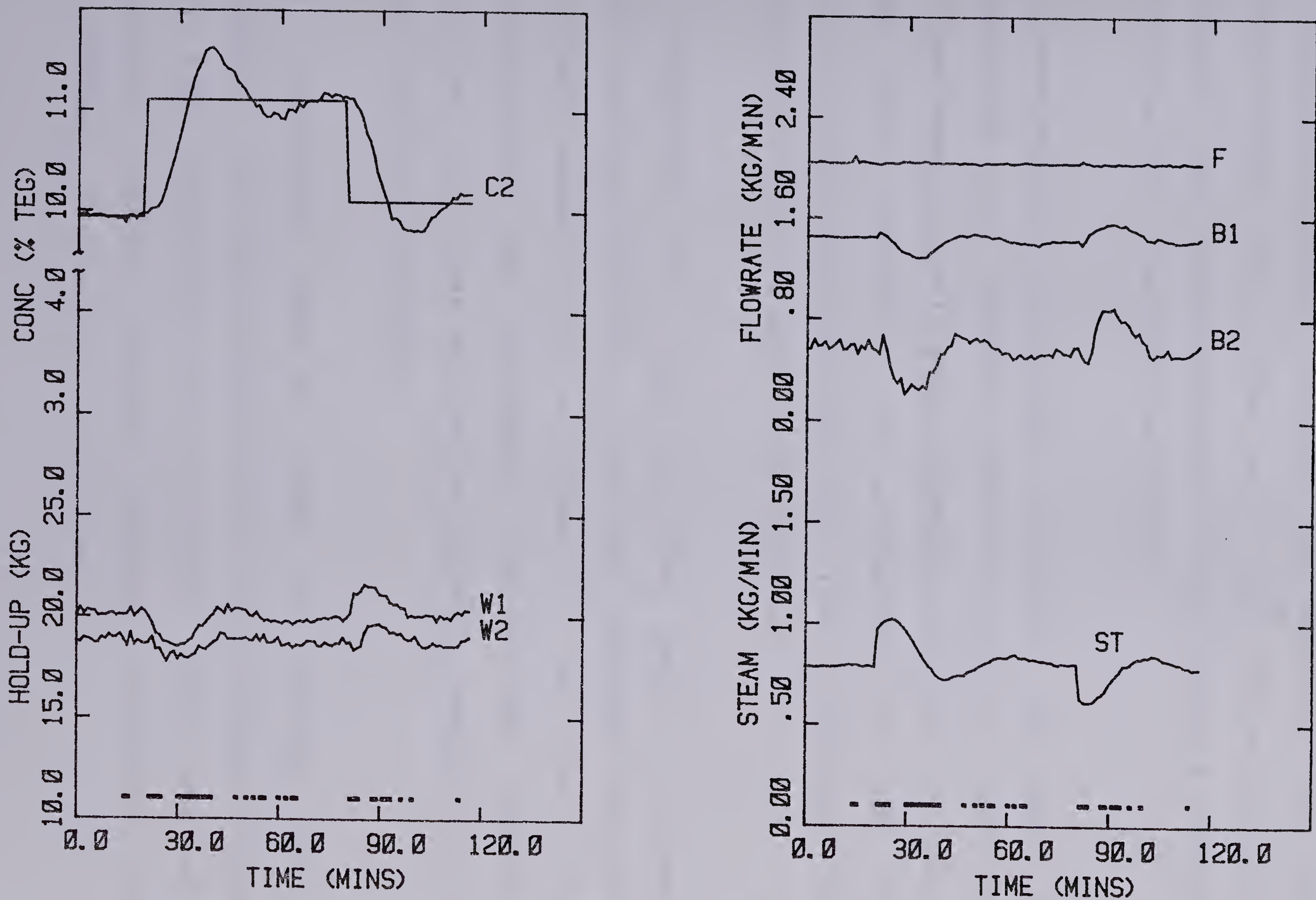


FIGURE 5.25 Evaporator Response Using APCS with Q-wt and setpoint Changes
(APCS/RP2008/ITDM/T64/M2/C1/D.005/P1/Q PI/ 10%SP/ Q-WT AND SP CHANGES)

5.7 Conclusions

1. Simulation studies using the basic APCS algorithm showed good output control even with poor initial conditions. However, they produced excessive control action as observed with the STR in the previous chapter.

2. When applied to the actual evaporator, the basic APCS algorithm also generated excessive control inputs which caused unstable oscillatory responses. Different values of the design variables such as sampling time, model order, initial model parameters and the error correcting factor were not helpful in solving this problem.

3. Reduction of the initial controller gain by increasing the leading coefficient of the input polynomial resulted in a stable response with an offset in the final product concentration. An incremental form of APCS was used to eliminate the offset but the corresponding response was also oscillatory and unstable.

4. A performance index was added to the basic APCS to moderate the control signal and to filter setpoint changes. PI type Q-weighting on the control input resulted in excellent control and good robustness to different initial conditions and external disturbances.

5. The order of the adaptive predictive model was important. The second order model performed better than the first order model with time delay or the third order model for the short term runs on the evporator.

6. An overall comparison suggests that the performance of weighted APCS in the evaporator application was equivalent to that achieved with the STC (chapter four).

6. The Self-Tuning Feedback Controller

6.1 Introduction

The development of an adaptive controller with strong theoretical properties such as stability, plus robust, practical performance has been one of the longstanding objectives of control engineers. During the past decade there have been a number of adaptive controllers proposed to meet this objective [Aström and Wittenmark, 1973; Landau, 1974; Monopoli, 1974; Clarke and Gawthrop, 1975; Narendra and Valavani, 1976; Martin-Sanchez, 1976; Feuer and Morse, 1978; Goodwin et al., 1978]. In recent years numerous experimental and simulated applications (cf. literature survey of chapter two and chapter three) have been reported that show the advantages and excellent performance of adaptive control over conventional (usually PID) controllers. Nevertheless not many of these adaptive control algorithms are being applied to the control of industrial processes. One key difficulty in having such controllers accepted and applied to the control of industrial processes is the unfamiliar and complicated structure of these adaptive control schemes. In contrast to this, continuous or discrete PID controllers are still being used extensively for the majority of industrial control problems even though in some applications a great deal of time and effort is required in the tuning of controller constants for such controllers. Consequently, there is considerable incentive

for the development of an adaptive algorithm which automatically tunes the conventional (PID) feedback controller coefficients or gains.

The adaptive controller presented in this chapter is defined as the Self-tuning Feedback Controller (SFC) and can be derived in a form that is mathematically and structurally equal to the widely used discrete, PID feedback algorithm. Its schematic diagram is shown in Figure 6.1.

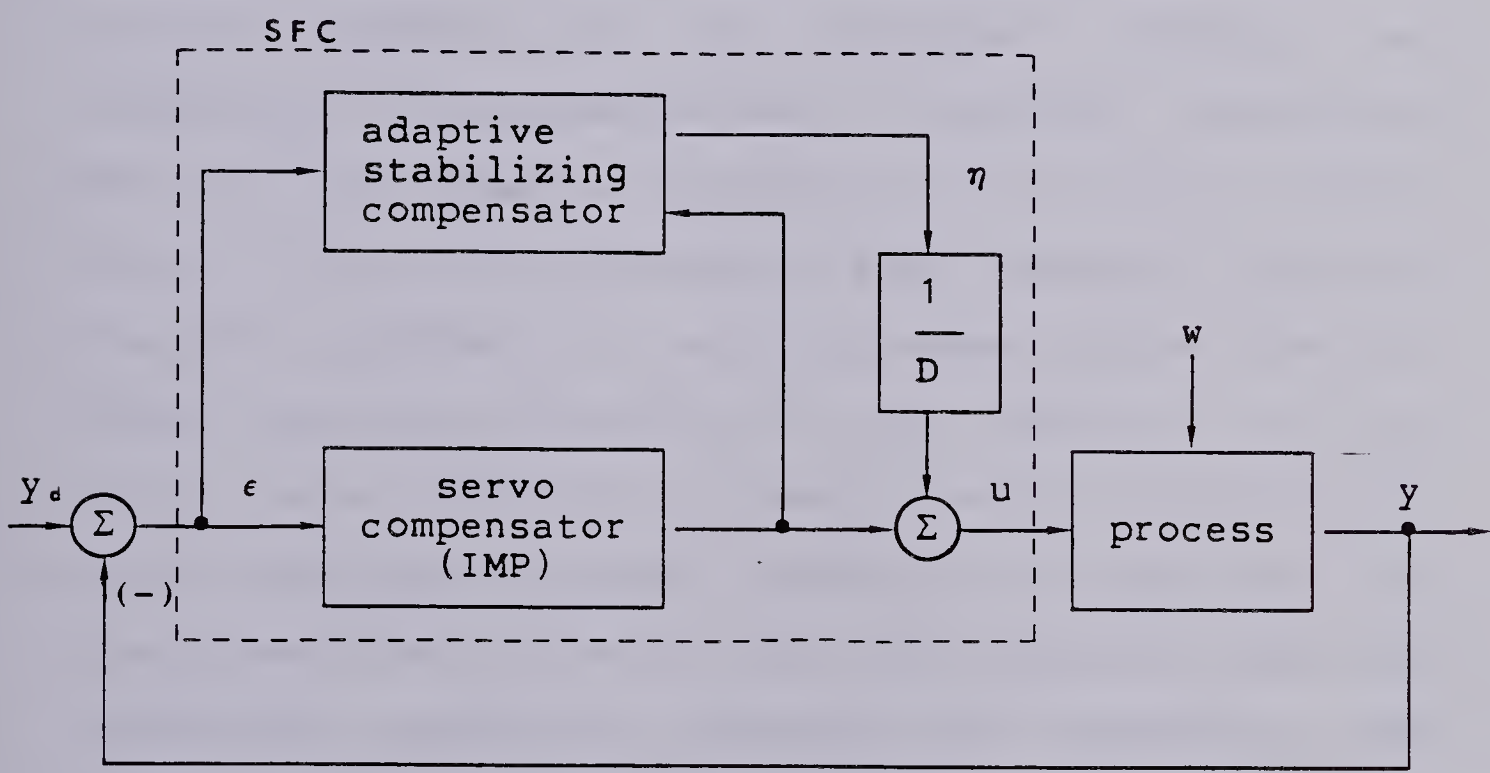


Figure 6.1 Schematic Diagram of the Self-Tuning Feedback Controller

Global stability of the overall system is proven and it is shown that SFC has a robust controller structure [Davison and Goldenberg, 1975; Davison, 1976a; Francis and Wonham,

1975] which means that once the controller parameters have converged sufficiently to stabilize the overall system then asymptotic tracking and/or regulation is achieved even in the presence of parameter perturbations, e.g. perturbations due to time-varying or nonlinear process characteristics.

According to the internal model principle of Francis and Wonham (1975,1976) a compensator can achieve stability and steady state regulation and/or tracking despite certain finite perturbations in the system and compensator parameters (i.e. it is robust) only if the controller utilizes feedback of the regulated variable, and incorporates in the feedback path a suitably reduplicated model of the dynamic structure of the exogenous signals which it is required to process. Most adaptive controller structures proposed in the literature differ from this 'robust' controller structure in that there is no error driven system which is an internal model of the reference and the disturbance signals. Davison has shown how this robust controller can be realized using separate 'servo' and 'stabilizing' compensators [Davison and Goldenberg, 1975; Davison, 1976a; Patel and Munro, 1982]. The 'servo compensator' design guarantees the robust properties of the system provided that the stabilizing compensator maintains overall system stability.

6.2 Literature Survey

Of the various adaptive control algorithms only a few are closely related to the conventonal PID feedback controller. Wittenmark(1979), Wittenmark and Aström(1980) and Isermann(1981) have proposed self-tuning PID controllers based on the pole assignment technique. In these controllers the PID coefficients depend on the placement of selected poles and the integral action comes from the prespecified controller transfer function. Furthermore, it is assumed that the plant to be controlled is governed by, at most, a dead time-free second order model. Banyasz and Keviczky(1982) have recently published results on self-tuning PID regulators which calculate the PID constants by a gradient search method based on the prescribed overshoot to a step input. More recently Cameron and Seborg(1982) have presented a design method based on the STC of Clarke and Gawthrop, where the PID controller has proportional and derivative action which act on the filtered measurements rather than the control error and the integral action is introduced by forcing the dynamics of the input variable. Gawthrop (1982), using his hybrid self-tuning controller (1980), has developed a continuous type self-tuning PI(PID) controller when the system to be controlled is first(second) order with no time-delay. In this algorithm the controller coefficients in continuous form are adaptively tuned by a discrete-time estimator and the integrator is incorporated with the assumption that the

external noise is a nonstationary or drifting process.

In this chapter it is shown that SFC has a robust controller structure [Davison,1976; Davison and Goldenberg, 1975; Francis and Wonham, 1975]. The idea of an adaptive, robust controller was proposed by Francis and Vidyasagar (1979) and a specific adaptive robust control strategy was presented by Silveira and Doraiswami (1981). Davison(1976a) has described how a robust controller can be designed for multivariable plants without any prior knowledge of the plant model. In the simplest of his cases the robust controller is initialized using the values derived from a steady state model of the plant developed using off-line identification. However, once the control is started there is no adaptation of the controller parameters. SFC as proposed here is a robust, adaptive controller.

One of the key difficulties in analysing adaptive control algorithms is the nonlinear, time-variant nature of the overall system due to the adaptive estimation law, even though the actual plant is linear and time-invariant. In 1974 Monopoli proposed a globally stable model reference adaptive control system designed by Lyapunov's direct method but his main claim of global stability has still not been justified for the general problem as claimed in his paper [Feuer and Morse 1978]. Ljung(1977) has presented a set of ordinary differential equations characterizing the behavior

of parameter adaptive algorithms and showed that the positive realness of the transfer functions plays a crucial role in certain recursive adaptation methods. This result supports Landau's work [Landau 1976] who used Popov's hyperstability criterion [Popov 1963] to design a model reference adaptive system. However, Ljung's method only provides a tool to test parameter convergence and does not answer the question of global stability. Moreover, his paper does not prove and account for the boundedness of the system I/O variables which is the most important problem in adaptive control [Goodwin et al., 1978,1981; Aström et al., 1977]. In 1978 Goodwin et al. presented a formal proof of global stability and parameter convergence for a certain class of discrete deterministic systems. This proof assumes that the process to be controlled is stable inverse and its structure is known. The results were then extended to the linear time-invariant stochastic process under the further assumption that the noise characteristic equation is strictly positive real [Goodwin et al., 1981]. Recently, Martin-Sanchez, Shah and Fisher (1981c) proved the global stability of an adaptive predictive control system (APCS) applicable to a delay-free, stable inverse, MIMO processes subject to bounded disturbances and/or noise sequences. More recently Martin-Sanchez (1982) extended these results to include time-delay(s). In this chapter global stability and parameter convergence of the SFC is accomplished using the same approach to stability and convergence as in APCS.

6.3 Theory

The theoretical development of SFC is pursued in the following sections in two steps. The global stability of an adaptive system is first established by showing that the combination of a particular form of controller and an estimation law when applied to an unknown SISO system yields a stable system. The robust property of the resulting control scheme is then shown by noting that the particular controller formulation adopted has a structure identical to that prescribed for a robust controller, i.e. an error driven servo compensator and a stabilizing compensator (cf. Figure 6.1).

6.3.1 Derivation of SFC

(1) Process Model

Let the single-input single-output process to be controlled be described by the following discrete equation:

$$y(k) = y_m(k) + \gamma(k) \tag{6.1}$$

that is, the actual process output, $y(k)$, consists of a purely deterministic component $y_m(k)$ which is defined below and a residual component $\gamma(k)$.

It is assumed that the deterministic part of the process output $y_m(k)$ is characterized by a finite structure ARIMA representation of the form:

$$A_m(z^{-1})y_m = B_m(z^{-1})u(k-d)+L_m(z^{-1})v(k-q)+H_m(z^{-1})w(k) \quad (6.2)$$

where $u(\cdot)$, $v(\cdot)$ and $w(\cdot)$ are the process input, measurable and unmeasurable but deterministic disturbance sequences respectively and d and q are the corresponding time-delays. z^{-1} is the backward shift operator and polynomials A_m, B_m, L_m and H_m are defined as follows.

$$\begin{aligned} A_m(z) &= 1 + a_1(z) + \cdots + a_n z^n \\ B_m(z) &= b_0 + b_1(z) + \cdots + b_m z^m \\ L_m(z) &= l_0 + l_1(z) + \cdots + l_i z^i \\ H_m(z) &= h_0 + h_1(z) + \cdots + h_n z^n \end{aligned} \quad (6.3)$$

The residual component $\gamma(k)$ is defined as the difference between the output of the actual process and the assumed model, i.e. it is the modelling residual that cannot be accommodated by the model y_m. For example $\gamma(\cdot)$ can include the effects of (i) unmeasured disturbances plus noise, and/or (ii) modelling errors, (iii) process nonlinearities, etc. For purposes of the stability proof, it is assumed that $\gamma(\cdot)$ is uncorrelated or independent of $y_m(\cdot)$ and it can be any bounded deterministic or stochastic signal. As a special example of case (i) the residual $\gamma(\cdot)$ may also be expressed as the output of a white noise input to a moving average (integrated) filter, i.e. $\gamma(k)$ can be a stationary or nonstationary stochastic process. However, once overall

stability is assured then the internal model principle guarantees asymptotic tracking and regulation despite parameters errors and/or disturbances of the class defined below.

The following assumptions are made about the system (6.1) and (6.2):

1) An upper bound for n, m and i is known.
2) The delays d and q are known.
3) The residual term, $\gamma(k)$, is bounded for all k.
4) $\gamma(k)$ is uncorrelated with present and past values of $y_s(k-i)$, $y(k-i)$, plus $\eta(k-i)$ and $\lambda(k-i)$ (defined later) for $i \geq 1$.

Now, define the control error $\epsilon(k)$ as:

$$\epsilon(k) = y_s(k) - y(k) \tag{6.4}$$

where $y_s(\cdot)$ is the desired setpoint or reference sequence.

(2) Disturbance Model

Let $w(k)$ and $y_s(k)$ be assumed to be the output of the following autonomous linear difference equation [Davison 1976, Silveira and Doraiswami 1981], i.e.

$$D(z^{-1})y_s(k) = 0 \quad \text{and}$$
$$D(z^{-1})w(k) = 0 \tag{6.5}$$

where $D(z^{-1})$ which is a polynomial with known coefficients that characterizes the dynamics of the reference input and the unmeasurable deterministic disturbances. $D(z^{-1})$ is a polynomial of the form:

$$D(z) = 1 + d_1 z + \cdots + d_j z^j \tag{6.6}$$

Note that it is not necessary for D(z) to have roots inside the unit circle. Therefore any setpoint or disturbance signals (bounded or unbounded) can be handled in the formulation provided that equation (6.5) holds. Multiplying equation (6.4) by $D(z^{-1})$ and substituting $y(k)$ and $y_m(k)$ from equations (6.1) and (6.2) gives a control error equation:

$$\begin{aligned} -A_m(z^{-1})D(z^{-1})\epsilon(k) &= D(z^{-1})B_m(z^{-1})u(k-d) + \\ &\quad D(z^{-1})L_m(z^{-1})v(k-q) + D(z^{-1})A_m(z^{-1})\gamma(k) \end{aligned} \tag{6.7}$$

The starting point of most I/O based adaptive system designs is an assumed ARMA model description of a plant with $A_m(z^{-1})$ and $B_m(z^{-1})$ polynomials in an irreducible form (cf. equation (6.2)), or in other words a system representation in minimal order form. If the $A_m(z^{-1})$ and $B_m(z^{-1})$ polynomials are assumed to be of a reducible form, i.e. there is a common factor between them and $B_m(z^{-1})$ then the process representation is nonminimal and these additional (common) modes are due to the uncontrollable and/or

unobservable modes of the system. Recently Aström (1983) has pointed out that it may be advantageous in some cases to consider such reducible or nonminimal system descriptions as the starting point in adaptive control. For example one can accomodate the internal model in the system description by having it appear as a common factor, i.e. the common factor between the input and output polynomials can be the polynomial, $D(z^{-1})$, which is the model of the external disturbance and setpoint signals. Such a representation (cf. equation (6.7)) would allow us to implicitly include in the system description the internal model of the exogenous signals entering the process. The adaptive controller design based on this nonminimal representation would then result in a compensator that would supply the right-half plane transmission zeros of the closed-loop system to cancel the unstable poles of the exogenous signal [Francis and Wonham, 1975, 1976]. This is one way of accomodating the internal model in the system representation with the servo compensator, $1/D(z^{-1})$, as a natural result (cf. Figure 6.1).

(3) Control Law and Performance Index

The main control objective of SFC is to generate a $u(k)$, such that (i) the closed loop system is asymptotically stable and (ii) asymptotic tracking and disturbance rejection is achieved. For these purposes consider the following control law (a robust controller design):

$$u(k) = \frac{P(z^{-1})}{D(z^{-1})}\epsilon(k) + \frac{1}{D(z^{-1})}\eta(k) \tag{6.8}$$

where $P(z^{-1})$ is an arbitrary polynomial defined by the user and $\eta(k)$ is an auxiliary signal which minimizes the chosen performance index and guarantees overall stability. Notice that the first term on the right hand side of equation (6.8) corresponds to the servo compensator which has $D(z^{-1})$ in the denominator to represent the dynamics of the disturbance and reference signals. The second term, $\eta(k)/D(z^{-1})$, corresponds to the output of the stabilizing compensator with $\eta(k)$ as the output of an auxiliary system. This strategy is shown in block diagram form in Figure 6.1 and has a robust controller structure, i.e. the required error driven internal model termed the 'servo compensator' and the 'stabilizing compensator'. In adaptive systems one has the freedom of choosing any controller structure. A particular controller structure is obviously chosen to give some desired properties. However, as a first step it will be shown that this controller structure in combination with the following strategy and estimation law is globally stable.

Substituting equation (6.8) into (6.7) yields:

$$-[A_m(z^{-1})D(z^{-1})+B_m(z^{-1})P(z^{-1})z^{-d}]\epsilon(k) = B_m(z^{-1})\eta(k-d)$$
$$+ D(z^{-1})L_m(z^{-1})v(k-q) + D(z^{-1})A_m(z^{-1})\gamma(k) \tag{6.9}$$

To facilitate further analysis equation (6.9) can be written more compactly by defining new polynomials and $\lambda(k)$ as follows:

$$
\begin{aligned}
A(z) &= -(A_m(z)D(z) + B_m(z)P(z)\, z^d) \\
B(z) &= B_m(z) \\
C(z) &= A_m(z)\, D(z) \\
L(z) &= L_m(z)\, D(z) \\
\lambda(k) &= v(k-q+d)
\end{aligned}
\tag{6.10}
$$

Then equation (6.9) can be written as:

$$\epsilon(k) = \frac{B(z^{-1})}{A(z^{-1})}\eta(k-d) + \frac{L(z^{-1})}{A(z^{-1})}\lambda(k-d) + \frac{C(z^{-1})}{A(z^{-1})}\gamma(k) \tag{6.11}$$

The following performance index is minimized by manipulating the auxiliary signal $\eta(k)$:

$$J = E\{[P(z^{-1})\epsilon(k+d)]^2 + [Q'(z^{-1})u(k)]^2\} \tag{6.12}$$

where $P(z^{-1})$ and $Q'(z^{-1})$ are weighting or design factors of polynomials in z^{-1}. In order to minimize the performance index $P(z^{-1})\epsilon(k+d)$ must be expressed in terms of known values at time k. Rewriting equation (6.11) in the form of weighted, predicted control error yields,

$$P\epsilon(k+d) = \frac{B}{A}\,\eta(k) + \frac{L}{A}\,\lambda(k) + \frac{C}{A}\,\gamma(k+d) \tag{6.13}$$

where the argument (z^{-1}) has been dropped for convenience. This equation is further manipulated by introducing the following additional identity:

$$\frac{P(z^{-1})C(z^{-1})}{A(z^{-1})} = G(z^{-1}) + \frac{F(z^{-1})}{A(z^{-1})}\,z^{-d} \tag{6.14}$$

where, with na, nc, np being the order of polynomials $A(z^{-1})$, $C(z^{-1})$, $P(z^{-1})$ respectively,

$$\begin{aligned} G(z) &= 1 + g_1 z + \cdots + g_{d-1} z^{d-1} \\ F(z) &= f_0 + f_1 z + \cdots + f_{s-1} z^{s-1} \\ s &= \max(\, na \,,\, nc + np - d + 1\,) \end{aligned} \tag{6.15}$$

Combining equations (6.13) and (6.14) then substituing $\gamma(k)$ from equation (6.11) results in the following equation for the predicted value of the weighted control error.

$$\begin{aligned} P(z^{-1})\epsilon(k+d) = {} & \frac{F(z^{-1})}{C(z^{-1})}\,\epsilon(k) + \frac{B(z^{-1})G(z^{-1})}{C(z^{-1})}\,\eta(k) \\ & + \frac{L(z^{-1})G(z^{-1})}{C(z^{-1})}\,\lambda(k) + G(z^{-1})\gamma(k+d) \end{aligned} \tag{6.16}$$

Let $\hat{\epsilon}(k+d|)$ be the estimate of $P(z^{-1})\epsilon(k+d)$ based on data up to and including time k. Then the variance of the estimation error is given by:

$$E\{P(z^{-1})\epsilon(k+d) - \hat{\epsilon}(k+d|)\}^2$$

$$= E\{[\frac{F(z^{-1})}{C(z^{-1})}\epsilon(k) + \frac{B(z^{-1})G(z^{-1})}{C(z^{-1})}\eta(k) +$$

$$\frac{L(z^{-1})G(z^{-1})}{C(z^{-1})}\lambda(k) - \hat{\epsilon}(k+d|)]^2 + [G(z^{-1})\gamma(k+d)]^2\}$$

$$= E\{\epsilon^* - \hat{\epsilon}(k+d|)\}^2 + E\{G(z^{-1})\gamma(k+d)\}^2$$

$$\geq E\{G(z^{-1})\gamma\}^2 \qquad (6.17)$$

where $E\{\}$ is the statistical expectation operator. Here, it is assumed that future values of $\gamma(\cdot)$ are uncorrelated with the present and the past values of $\epsilon(k-i)$, $\eta(k-i)$ and $\lambda(k-i)$ for $i \geq 0$, and $\epsilon^*(k+d)$ is defined as:

$$\epsilon^*(k+d) = [F(z^{-1})\epsilon(k) + B(z^{-1})G(z^{-1})\eta(k)$$
$$+ L(z^{-1})G(z^{-1})\lambda(k)] / C(z^{-1}) \qquad (6.18)$$

The equality in expression (6.17) holds if $\hat{\epsilon}(k+d|) = \epsilon^*(k+d)$, which is the best estimate of $\epsilon(k+d)$ in the sense that the variance is minimized. Using equation (6.18), $P(z^{-1})\epsilon(k+d)$ can also be expressed as:

$$P(z^{-1})\epsilon(k+d) = \epsilon^*(k+d) + \xi(k+d) \qquad (6.19)$$

where $\xi(k+d)$ is the estimation error, i.e.

$$\xi(k+d) = G(z^{-1})\gamma(k+d) \tag{6.20}$$

By substituting equations (6.19) and (6.8) into (6.12), the performance index can now be expressed as a function of the control error $\epsilon(\cdot)$ and the auxiliary signal $\eta(\cdot)$ up to and including time k:

$$J = E\{[\epsilon^*(k+d)]^2 + [Q'(\eta(k) + P\epsilon(k))/D]^2\} + [E\{\xi(k+d)\}]^2 + \sigma^2 \tag{6.21}$$

where σ^2 denotes the variance of $\xi(k+d)$.

The auxiliary signal $\eta(k)$ is determined such that the performance function is minimized, i.e.

$$\frac{\partial J}{\partial \eta(k)} = 0$$

or, since the last two terms in equation (6.21) are not functions of $\eta(k)$:

$$\epsilon^*(k+d) + Q(z^{-1})[\eta(k) + P(z^{-1})\epsilon(k)] = 0 \tag{6.22}$$

where $Q = q'_0Q'/b_0D$ [Appendix B]. Sustituting equation (6.18) into (6.22) gives:

$$[F'(z^{-1}) + C(z^{-1})Q(z^{-1})]\ P(z^{-1})\epsilon(k) +$$
$$[B'(z^{-1})G(z^{-1}) + C(z^{-1})]\ Q(z^{-1})\eta(k) +$$
$$[L(z^{-1})G(z^{-1})]\ \lambda(k) = 0 \tag{6.23}$$

where $B'(z^{-1}) = B(z^{-1})/Q(z^{-1})$, $F'(z^{-1}) = F(z^{-1})/P(z^{-1})$. Define new polynomials $T(z^{-1})$, $V(z^{-1})$ and $W(z^{-1})$ as:

$$\begin{aligned} T(z) &= F'(z) + C(z)Q(z) \\ V(z) &= B'(z)G(z) + C(z) \\ W(z) &= L(z)G(z) \end{aligned} \tag{6.24}$$

then equation (6.23) can be written as:

$$T(z^{-1})\tilde{\epsilon}(k) + V(z^{-1})\tilde{\eta}(k) + W(z^{-1})\lambda(k) = 0 \tag{6.25}$$

or in vector notation as:

$$\Theta_0^t\ \Psi(k) = 0 \tag{6.26}$$

with $\tilde{\epsilon}(k) = P\epsilon(k)$, $\tilde{\eta}(k) = Q\eta(k)$ and Θ_0 and $\Psi(k)$ defined as:

$$\Theta_0^t = [t_0, t_1, \cdots t_x, v_0, v_1, \cdots v_y, w_0, w_1, \cdots w_z]$$

$$\Psi^t(k) = [\tilde{\epsilon}(k), \tilde{\epsilon}(k-1), \cdots \tilde{\epsilon}(k-x),$$
$$\tilde{\eta}(k), \tilde{\eta}(k-1), \cdots \tilde{\eta}(k-y), \lambda(k), \lambda(k-1), \cdots \lambda(k-z)]$$

where superscript 't' denotes the transpose and subscript variables x,y and z are integers to denote the order of the corresponding coefficient polynomials. The auxiliary signal $\eta(k)$ can be obtained from equation(6.26).

For minimum variance control of the auxiliary system, i.e. $P(z^{-1})=1$ and $Q(z^{-1})=0$, from equations (6.22) and (6.18) the auxiliary signal $\eta(k)$ is given by:

$$\eta(k) = \frac{-F(z^{-1})}{B(z^{-1})G(z^{-1})}\,\epsilon(k) + \frac{-L(z^{-1})}{B(z^{-1})}\,\lambda(k) \tag{6.27}$$

and the control error, $\epsilon(k)$ becomes the output of a moving average process of order (d-1) whose input is $\gamma(k)$.

$$\epsilon(k) = G(z^{-1})\gamma(k)$$

Remark: If, as mentioned in the beginning of this section, $\gamma(\cdot)$ is a stochastic residual term, then the mean of the control error of the above illustration can be expressed as:

$$E\{\epsilon(k)\} = G(z^{-1})\ E\{\gamma(k)\}$$

Thus, if $\gamma(\cdot)$ is a zero mean stationary sequence, the mean of the control error will be zero. However, if $\gamma(\cdot)$ is a nonstochastic type residual term, e.g. due to unmeasured disturbances and/or modelling error then one cannot make any

conclusions regarding the control error except for the fact that it will be bounded. (As discussed later, this bound is a direct function of the minimal upper bound on the unknown components.)

For purposes of the stability proof, $\gamma(\cdot)$ can be any bounded deterministic or stochastic signal. However, once stability is assured then the internal model principle guarantees asymptotic tracking and regulation for the class of disturbances and setpoint signals defined by equation (6.5) despite perturbations in the system parameters.

(4) Adaptive Algorithm

In the previous section a feedback control law based on the minimization of a certain cost function, J, was derived for systems with known parameters. However, in many real situations this is not the case. The process and hence the controller parameters, Θ_o, in addition to the structure of the system to be controlled are usually not known exactly. In this section an adaptive law is established to estimate Θ_o. Recalling equation (6.22), let the controller output function be defined with estimated parameters as:

$$\Phi^*(k+d) = \theta^t(k)\Psi(k) \;/\; C(z^{-1}) \tag{6.28}$$

where $\theta^t(k) = [\hat{t}_o, \hat{t}_1, \cdots \hat{t}_x, \hat{v}_o, \hat{v}_1, \cdots \hat{v}_y, \hat{w}_o, \hat{w}_1, \cdots \hat{w}_z]$ are the estimates of the controller parameters in Θ_o.

Since the actual future value $\epsilon(k+d)$ is unknown the prediction $\epsilon^*(k+d)$ is used to calculate the control law in equation (6.22). Let $\Phi(k+d)$ represent the controller output function defined by equation(6.22) when the actual weighted error $P(z^{-1})\epsilon(k+d)$ is used:

$$\Phi(k+d) = P(z^{-1})\epsilon(k+d) + Q(\eta(k) + P(z^{-1})\epsilon(k)) \tag{6.29}$$

then combining equations (6.19) and (6.29) gives

$$\Phi(k+d) = \Phi^*(k+d) + \xi(k+d) \tag{6.30}$$

Adding equation (6.28) and (6.30), and then using the fact that $\Phi^*(k)$ is zero due to minimization yields the following equation.

$$\Phi(k) = \theta^t(k-d)\Psi(k-d) + \xi(k) \tag{6.31}$$

It is obvious that the actual controller output function, $\Phi(k+d)$ will achieve its best possible value, $\xi(k+d)$, if $\theta^t(k-d)\Psi(k-d)$ is equal to zero. The adaptive law for estimating $\theta(k)$ is given by:

$$\theta(k) = \theta(k-d) + \frac{a(k)\Psi(k-d)}{1 + a(k)\Psi^t(k-d)\Psi(k-d)}\,\delta(k) \tag{6.32}$$

where $\delta(k) = \Phi(k) - \theta^t(k-d)\Psi(k-d)$. This is a d-sample time

interlaced recursion algorithm. The adaptive algorithm is not an intuitive one but one that is dictated by the proof of stability and convergence analysis.

The auxiliary signal $\eta(k)$ can now be adaptively calculated from the estimated parameters at each sampling time by the equation.

$$\theta^{t}(k)\ \Psi(k) = 0 \tag{6.33}$$

where $\Psi(k)$ is the vector defined in equation (6.26). Since the recursive parameter estimation is driven by the tracking error $\delta(k)$, the algorithm is a d-sample time interlaced multiple recursion type [Goodwin et al., 1978,1981].

The overall control scheme can be recast as a nonlinear feedback problem where the input, $\eta(k)$ is adapted such that the output of the linear block, $\Phi(k+d)$ is bounded under the unmeasurable disturbance $\xi(k)$ (Figure 6.2).

The scalar quantity $a(k)$ that appears in equation (6.32) is part of a criterion to stop adaptation when necessary and is required to prove stability and parameter convergence of the algorithm [Martin-Sanchez et al., 1981c]. It is defined as:

i) $a(k) = 0$ if and only if

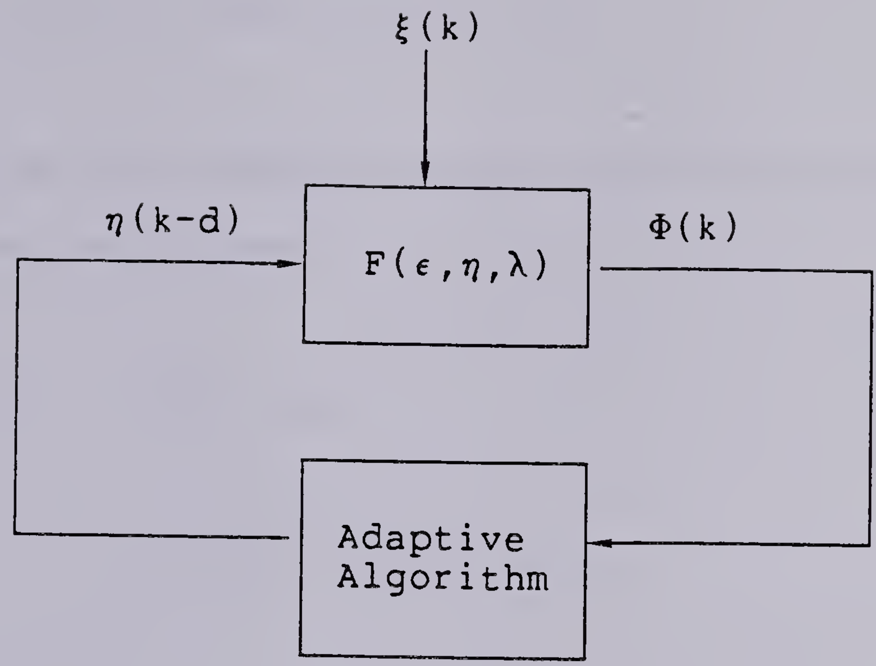


Figure 6.2 Equivalent Nonlinear Feedback System

$$|\delta(k)| \leq \Delta_d'(a_0,\Delta_d,k) \leq 2\Delta_d < \infty \tag{6.34}$$

where function Δ_d' is defined as:

$$\Delta_d'(a(k),\Delta_d,k) = \frac{2 + 2a(k)\Psi^t(k-d)\Psi(k-d)}{2 + a(k)\Psi^t(k-d)\Psi(k-d)} \Delta_d \tag{6.35}$$

and a positive constant a_0 denotes the lower limit of $a(k)$. Δ_d is an estimate of the upper bound on the absolute value of the nonmodelling residual and unmeasurable disturbance, $\xi(k)$, i.e.

$$\Delta_d \geq \Delta_m = \sup_{0<k\leq\infty} |\xi(k)| \tag{6.36}$$

ii) $a_0 < a(k) \leq a_d(k) \leq a_1 < \infty$ if and only if

$$|\delta(k)| > \Delta_d'(a_0,\Delta_d,k) \geq \Delta_d \tag{6.37}$$

where a_1 is a constant upper bound for $a(k)$ and $a_d(k)$ is defined as follows:

$$(1) \quad a_d(k) = a_1 \tag{6.38}$$

$$\text{if } |\delta(k)| > \Delta_d'(a_1,\Delta_d,k)$$

$$(2) \quad a_d(k) = \frac{2(|\delta(k)| - \Delta_d)}{(2\Delta_d - |\delta(k)|)\Psi^t(k-d)\Psi(k-d)} \tag{6.39}$$

$$\text{if } \Delta_d'(a_0,\Delta_d,k) < |\delta(k)| \leq \Delta_d'(a_1,\Delta_d,k)$$

Then, for all nonzero $a(k)$ the following inequality is followed

$$|\delta(k)| \geq \Delta_d'(a(k),\Delta_d,k) \tag{6.40}$$

Consequently, the adaptive mechanism defined along equation (6.32) to (6.39) will be stopped at sampling time k if the magnitude of the a priori estimation error $|\delta(k)|$ is less than or equal to $\Delta_d'(a_0,\Delta_d,k)$. When the adaptation is not stopped the weighting factor $a(k)$ is chosen in an interval greater than a selected value a_0 and less than or equal to $a_d(k)$ which is calculated according to equation (6.38) or

(6.39) so that the inequality (6.40) is satisfied.

6.3.2 Stability and Convergence Analysis

This section establishes the global stability of the SFC shown in Figure 6.2 and analyzes the parameter convergence along the trajectory of the adaptive scheme described by equations (6.32) to (6.39). The main results are summarized in the following theorem.

Theorem 6.1: Subject to the following assumptions

i) The minimum upper bound, Δ_m, of $|\xi(k)|$ is known.

ii) The system represented by equation (6.31) is stable inverse, i.e.

$$|\Phi(k)| \geq \alpha_1 ||\Psi(k-d)|| - \alpha_2 \tag{6.41}$$

where α_1 and α_2 are positive constants.

iii) The measurable disturbance v(k) is bounded.

iv) The model structure is known, i.e. an ARIMA model with known time-delays d, q and known orders for polynomials $A_m(z^{-1})$, $B_m(z^{-1})$ and $L_m(z^{-1})$.

v) Polynomials $A_m(z^{-1})$, $B_m(z^{-1})$ and $D_m(z^{-1})$ are irreducible.

Then the following properties are true if the adaptive law and the control law given by equations (6.32) to (6.39) are applied to the system depicted by equation (6.31).

i) The norm of the I/O vector $\Psi(k)$ is finite, or in other words SFC will 'stabilize' the overall system.

$$||\Psi(k)|| < \infty \ , \ \forall \ k \geq 0 \tag{6.42}$$

ii) The norm of parameter vector is a nonincreasing function and the tracking error is bounded.

$$\text{a)} \lim_{k\to\infty} [||\theta(k+d)||^2 - ||\theta(k)||^2] = 0 \tag{6.43}$$

$$\text{b)} \lim_{k\to\infty} |\delta(k)| \leq \Delta_d'(a_0,\Delta_d,k) < 2\Delta_d \tag{6.44}$$

where $\theta(k)$ is defined as the parameter estimation error, i.e. the difference between the estimated parameter values and the values that would minimize the specified performance index.

The proof of this theorem is based on the APCS convergence analysis outlined by Martin-Sanchez et al. (1981c) and is included in Appendix C.

6.3.3 Adaptive PID Contoller

As a special case of the previous derivation, if a plant satisfies the following conditions:

i) It can be modelled by second order ARMA model with a finite residual term, i.e.

$$y(k) = y_m(k) + \gamma(k) \tag{6.45}$$

where

$$y_m(k) = -a_1 y_m(k-1) - a_2 y_m(k-2) + b_o u(k-1) + H_m w(k) \tag{6.46}$$

ii) The external inputs are such that the following conditions are satisfied

$$(1-z^{-1})y_s(k) = 0$$
$$(1-z^{-1})w(k) = 0 \tag{6.47}$$

iii) The controller design is based on the performance index, J, with $P(z^{-1}) = 1$ and $Q(z^{-1}) = 0$

then in this case the auxiliary signal, $\eta(k)$, is:

$$\eta(k) = -\frac{f_o + f_1 z^{-1} + f_2 z^{-2}}{b_o} \epsilon(k) \tag{6.48}$$

and the control law u(k) is as follows

$$u(k) = -\frac{(f_o + b_o) + f_1 z^{-1} + f_2 z^{-2}}{b_o(1 - z^{-1})} \epsilon(k) \tag{6.49}$$

which is identical to the structure of a conventional, discrete three term, PID controller.

6.3.4 Robust Controller Structure

Having proved overall stability, the robust property of SFC are now highlighted. The overall SFC design is shown schematically in Figure 6.1. In the SFC control law shown in equation (6.8) the first term on the right-hand side, $P(z^{-1})/D(z^{-1})$, corresponds to the servo compensator [Davison, 1976], that is driven by the measured error, $\epsilon(k)=y_s(k)-y(k)$. This error driven servo compensator with $D(z^{-1})$ in the denominator to represent the unstable modes of the disturbances and reference signals is an essential part of a robust controller. The second term, $\eta(k)/D(z^{-1})$, in equation (6.8) corresponds to the output of a stabilizing compensator with $\eta(k)$ as the output of an auxiliary system. As the adaptive parameters in the stabilizing compensator converge to a point where the overall system is stable then the SFC scheme takes on the properties of a robust controller due to the presence of the error driven servo compensator [Davison and Goldenberg, 1975; Davison, 1976]. The robust controller property ensures that $\epsilon(k)\rightarrow 0$ asymptotically even in the presence of finite changes in the system or compensator parameters. The importance of this property to the overall performance of SFC is particularly obvious when SFC parameter adaptation is stopped.

6.4 Implementation

Implementation of the SFC is very staightforward and simple. It requires only a few algebraic equations to calculate the adaptive parameters and implement the control law. No matrix inversion or trial and error type iterative calculation is required so that this scheme can be easily programed on a micoprocessor. However, its control performance and the parameter convergence are very much influenced by the choice of the initial values for the estimation routine and the weighting functions of the control law. This section will describes how to pick initial values for the SFC algorithm and the influence of this choice on the control performance will be discussed.

The basic initial parameters for a SFC, which must be supplied before the algorithm can be started, are as follows:

i) The sampling interval (cf. chapter 2)

ii) The initial parameter values for the estimation routine

(1) Order of controller polynomial

(2) Initial values of the controller coefficients

(3) Error correcting factor, a(k),(cf. chapter 5)

(4) Upper bound on disturbances Δ_d (cf. chapter 5)

iii) The weighting functions of the control law

(1) Polynomial $P(z^{-1})$

(2) Polynomial $Q(z^{-1})$

Note that P=1 and Q=0 for SFC with PID structure.

6.4.1 Initial Parameter Values

The number of coefficients in the controller polynomials $T(z^{-1})$, $V(z^{-1})$ and $W(z^{-1})$ must be determined before control calculations start. These can be calculated from the number of model parameters and the order of weighting function $P(z^{-1})$ and $Q(z^{-1})$,i.e.

$$\begin{aligned} x &= \max(s-1-np,\ n+r+nq) && \text{if } Q(z^{-1}) \neq 0 \\ &= s-1-np && \text{if } Q(z^{-1}) = 0 \end{aligned} \tag{6.50}$$

$$\begin{aligned} y &= \max(m-nq+d-1,\ n+r) && \text{if } Q(z^{-1}) \neq 0 \\ &= m+d-1 && \text{if } Q(z^{-1}) = 0 \end{aligned} \tag{6.51}$$

$$z = j+d-1 \qquad \text{if } L(z^{-1}) \neq 0 \tag{6.52}$$

The total number of coefficients, $n\theta$, to be estimated by the etimation routine is given by:

$$\begin{aligned} n\theta &= (x+1) + (y+1) + (z+1) && \text{if } L(z^{-1}) \neq 0 \\ &= (x+1) + (y+1) && \text{if } L(z^{-1}) = 0 \end{aligned} \tag{6.53}$$

Usually the exact order of the plant is not known and instead an approximate model (usually of a lower order) is introduced to describe its dynamics. An approximate model which can be found by time series analysis, open loop identification tests, or simple modelling via heat and material balances is useful in choosing initial values for

the controller polynomials. The choice of process model and its effect have already been discussed in the previous two chapters.

As in many other adaptive algorithms the choice of initial parameter values, $\theta(0)$, for SFC is very important to the overall performance since they not only determine the initial control action but also affect the trajectory of future parameter estimates. In the actual application an adaptive controller is often initialized with reasonable values to eliminate or reduce the uncertainty of the control action which possibly causes unacceptable I/O variation or even closed loop instability. For STC and APCS the initial parameters were calculated based on the identified process model. However, the parameters adapted in SFC are not the coefficients of the transfer function repesenting the process input/output but those of the transfer function between the control error, $\epsilon(k)$, and the auxiliary signal, $\eta(k)$, which is part of the control action. The parameter estimates of SFC are equivalent to the conventional controller coefficients rather than the coefficients of the process model. Therefore, the initial parameters of SFC can be obtained directly from the controller settings. For example, PID constants currently being used can be used as initial parameter values and SFC will generate an equal or better set of the PID constants.

6.4.2 Weighting Functions

Using equations (6.1), (6.2), (6.4) and (6.22) the closed loop response is given by:

$$y(k) = \frac{-B_m P y_s(k) + DQL_m v(k-q) + B_m \xi(k) + DQA_m \gamma(k)}{A_m DQ - B_m P} \tag{6.54}$$

Stability of the optimally controlled closed loop system is thus dependent upon the roots of the following chracteristic equation.

$$A_m(z^{-1})D(z^{-1})Q(z^{-1}) - B_m(z^{-1})P(z^{-1}) = 0 \tag{6.55}$$

Proper choice of the weighting functions enables the closed loop poles to be relocated and the transient response improved. This section will briefly describe the properties of the weighting functions.

(1) **P-Weighting:** The main purpose of the P polynomial is to control or manipulate the dynamic response of the control error. The performance index equation (6.12) is minimum when $P(z^{-1})\epsilon(k+d)=0$ if there is no penalty on the control action. In this case the polynomial $P(z^{-1})$ governs the error trajectory. For instance if $P(z^{-1})$ is a first order polynomial,

$$P(z^{-1}) = 1 - az^{-1}$$

then the corresponding error sequence will be forced to follow the exponential function.

$$\epsilon(k+d) = \epsilon(0)\cdot a^k, \qquad k=0,1,2,\cdots$$

where $\epsilon(0)$ is the control error at time k equal to zero. Obviously, if a is positive but less than unity the error will decay exponentially and if it is negative but less than unity the response will be a damped oscillation. $P(z^{-1})$ can also take the form of a lead or lag digital filter to filter out the control error. In any case it is desirable to choose a $P(z^{-1})$ polynomial so as to result in a satisfactory, stable, closed loop response.

Since the polynomial $P(z^{-1})$ is acting on the control error its weighting is effective not only on the error caused by setpoint changes but also that due to external disturbances. One guideline for choosing $P(z^{-1})$ is to think of it as a reference model. For example, for a step change in setpoint, $P(z^{-1})$ could be chosen to produce an output $P(z^{-1})(y_d-y)$ that represents the desired performance of the actual process. (Note, this model reference analogy is not exact because of modelling errors, etc. that result in feedback action.)

(2) **Q-Weighting:** The Q-weighting enables the design of SFC to be more flexible since including the $Q(z^{-1})$ polynomial gives the control law a structure indentical to that of a general type of discrete controller. From equation (6.25) when the measurable disturbance is not considered the auxiliary signal, $\eta(k)$, is calculated as

$$\eta(k) = \frac{T(z^{-1})P(z^{-1})}{V(z^{-1})Q(z^{-1})} \epsilon(k)$$

Substituting $\eta(k)$ into the control law, (6.8), gives the following controller equation.

$$u(k) = \frac{P(z^{-1})}{D(z^{-1})Q(z^{-1})} \left[\frac{T(z^{-1})+V(z^{-1})Q(z^{-1})}{V(z^{-1})} \right] \epsilon(k)$$

$$= \frac{b_0' + b_1' z^{-1} + \cdots}{1 + a_1' z^{-1} + \cdots} \epsilon(k) \qquad (6.56))$$

Therefore, appropriate choice of $Q(z^{-1})$ and $P(z^{-1})$ can make the control law structure identical to one of many forms of a general discrete controller. For example, if the desired controller form, e.g. Smith predictor, is expressed in the form of equation (6.56), then the design problem is to find the values of P and Q (and/or other parameters such as model order) that will produce the desired structure. Note that the result is an adaptive form of the specified controller. One disadvantage is that introducing $Q(z^{-1})$ increases the

number of controller parameters to be estimated and may slow down the parameter convergence rate. To increase the adaptation rate RLS or RAML estimation scheme can be used with the same controller structure.

6.5 Properties and Features of SFC

The purposes of this section is to summarize the major **properties** and **features** of SFC for convenient reference. (The **performance** of SFC on the evaporator application will be documented later in this chapter.) For convenience the properties and features of SFC are grouped into the following categories;

1) Structure
2) Robustness
3) Parameter estimation
4) Performance criterion

6.5.1 Structure

The structure or formulation of SFC is characterized by the following;

i) a classical, error driven SISO feedback structure (cf. Figure 6.1)

ii) as a special case, mathematically and structurally equal to the conventional, discrete PID feedback controller.

iii) global stability in the presence of any bounded stochastic or deterministic input

iv) for the deterministic case the control error converges to zero in a finite time (cf. theorem 6.1)

v) for the general case the control error converges to within a bound that corresponds to the minimum upper bound on the unmeasured external inputs (cf. equations (6.34) to (6.40))

vi) parameter convergence to the actual optimal values is not required but it is shown that the norm of the parameter error vector is a non-increasing function, e.g. if the adapted parameters attain or are initialized to 'reasonable values' then they will not 'blow-up' in the interim before convergence is attained (cf. equation 6.43)

vii) can be applied to 'nonminimal' system representations provided that the common factor between the input and the output polynomials is $D(z^{-1})$ which is the model of the external setpoints and disturbances (cf. section 6.3.1(2))

6.5.2 Robustness

SFC meets the necessary and sufficient conditions for a 'robust controller' (internal model principle) as defined by Francis, Wonham and Davison. Assuming that the adaptive SFC controller maintains the stability of the overall system then this 'robust structure' results in the following properties;

i) asymptotic tracking and regulation can be achieved in

the presence of unmeasured, bounded or **unbounded** external inputs of the type defined by $D(z^{-1})$ (cf.equation 6.5). This can be reguarded as a generalization of the familiar integral control feature of PID controllers, i.e. integral action results in zero offset (asymptotic tracking and regulation) for step inputs.

ii) asymptotic tracking and regulation can be achieved in the presence of modelling errors. The modelling errors do not have to be arbitrarily small and can include:

- nolinearities
- model order
- process or other system parameter errors
- time varying systems

To the best of author's knowledge, other adaptive controllers such as STR/C, APCS etc. do not have this error-driven robust structure. Therefore 'robustness' is a key distinguishing feature of SFC. (The practical significance of this robust structure will have to be established by extensive evaluations in a number of different applications.) SFC has an explicit error driven servo-compensator. In comparison, even for the special case of the STC algorithm with an incremental control signal, the implicit controller is driven by the predicted error and not the measured one.

6.5.3 Parameter Estimation

All adaptive controllers include some type of adaptive mechanism (e.g parameter estimation) plus a basic control strategy (e.g. predictive control). Many combinations of adaptive mechanisms and control strategy are possible and unfortunately there is no 'separation theorem' that allow them to be evaluated separately. The parameter estimation law used in SFC is the same projection type algorithm used by APCS and was selected primarily to facilitate proof of the stability and convergence theorems. However, the SFC adaptive mechanism:

i) directly estimates the controller parameters (in the special case these are the parameters in a conventional discrete PID controller)

ii) does not turn off as time increases (cf. RLS without a forgetting factor) and can therefore be applied to slowly time varying systems

iii) is simple and requires less computational time than recursive least squares (cf. section 7.2.5)

iv) switches off when the estimation error is small. This reduces computational load during normal steady state operation and appears to prevent problems like parameter windup or drift during extended periods of steady state operation (This on/off switching is part of the formal stability proof.)

v) because of the robust structure (see above) SFC can handle disturbances of the assumed class without

restarting parameter estimation. Thus disturbances do not destroy the process input/output relationship needed for good predictive control

vi) when parameter estimation is off, SFC is exactly equal (as a special case) to the conventional discrete PID feedback controller and/or robust controller. Thus its operation is easily understood by plant personnel. Instrument board and/or computer console displays can be made identical to conventional PID forms. (Initialization of the adaptive mechanism can be 'hidden' and left to the control engineer since it seldom requires human intervention.)

There is no guarantee that the performance of SFC in a given application will be better than other techniques. For example in the evaporator application there is some evidence that the SFC projection algorithm gives slower parameter convergence than the widely used recursive least squares. (However, to date a formal proof of SFC stability using RLS has not been completed.)

6.5.4 Performance Criterion

SFC includes a user-specified quadratic performance index with $P(z^{-1})$ weighting on the control error and $Q(z^{-1})$ weighting on the control action. (Similar to STC but added as part of this work to APCS.) The inclusion of this performance index provides;

i) a means of reducing the excessive control action that

characterizes many adaptive systems by selecting $Q(z^{-1})$

ii) a means of filtering or shaping the error signal by proper choice of $P(z^{-1})$. Notice, since SFC is error-driven, there is no separate weighting on the setpoint.

iii) proper choice of $P(z^{-1})$ and $Q(z^{-1})$ can result in a final control law for calculating the control action $u(k)$, that can be interpretted as the adaptive version of one of the familiar conventional controllers, e.g. SFC-PID other choices of P and Q could lead to 'Dahlin' or 'Smith Predictor' type compensators.

iv) a means of handling nonminimum phase systems

The inclusion of P and Q weighting provides desirable design flexibility but further work is required to develop design guidelines for the selection of P and Q for a specific application.

6.6 Simulation Study

The purpose of simulation study is to illustrate some of the properties of SFC discussed in the previous section and to evaluate and identify the conditions and the choice of initial parameter values that are required to control the pilot scale double effect evaporator. The simulation conditions were chosen to facilitate comparion of SFC with the STR/C, APCS and fixed gain PID runs described in the previous chapters.

Like any other controllers the choice of initial parameters and design constants of SFC is critical to the performance. Several runs were made to demonstrate the effect of:

1) Model order
2) Initial model parameters
3) Adaptive mechanism
4) Weighting functions.

A summary of SFC simulation runs is given in Table 6.1 and can be compared directly with the STC runs in Table 4.1 and the APCS runs in Table 5.1. The next section will discuss the individual simulation runs and the general results will be applied to the experimental runs. Since SFC uses the APCS adaptive law the adaptive portion of SFC can be turned on/off at any time. The straight line or dots just above the time axis in the figures, e.g. Figure 6.3, indicate whether adaptation is on (dots) or off (blank space).

6.6.1 Model Order

As in section 6.3.3 if the process model is second order, SFC takes the familiar conventional PID form and simliarly if the process is approximated by a first order model then SFC is identical to a conventional PI controller. Figure 6.3. and 6.4 show the adaptive PID and PI of SFC respectively. The solid dots at the bottom of figures indicate periods during which parameter adaptation is turned on. The output performance of the adaptive PID in Figure 6.3

Table 6.1 List of Simulation Runs Using SFC

Figure No.	Run No.	Intial O(O)	Ts (sec)	Model order	a(k) (upper)	Noise Bound	P wt	Q wt	Comments
	SF2001	$O_0/5$	180	2	0.1	.005	1	0	zero initial parameters
6.3	SF2002	O_1	64	2	1	.005	1	0	adaptive PID
6.4	SF2003	O_2	64	1	1	.005	1	0	adaptive PI
	SF2004	O_1	64	2	1	.005	$(1-.5z^{-1})$	0	Feed change and P-wt
	SF2005	O_1	64	2	1	.005	$(1-.5z^{-1})$	0	setpiont change and P-wt
6.5	SF2006	O_1	64	2	1	.005	1	0	setpoint change P-wt
	SF2007	O_1	64	2	1	.005	$(1-.2z^{-1})$	0	P-wt. cf.Sf2005 SF2006
	SF2008	O_1	64	4	1	.005	1	0.1	feed change and Q-wt
	SF2009	O_1	64	4	1	.005	1	1	setpoint change and Q-wt
	SF2010	O_0	64	2	10000	.005	1	0	RLS estimator cf. SF2011
	SF2011	O_0	64	2	10000	.005	1	0	APCS estimator cf. SF2010
	SF2012	O_3	64	2	10000	.005	1	0	RLS estimator cf. SF2010 SF2011
	SF2013	O_1	64	2	1	.005	$(1+.5z^{-1})$	0	Feed change and P-wt cf. SF2004
6.6	SF2014	O_1	64	2	1	.005	$(2+ 2z^{-1})$	0	Feed change and P-wtcf. SF2013
6.8	SF2015	O_1	64	4	1	.005	1	1	Feed change and Q-wt cf. SF2008
	SF2016	O_1	64	5	1	.005	1	$1+.5z^{-1}$	Feed change and Q-wt cf. SF2017
6.9	SF2017	O_1	64	5	1	.005	1	$1-.5z^{-1}$	Feed change and Q-wt cf. SF2016
	SF2018	O_1	64	5	1	.005	1	$1-.5z^{-1}$	setpoint change and Q-wt
	SF2019	O_1	64	2	1	.005	$1/(1-.5z^{-1})$	0	setpoint change and P-wt cf. SF2020
6.7	SF2020	O_1	64	2	1	.005	$1/(1-.3z^{-1})$	0	setpoint change and P-wt cf. SF2019
	SF3001	$O_1+0.0$	64	3	1	.005	1	0	third oder model cf. SF2002
	SF3002	$O_1+0.0$	64	3	1	.005	$(1-.5z^{-1})$	0	third oder model and P-wt cf. SF2002
	SF3003	$O_1+0.0$	64	4	1	.005	1	0.1	third oder model and Q-wt cf. SF2008

Note (i): O_0 =[0.0 0.0 0.0 0.01]
O_1 =[9.88 -15.48 5.42 1.0]
O_2 =[3.64 -4.18 1.0]
O_3 =[0.0 0.0 0.0 0.10]

Note (ii): Measurement noise is added to all runs.

is slightly better than the result obtained by PI settings in Figure 6.4. It is worthwhile to note that both gave good behaviour of the control action which is comparable to the control performance of STC and APCS with the PI type Q-weighting represented in Figure 4.17 and 5.12 respectively. (Note that Q=0 for SFC PID).

6.6.2 Initial Controller Parameters

The initial parameters of SFC can be obtained directly from the controller coefficients and a number of methods can be used to determine suitable parameter values: experience, simulation, tuning, etc. In this simulation study the initial parameter values were obtained from the PID or PI constants calculated from the evaporator model, equation (3.3). The detailed calculation of these PID or PI constants is given in chapter three. In this manner the choice of initial parameter is assumed to be on the same basis with STR/C and APCS, where the same model was used to determine the initial parameter values. Figure 6.3 and 6.4 are the results obtained from the PID and PI parameters respectively. In fact the simulated, linear evaporator behaves like a pseudo-first order process (cf. Figure 3.2) and thus a conventional PID controller with a large controller gain produced excellent control on this simulated evaporator. However, the real evaporator was very sensitive to the controller gain, as in section 3.5, and the effect of initial parameters on the control performance was therefore

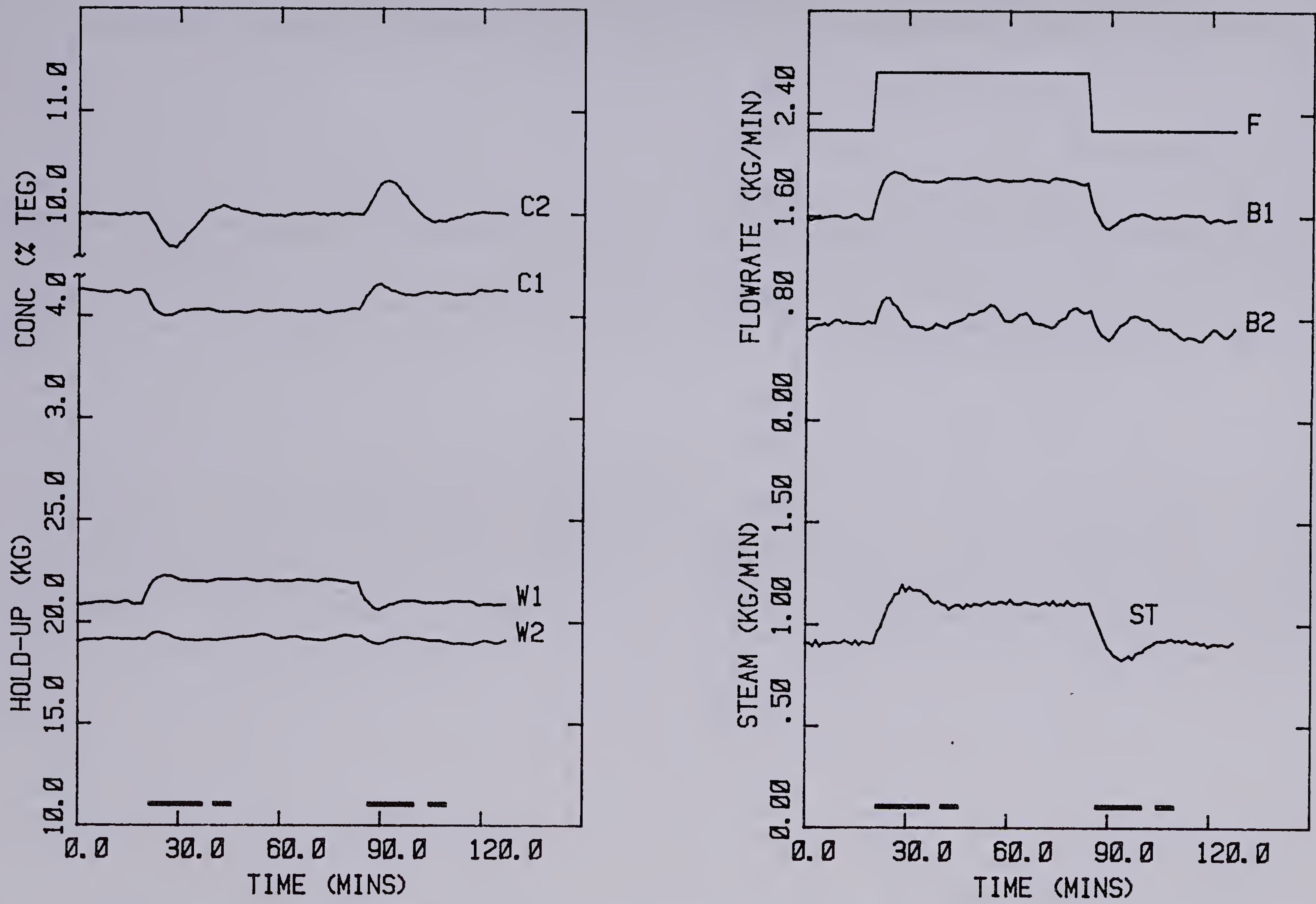


FIGURE 6.3 Simulated Evaporator Response Using SFC with Adaptive PID Mode
(SFC/SF2002/ITDM/T64/M2/C1/D.005/P1/Q0/ 20%FD/ ADAPTIVE PID CONTROL)

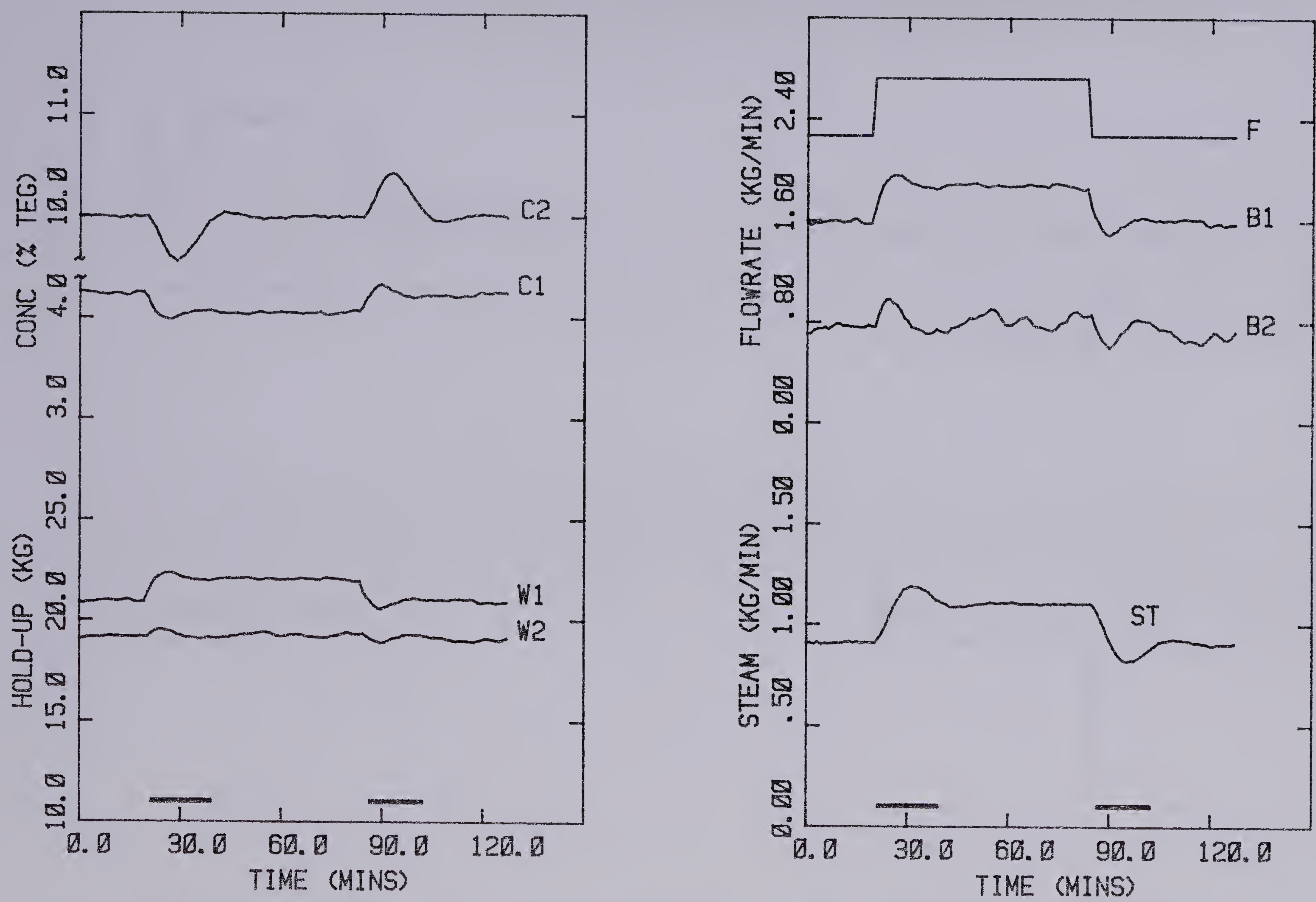


FIGURE 6.4 Simulated Evaporator Response Using SFC with Adaptive PI Mode
(SFC/SF2003/ITDM/T64/M1/C1/D.005/P1/Q0/ 20%FD/ ADAPTIVE PI CONTROL)

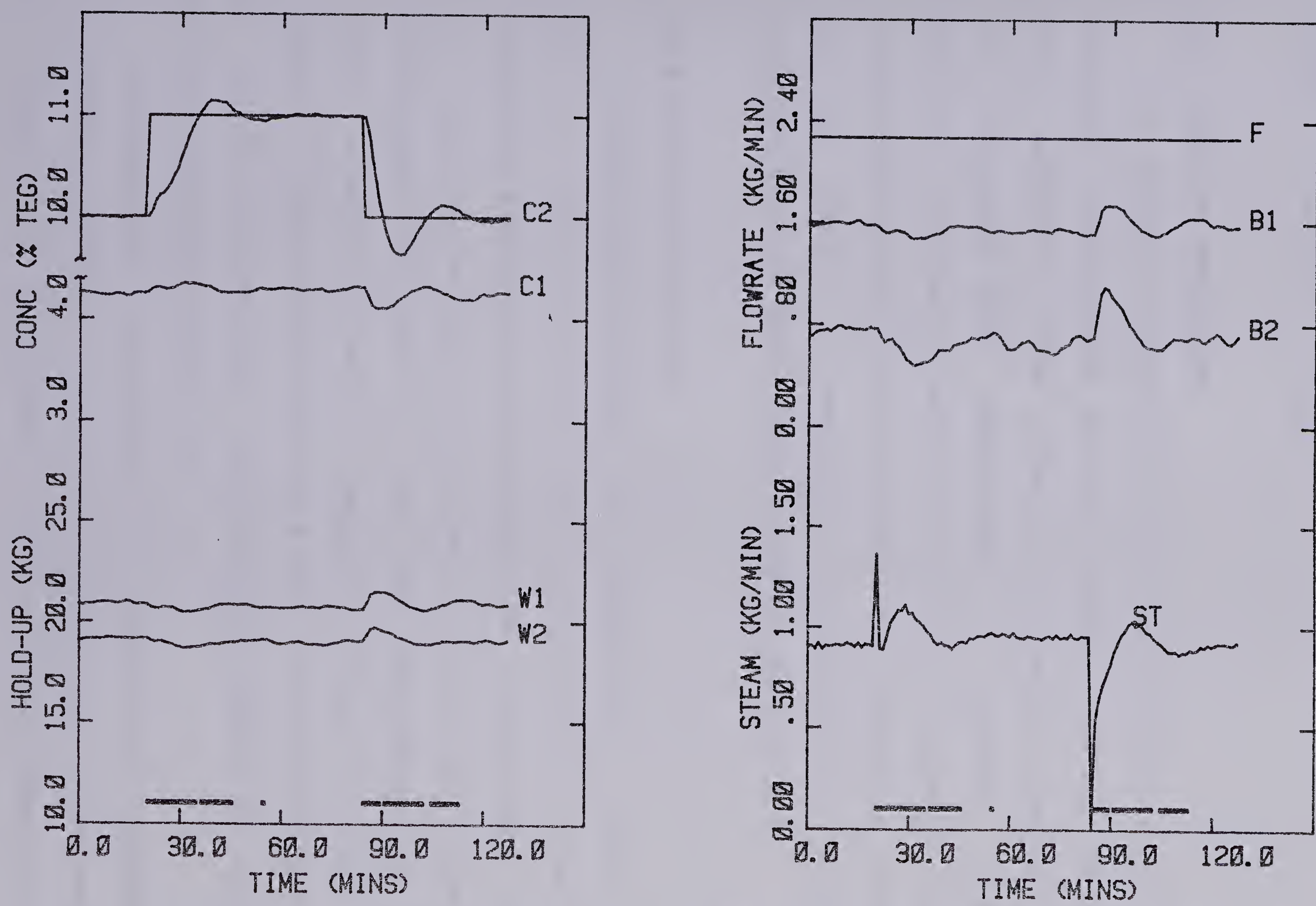


FIGURE 6.5 Simulated Evaporator Response by SFC PID to Setpoint Changes (SFC/SF2006/ITDM/T64/M2/C1/D.005/P1/Q0/ 10%SP/ ADAPTIVE PI SETPOINT CHANGE)

investigated further by experiment runs.

Figure 6.5 shows the adaptive PID control in the presence of setpoint changes.

6.6.3 Adaptive Mechanism

The adaptive algorithm used in this simulation is the one chosen based on APCS stability analysis and has the same design parameters. The effect of the design constants was fully discussed and demonstrated in section 5.5.3. Therefore, additional simulation runs are not presented here but the design factors such as the error correcting factor and the bound on unmeasured disturbances is further illustrated by the experimental study in section 6.6.

6.6.4 Weighting Functions

P-Weighting: The effect of P-weighting is shown in Figure 6.6, where a critical value (ringing pole) is used as a weighting function and as a result the control signal and the output becomes oscillatory due to the critical value of P-polynomial. The effect of the feed disturbance on the output is very much reduced compared to Figure 6.3 and 6.4. Figure 6.7 shows the P-weighting effect on setpoint changes. It reduces the overshoot significantly compared to the the non-weighted case (Figure 6.5)

Q-Weighting: Introducing a Q-weighting polynomial increases the order of controller polynomials. For example, in Figure 6.8, a constant Q-weighting was used and the

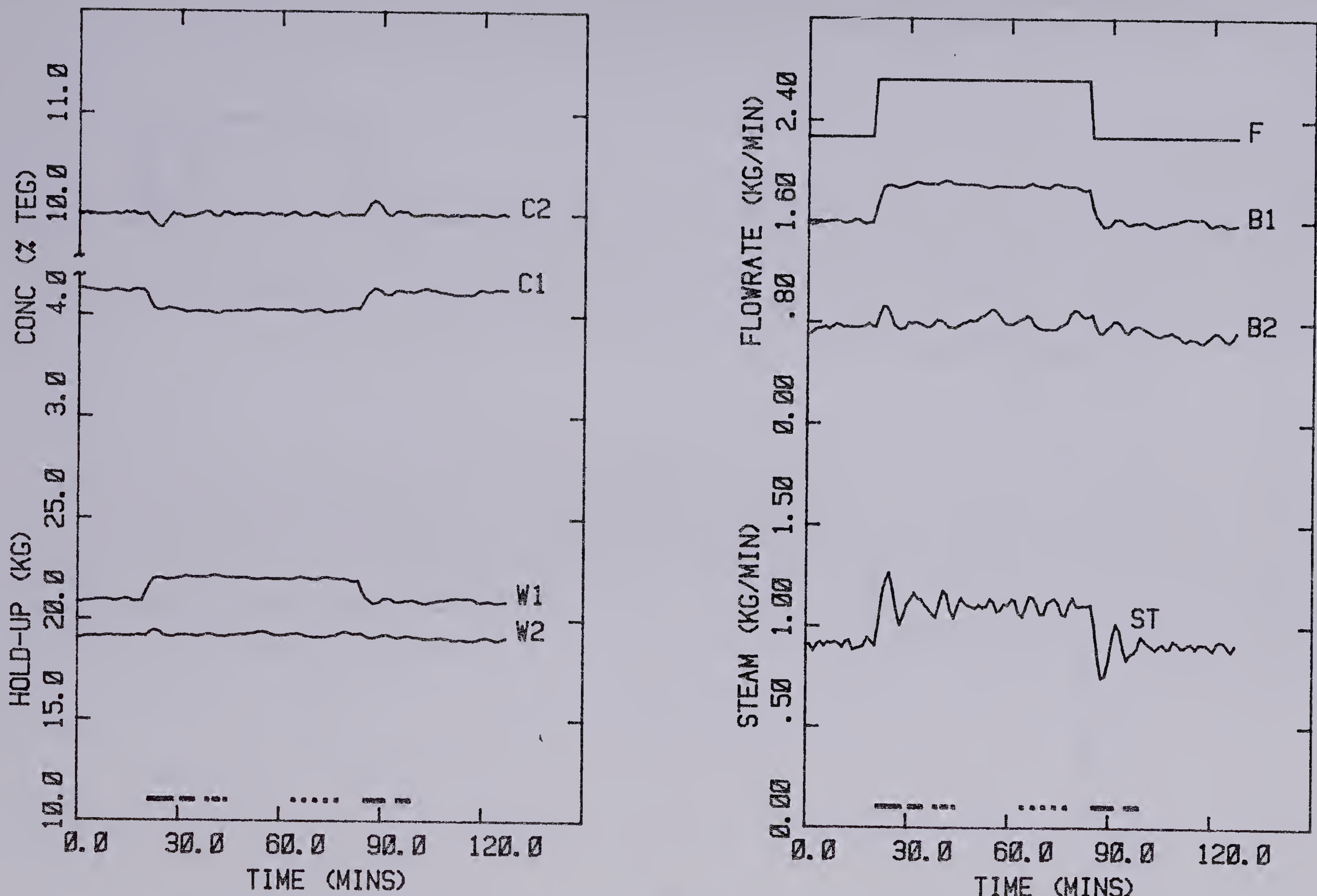


FIGURE 6.6 Simulated Evaporator Response by SFC PID with P-Weighting (SFC/SF2014/ITDM/T64/M2/C1/D.005/P2(1+z)/Q0/ 20%FD/ P-WEIGHTING)

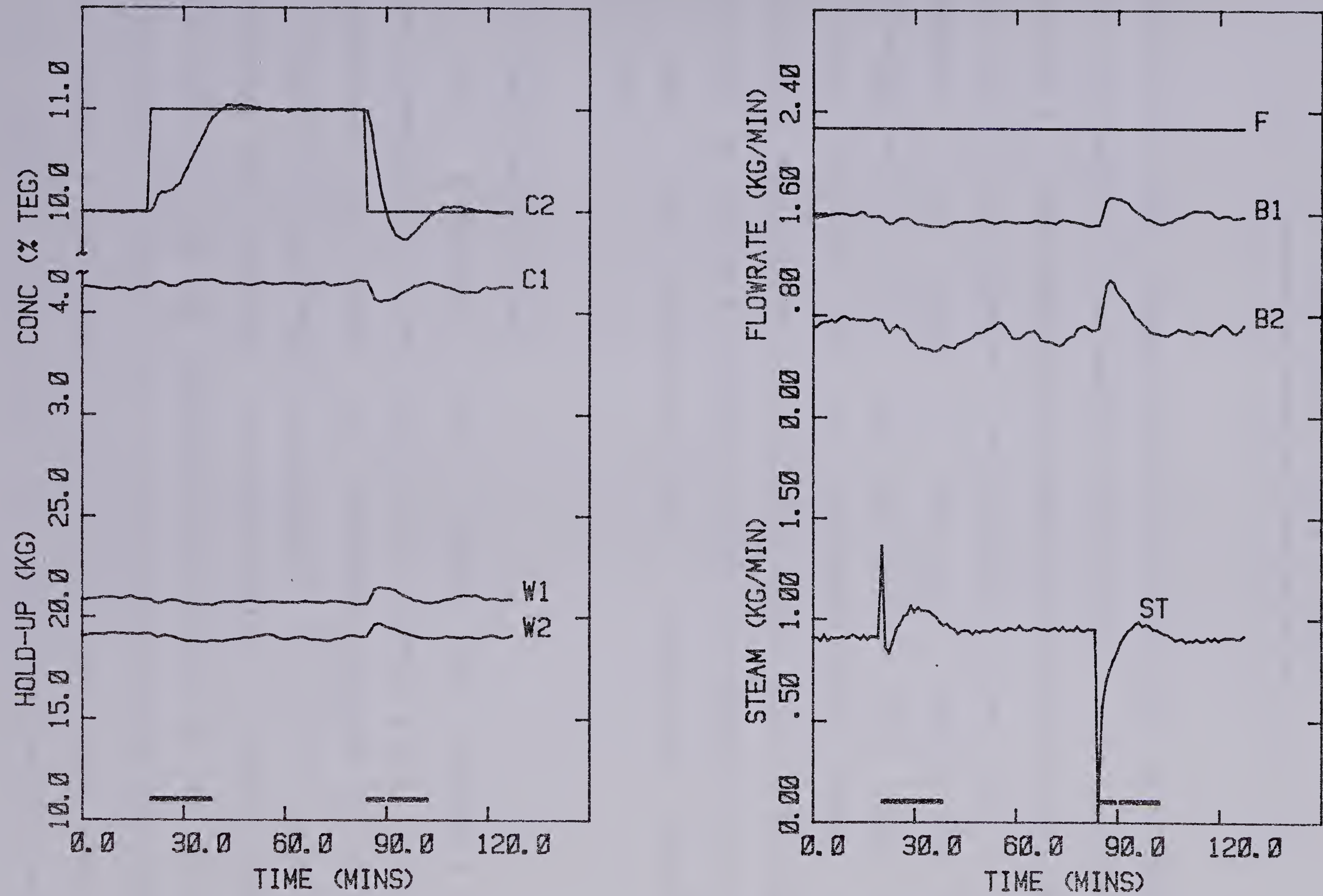


FIGURE 6.7 Simulated Evaporator Response Using SFC with P-Weighting
(SFC/SF2020/ITDM/T64/M2/C1/D.005/P$(1-.3z)^{-1}$/Q0/ 10%SP/ P-WEIGHTING)

corresponding controller structure becomes

$$u(k) = \frac{a_0' + a_1'z^{-1} + a_2'z^{-2} + a_3'z^{-3}}{(1-z^{-1})(b_0' + b_1'z^{-1} + b_2'z^{-2} + b_3'z^{-3})} e(k)$$

Therefore, seven parameters with b_0' fixed have to be updated at each sampling time and the solid line at the bottom of Figure 6.8 indicates the increased estimation interval while the control performance is not different from that of the adaptive PID (Figure 6.3 and 6.4). Figure 6.9 uses Q-weighting of a first order polynomial which shows the improved control and also increased time interval for parameter estimation. It can be concluded that Q-weighting provides an option to design an adaptive feedback controller of higher order structure and hence requires more parameter estimation.

6.7 Experimental Study

The simulation study served to illustrate the performance of SFC when applied to a linear plant but its capability to control real, nonlinear systems can only be evaluated by experimental application. To verify some of its features and evaluate its applicability to real processes the SFC algorithm was implemented and tested on the product concentration/steam loop of the pilot scale double effect evaporator at the University of Alberta. As in the case of other adaptive controllers the effects of various design

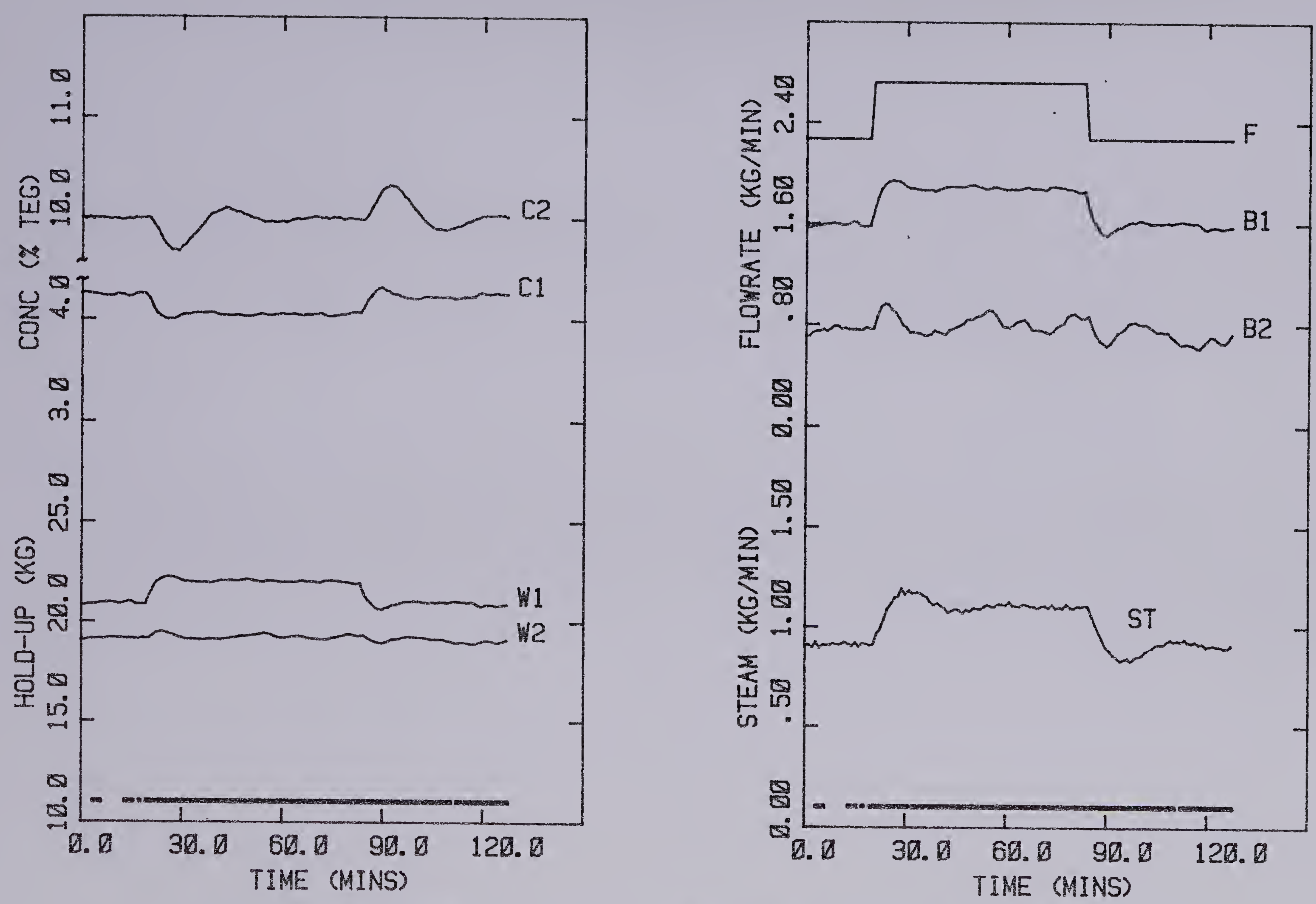


FIGURE 6.8 Simulated Evaporator Response Using SFC with Q-Weighting
(SFC/SF2015/ITDM/T64/M4/C1/D.005/P1/Q1/ 20%FD/ Q-WEIGHTING)

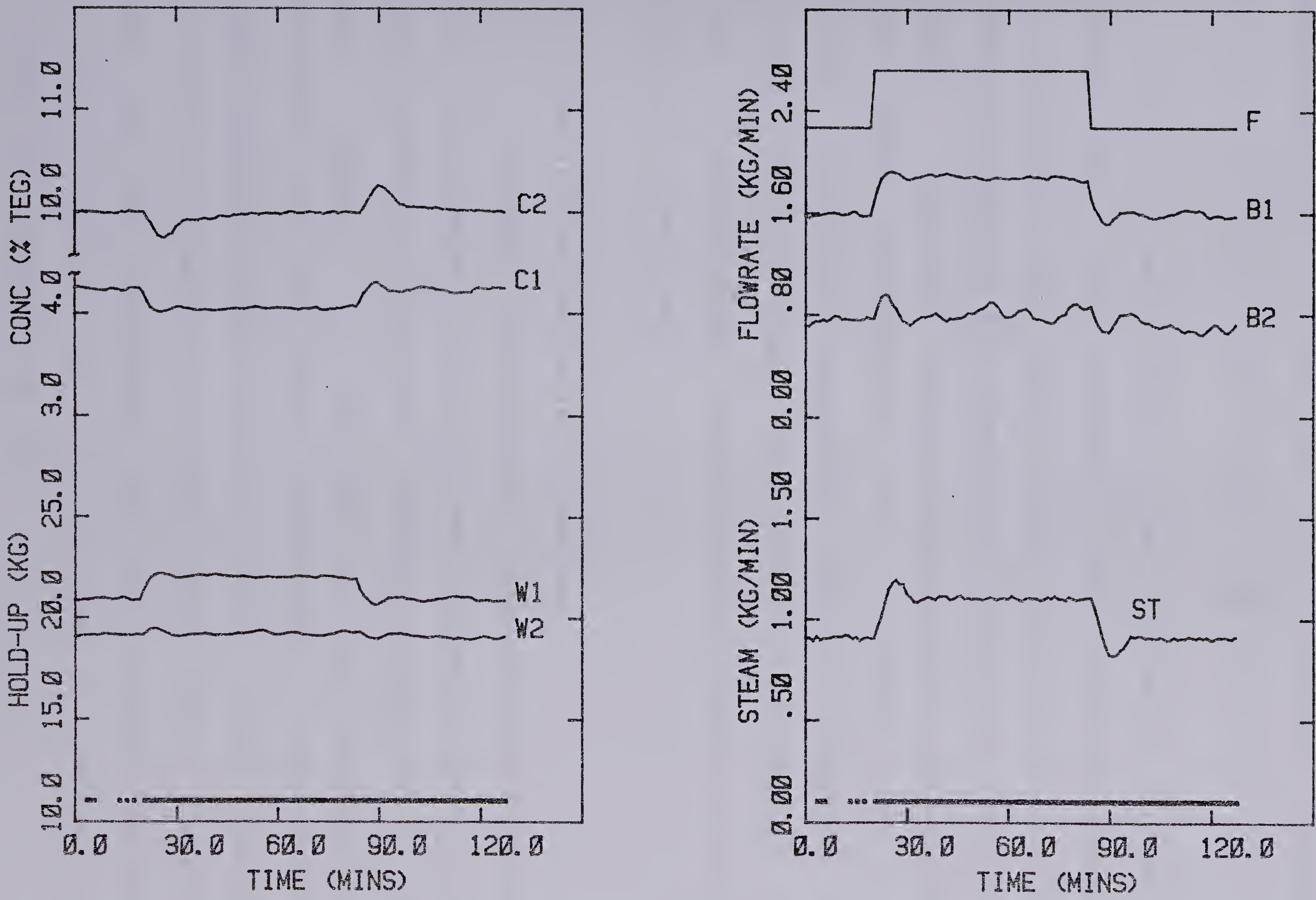


FIGURE 6.9 Simulated Evaporator Response Using SFC with Q-Weighting
(SFC/SF2017/ITDM/T64/M5/C1/D.005/P1/Q(1-.5z)/ 20%FD/ POLYNOMIAL Q-WT)

factors and parameters were examined and the results were also compared with the results from other adaptive controllers and conventional, PID controllers. Table 6.2 presents a summary of experimental runs conducted.

Since it was judged to be of greatest interest to applications the 'PID' form of SFC was the primary objective for evaluation and hence the most experimental runs were made using adaptive PID. However, for comparison with other adaptive controllers some runs were conducted using higher order models. The important areas for evaluation are: the initial controller parameters, adaptive mechanism, the choice of design constants (e.g. weighting functions), the order of the controller. The following discussion therefore emphasizes these areas.

6.7.1 The Initial Controller Parameters

In this study the open-loop, response of the evaporator to a step change in feed flow was recorded as shown by the dots in Figure 3.2. This was fitted by a simple first order model, equation (3.3) and PID controller constants were estimated using IAE technique as described in chapter three. These estimated values, KC=5.04, τ_i=6.06, τ_d=1.08, were used for initial estimates for the PID form of SFC. It should be noted that any technique that can be used to determine controller constants can also be used to generate the initial values required by SFC.

Table 6.2 List of Experimental Runs Using SFC

Figure No.	Run No.	Intial O(O)	Ts (sec)	Model order	a(k) (upper)	Noise Bound	P wt	Q wt	Comments
	RR2001	$O_0/2$	180	2	0.1	.005	1	0	low gain, long sampling time
6.18	RR2002	O_0	128	2	0.1	.005	1	0	adaptive PID, sampling time 128s
6.17	RR2003	O_0	64	2	0.1	.005	1	0	effect of a(t), cf. RR2002 RR2004
6.10	RR2004	O_0	64	2	1	.005	1	0	basic adaptive PID
6.22	RR2005	O_0	64	2	1	.005	$(1+z^{-1})$	0	critical P-wt, oscillation
6.21	RR2006	O_0	64	2	1	.005	$(1+.5z^{-1})$	0	stable P-wt cf. RR2005
6.25	RR2007	$O_0+0.0$	64	2(7)	1	.005	1	1	Q-wt higher order controller
6.11	RR2008	O_0	64	2	0.0	.005	1	0	fixed gain PID
6.14	RR2010	$O_0/2$	64	2	1	.005	1	0	low gain initial parameters
	RR2011	$3O_0/4$	64	2	1	.005	1	0	low gain initial parameters
6.15	RR2012	$3O_0/2$	64	2	1	.005	1	0	high gain initial parameters
6.16	RR2013	Ti=30.32	64	2	1	.005	1	0	large integral time initial
	RR2015	O_1	64	2	1	.005	1	0	PI initial parameters cf. RR2004
	RR2016	O_0	64	2	1	.005	1	0	initial parameters
6.20	RR2017	O_0	64	2	1	0.0	1	0	zero disturbance bound
6.19	RR2018	O_0	64	2	1	.015	1	0	large disturbance bound
6.27	RR3001	$O_0+0.0$	64	3	1	.005	1	0	third order process model
	RR3002	$O_0+0.0$	180	3	1	.005	1	0	long sampling time cf. RR3001
	RR3003	$O_0+0.0$	64	3	1	.005	$(1+z^{-1})$	0	critical P-wt cf. RR2005
6.28	RR3004	$O_0+0.0$	64	3	1	.005	$(1+.5z^{-1})$	0	stable P-wt cf. RR2006
6.26	RR3005	$O_1+0.0$	64	3	1	.005	1	0	third oder model, Q-wt cf. RR2007
6.23	RS2001	O_0	64	2	1	.005	1	0	setpoint change, adaptive PID
	RS2002	$O_0/3$	64	2	1	.005	1	0	setpoint change, sluggish
6.24	RS2003	O_0	64	2	1	.005	$(.2-.5z^{-1})$	0	setpoint change with P-wt

Note: O_0 =[10.88 -15.48 5.42 1.0]
O_1 =[4.64 -4.18 0.00 1.0]

The performance of the SFC using the PID values discussed above as initial values is shown in Figure 6.10. The process variables can be identified by reference to Figure 3.1. Comparison with previous work on the evaporator indicates that the control of C2 shown in Figure 6.10 is excellent for a SISO controller. The manipulation of the steam (ST) is moderate and does not include the spikes or rapid cycles often produced by minimum variance type adaptive controllers. The three adapted parameter values are plotted in Figure 6.13(a) and the equivalent continuous parameters (cf. equation 3.14) are shown in Table 6.3 for t=0 and t=120. Note that the period of rapid parameter change in Figure 6.13(a) coincides with the period having the large error and the largest perturbations in the I/O vector. This is as expected from an examination of the adaptive law in equation (6.32). The absolute change in parameter values as shown in Table 6.3 is small. However, the changes are significant when measured in terms of evaporator performance. Figure 6.11 shows the results of fixed parameter PID control using the same PID constants used at t=0 in Figure 6.10. Obviously the results in Figure 6.11 are unsatisfactory. Figure 6.11 also indicates the highly interative nature of the evaporator; e.g. changes in ST affect C2 but also W1 and hence B1 which affects C2. The fixed parameter PID controller could be retuned in a number of ways. However, Figure 6.12 shows the performance of a fixed gain PID controller using the parameters obtained by

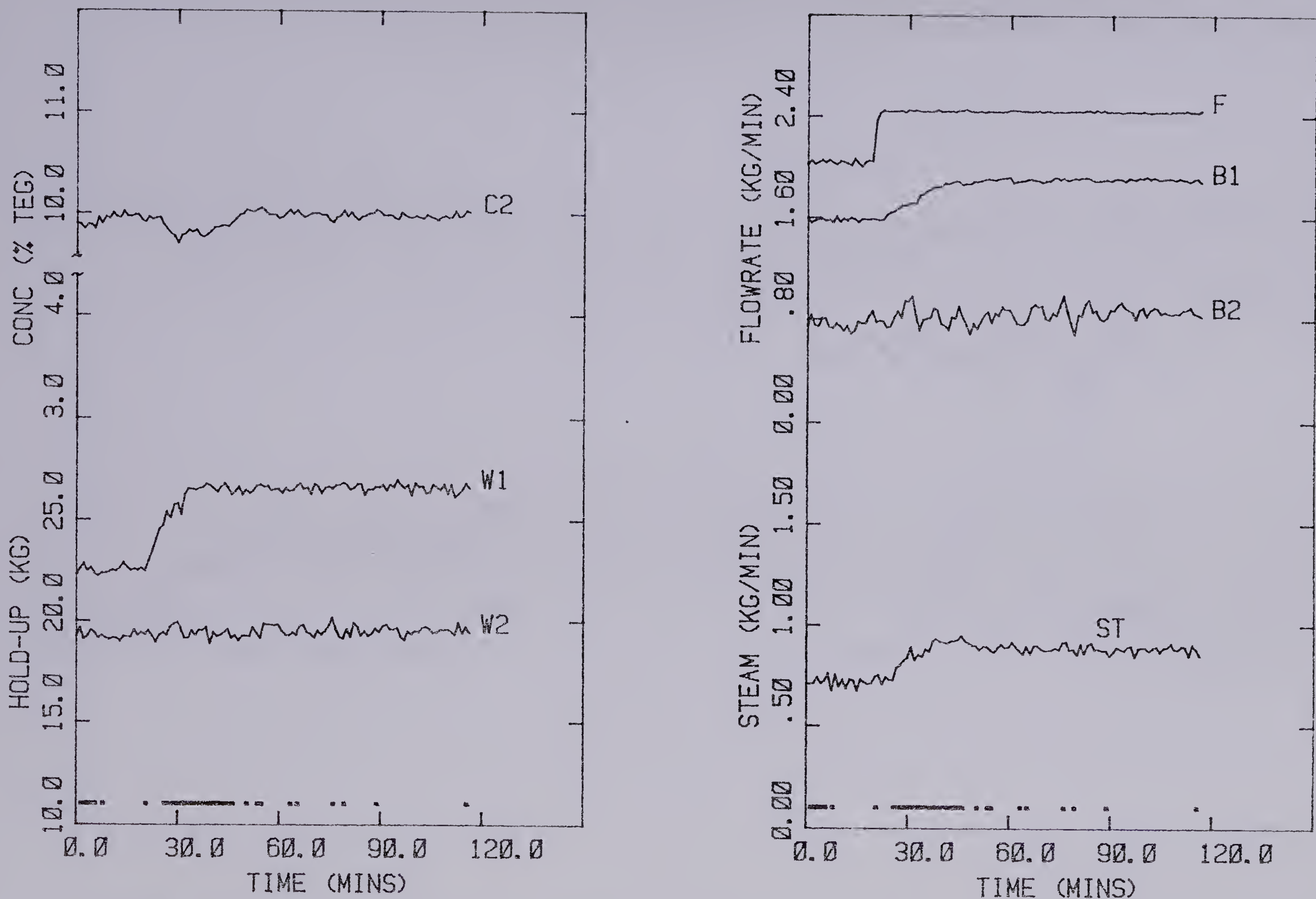


FIGURE 6.10 Evaporator Response Controlled by SFC with Adaptive PID
(SFC/RR2004/ITDM/T64/M2/C1/D.005/P1/Q0/ 20%FD/ SFC ADAPTIVE PID)

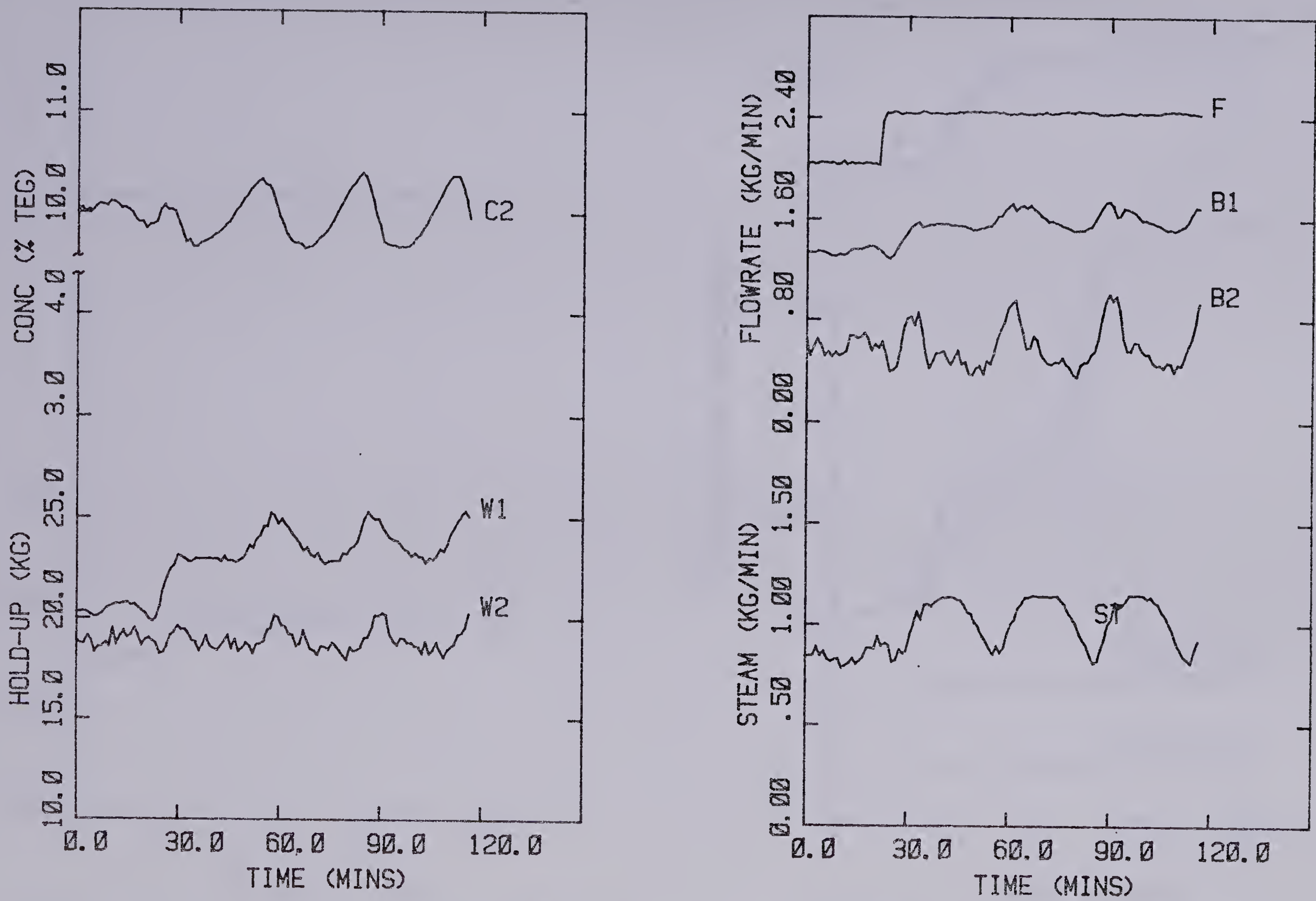


FIGURE 6.11 Evaporator Response Using Fixed Gain PID with Gains from Step Test
(PID/RR2008/ITDM/T64/PID/ 20%FD/ PID TEST WITH SETTINGS FROM STEP TEST)

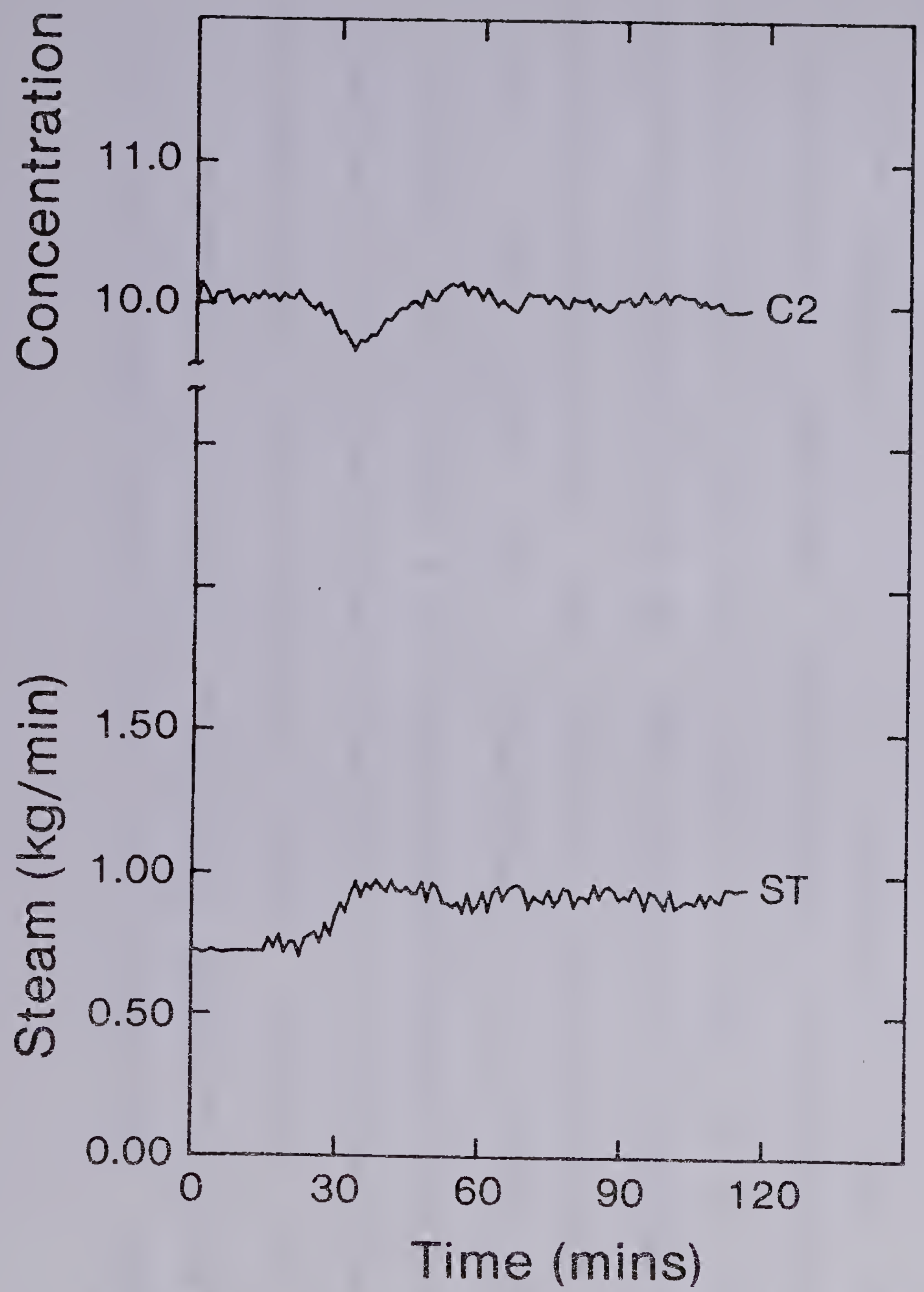


FIGURE 6.12 Fixed Gain PID Control with PID Gains Obtained from Fig. 6.10

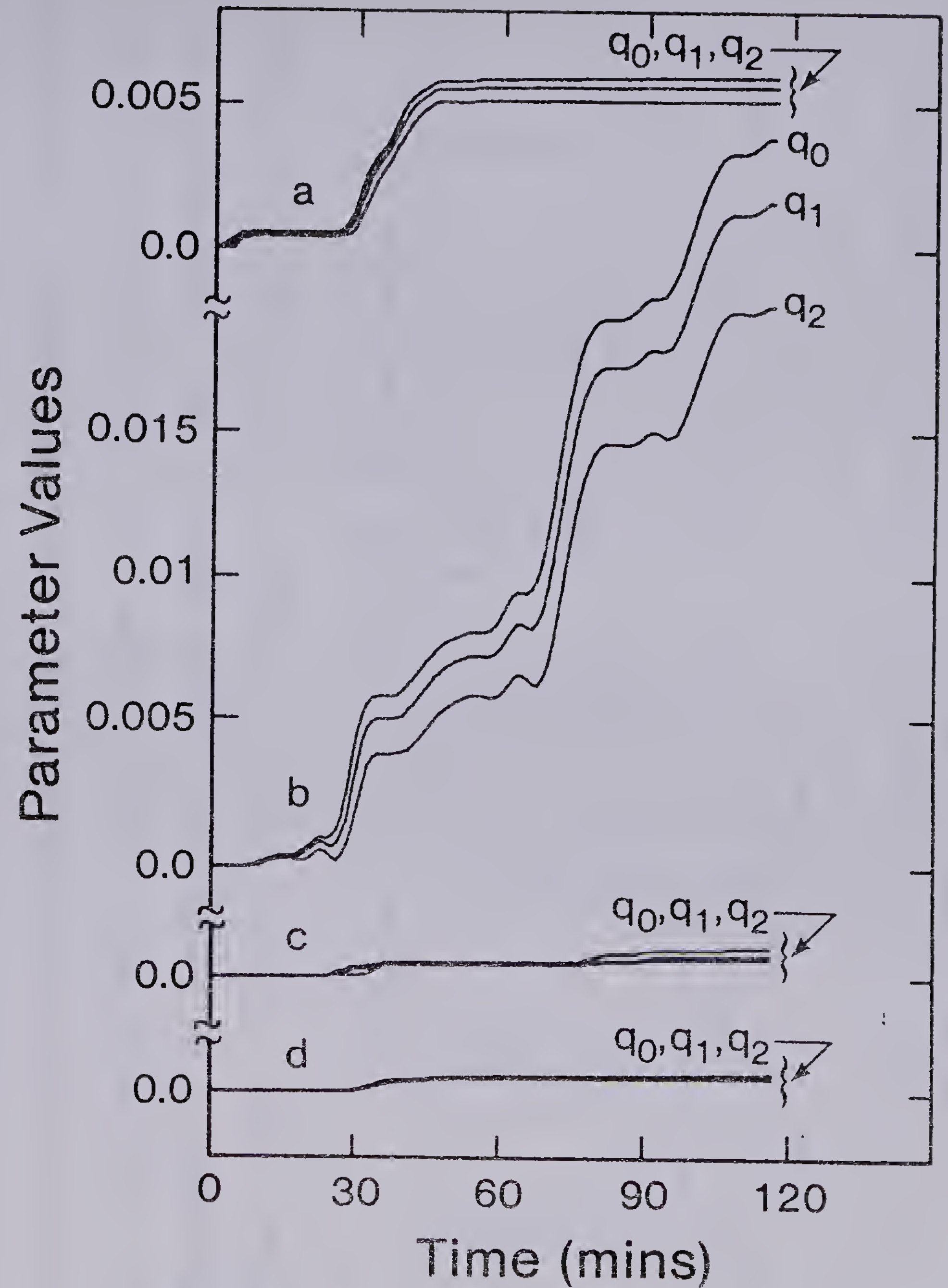


FIGURE 6.13 Parameter Trajectory of PID Controller Constants

Table 6.3 Initial and Final Values of Adaptive Gains
(Equivalent Continuous PID Parameters (cf. Eqn 3.14))

Figure No.	KC		τ_i (min)		τ_d (min)	
	t=0	t=120	t=0	t=120	t=0	t=120
6.10	5.04	4.96	6.06	5.55	1.08	1.11
6.11	5.04	5.04	6.06	6.06	1.08	1.08
6.12	4.96	4.96	5.55	5.55	1.11	1.11
6.13	2.52	2.44	6.06	1.54	1.08	1.20
6.14	7.56	7.53	6.06	5.70	1.08	1.09
6.15	5.04	5.01	30.76	21.40	1.08	1.09

Note experimental period, t, is in minutes.

SFC during run 6.10 (cf. Table 6.3 at t=120). The results are comparable to the SFC result in Figure 6.10 suggesting that if the evaporator were truly time-invariant then adaptation could be shut off permanently after about one hour. These results suggest that SFC can improve a marginal set of initial PID controller constants.

Two points are worthy of emphasis. First, for time-invariant processes the perfomance of the PID form of SFC will always be less than or equal to that of a fixed gain PID controller with the 'best' controller settings. The question is how can the 'best' PID controller constants be found! (SFC starts with the user-specified initial values and adapts them in such a way that the performance index in equation (6.12) is minimized). Secondly, it would be nice to be able to start with a very poor initial estimate of the PID parameters, e.g. zero, and have an adaptive controller that would maintain good control of the process and rapidly

adapt the parameters so they quickly reached the 'best' values. However, examination of adaptive mechanisms shows that rapid parameter adaptation occurs when the estimation error and/or the elements of the I/O vector are large. Thus poor initial estimates will result in poor control initially (certainty-equivalency principle) and/or a long adaptation period. Note that more sophisticated forms of SFC than the PID version and/or a different choice of adaptive mechanism could result in better performance.

The effect of selecting different initial values for the PID constants used by SFC can be seen by comparing Figures 6.14, 6.15 and 6.16 versus Figure 6.10. Note that the set of PID constants that produce a given process response, e.g. C2 in Figure 6.10, is not unique and hence an adaptive controller may not converge to the same set of values when started from different initial conditions. The worst results are in Figure 6.15 which started with an initial gain 50% larger than the value (5.04) that Figure 6.11 showed was already too large. The parameters for run 6.15 changed significantly and rapidly as shown in Figure 6.13(b) and the control of C2 seemed to be improving with time. (Unfortunately, run 6.15 could not be extended because of a film which forms on the glass prism of the on-line refractometer and introduces a bias into the measurement of C2 after 2.5 or 3 hours.) Figure 6.14 shows the performance when the proprotional gain was 50% lower than that used in Figure 6.10. In the previous adaptive PID experiment it was

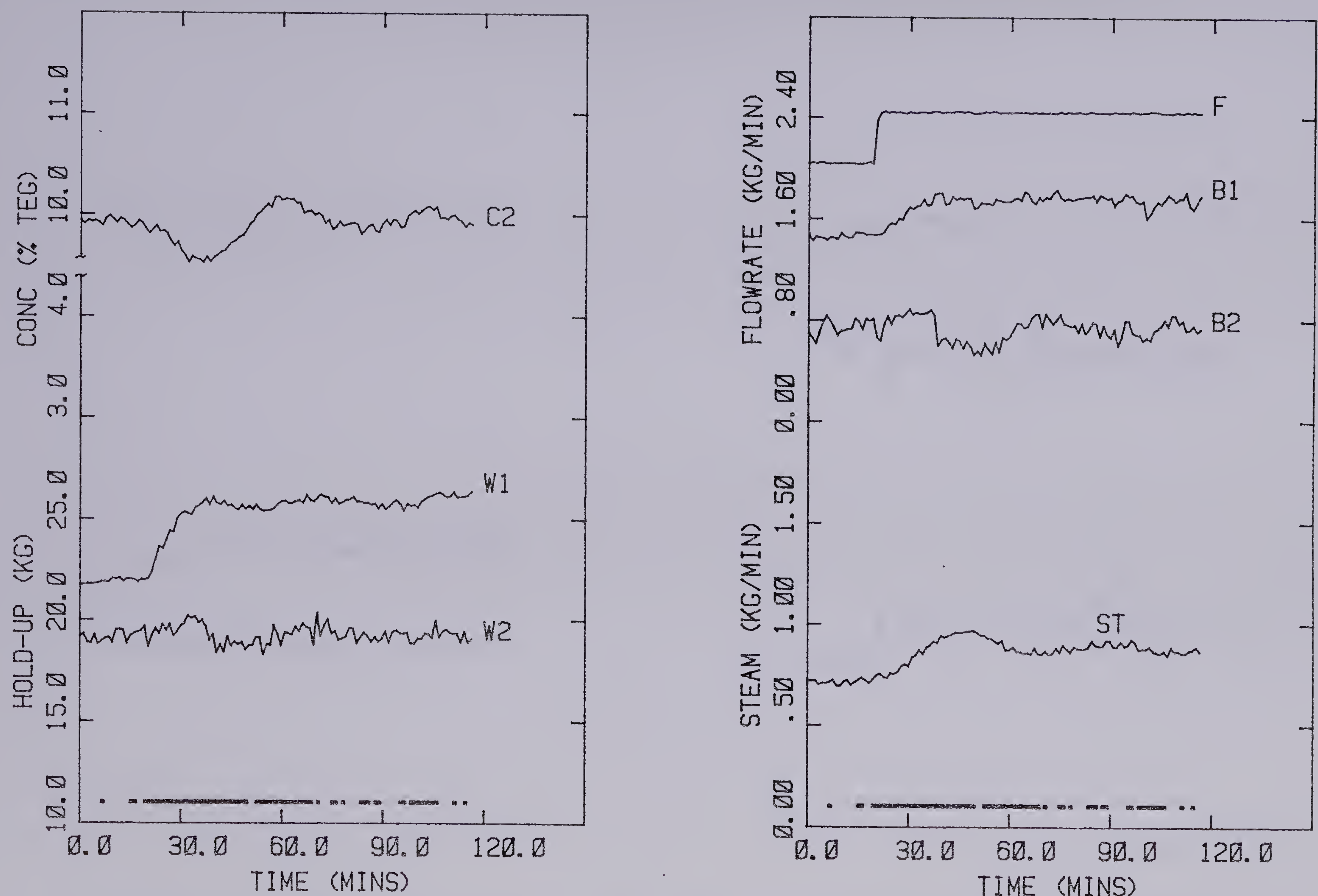


FIGURE 6.14 Evaporator Control Using SFC in PID Mode with Low Parameter Values
(SFC/RR2Ø1Ø/.5ITDM/T64/M2/C1/D.ØØ5/P1/QØ/ 2Ø%FD/ .5*GAIN OF RR2ØØ4)

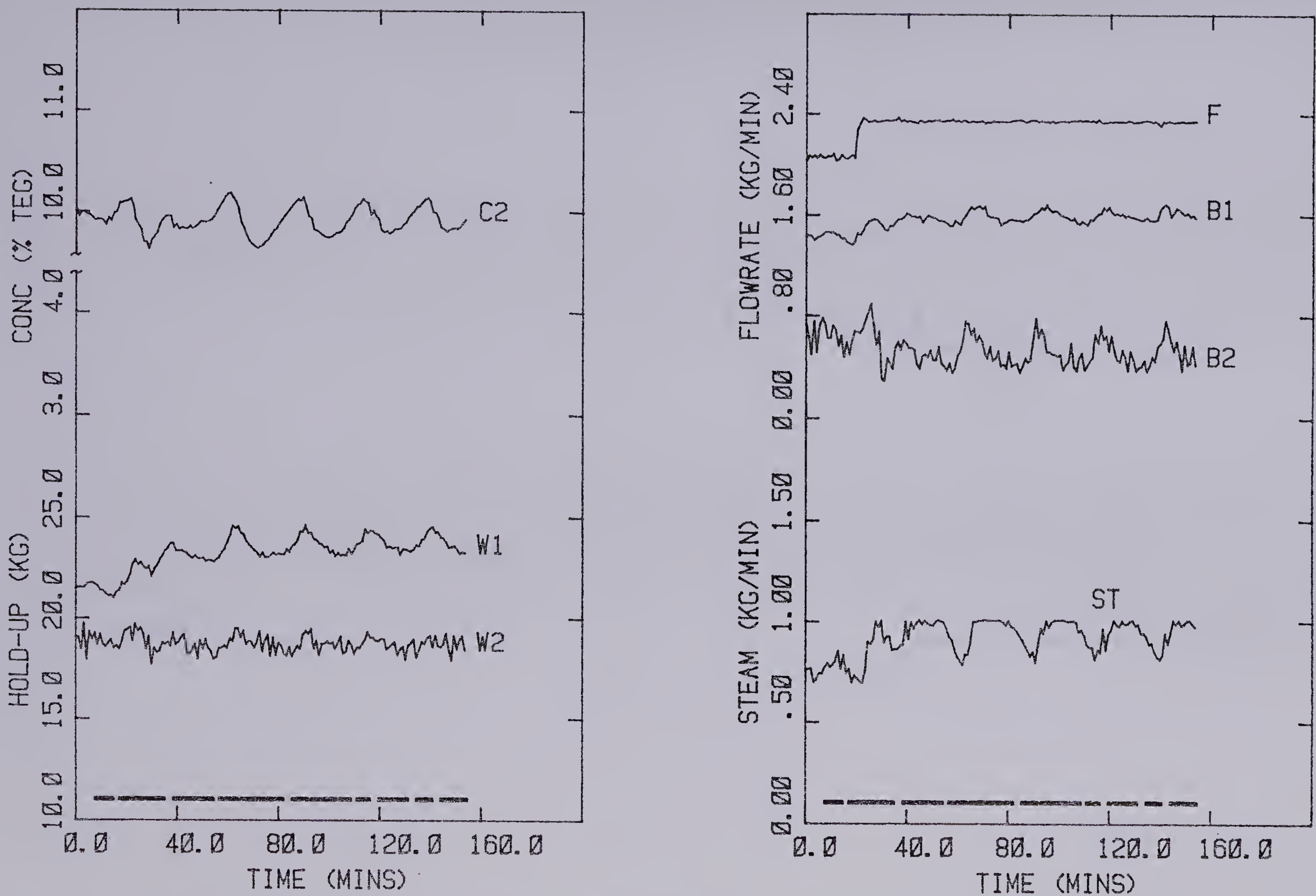


FIGURE 6.15 Evaporator Response Using SFC in PID with High Parameter Values
(SFC/RR2012/1.5ITDM/T64/M2/C1/D.005/P1/Q0/ 20%FD/ 1.5*GAIN OF RR2004)

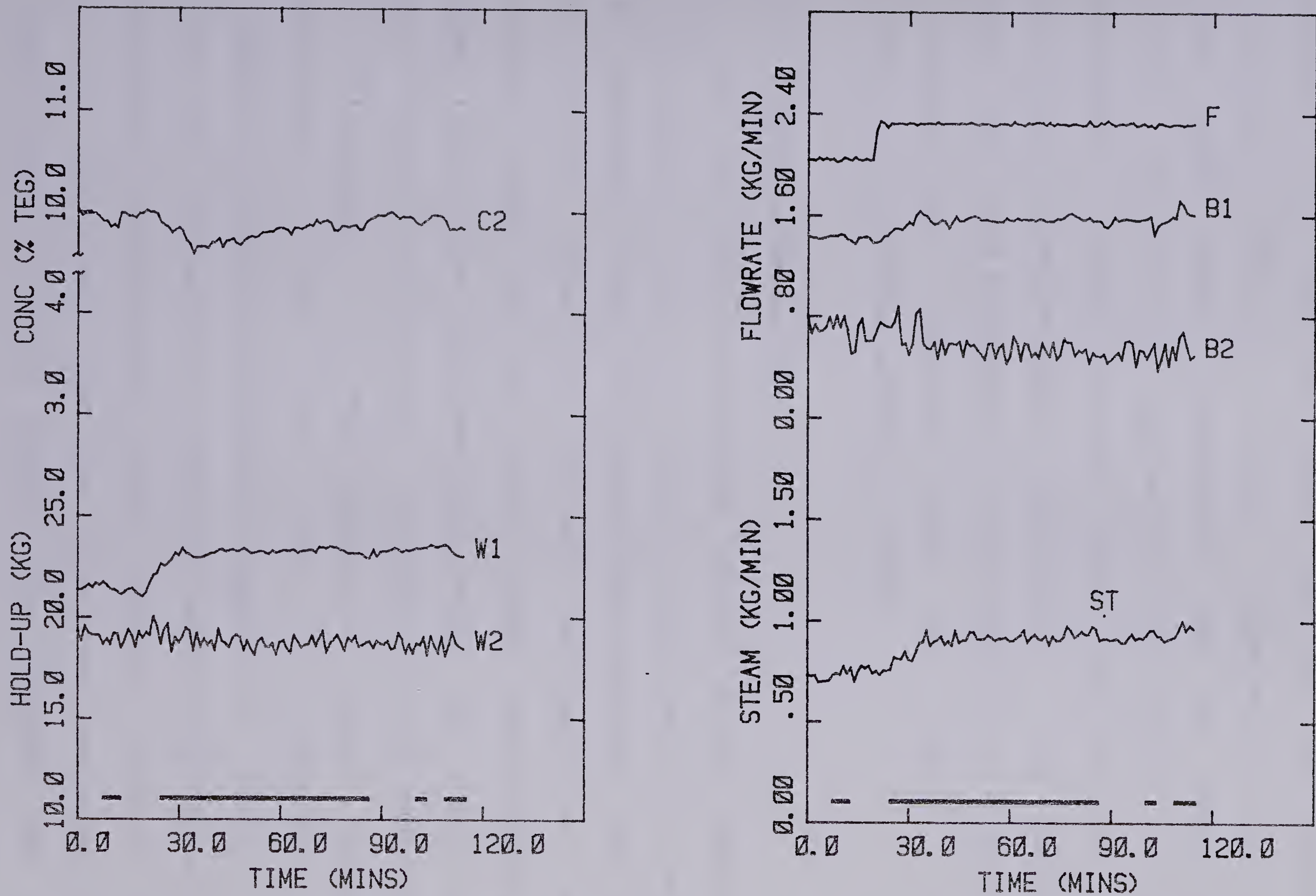


FIGURE 6.16 Evaporator Response Using SFC in PID with Large Integral Time
(SFC/RR2013/ITDM/T64/M2/C1/D.005/P1/Q0/ 20%FD/ 5*Ti (=30.3 MINS) OF RR2004)

noticed that the integral time constant τ_i changed most. In Figure 6.16 the parameters were initialized with an integral constant five times the original value with other constants unchanged. It shows slow integral compensation but still gives better overall performance than the constant PID (Figure 3.7) and it was observed that the integral constant decreased to about 25% of its initial value (30 minutes to 22 minutes). From the above experiments it was concluded that SFC could be used to tune PID settings for a specific application and is reasonably robust with respect to the choice of its initial parameter values.

6.7.2 Adaptive Mechanism

(1) **The Error Correcting Factor:** The user specified limit placed on the error correcting factor a(k) is one of key variables of SFC to be given before startup. Also since the initial parameters used were pre-identified, small values (usually less than two and greater than 0.1) for the upper limit of the factor were used while the lower limit set to zero or close to zero. In Figure 6.17 the upper limit of a(k) are set to 0.1, which results in slower parameter adaptation (Figure 6.13(d)) and as a consequence the response is slightly oscillatory compared to Figure 6.12 where the limit was unity. This oscillatory response was not improved by increasing the control interval in Figure 6.18. On the other hand when the upper limit was equal to or greater than five it was observed the response was more

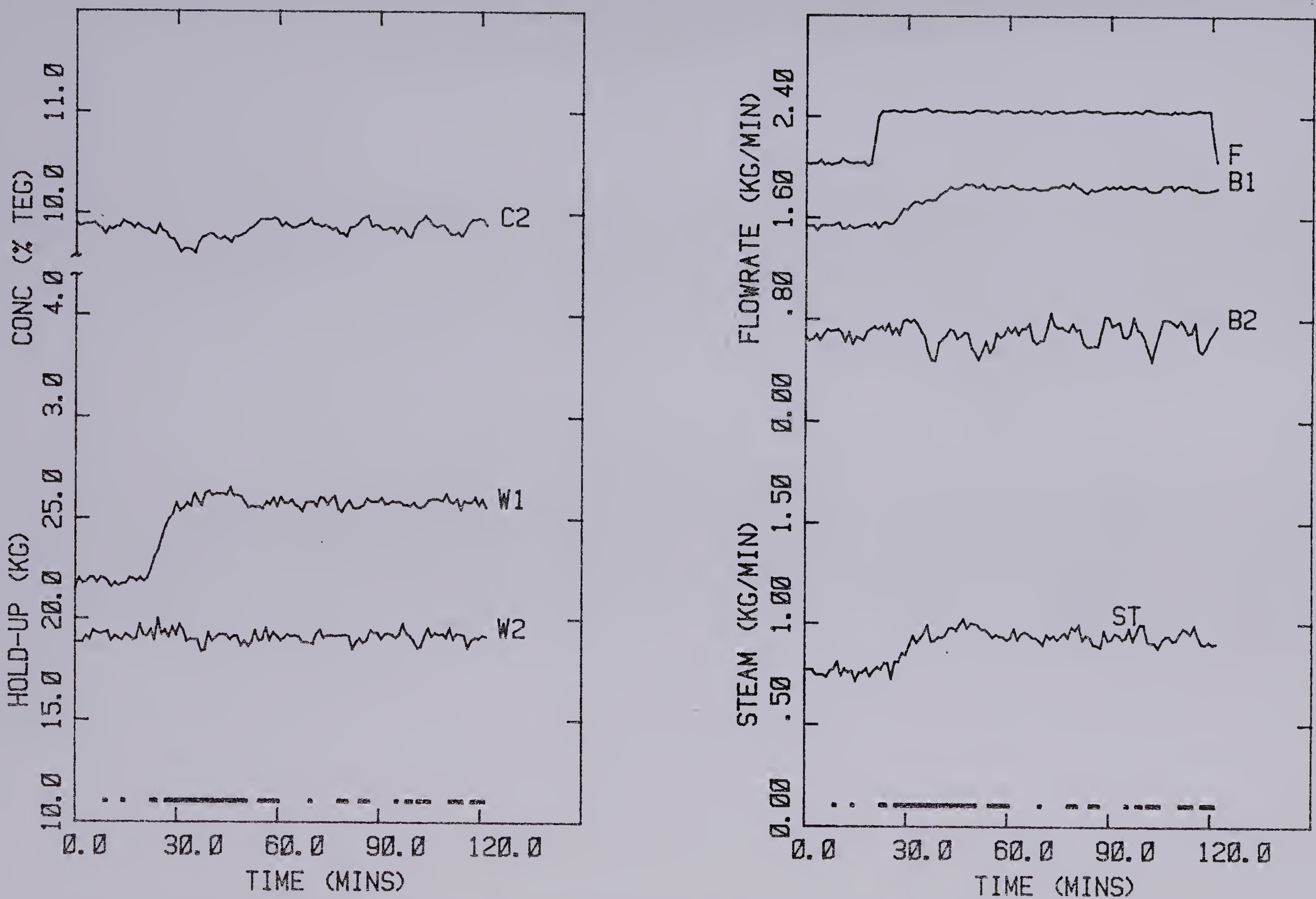


FIGURE 6.17 Evaporator Response Using SFC in Adaptive PID with Small a(k) (=0.1)
(SFC/RR2003/ITDM/T64/M2/C.1/D0.005/P1/Q0/ 20%FD/ WEIGHTING of. RR2004)

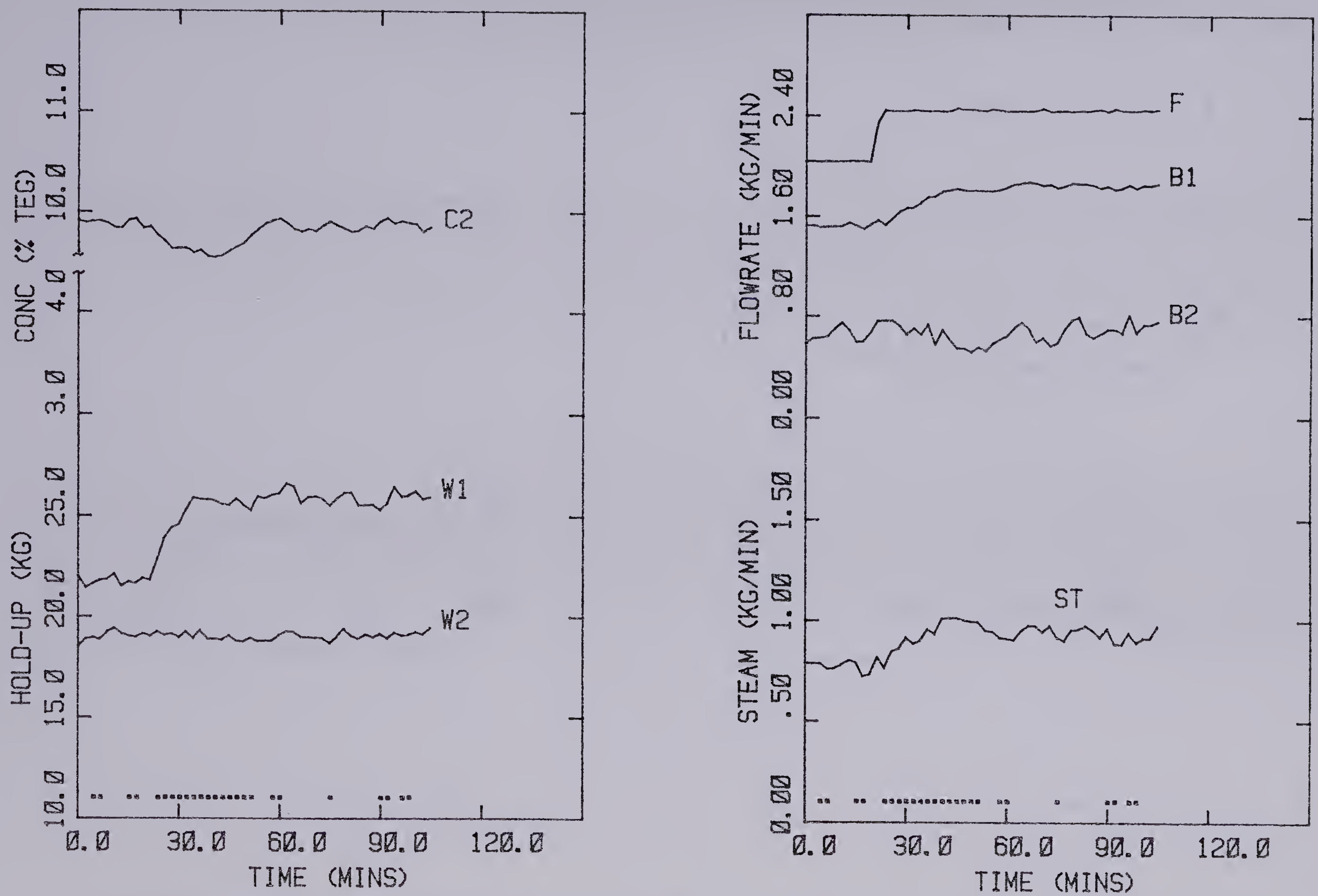


FIGURE 6.18 Evaporator Response Controlled by SFC with PID Structure (Ts=128sec)
(SFC/RR2002/ITDM/T128/M2/C.1/D0.005/P1/Q0/ 20%FD/ SAMPLING TIME cf. RR2001)

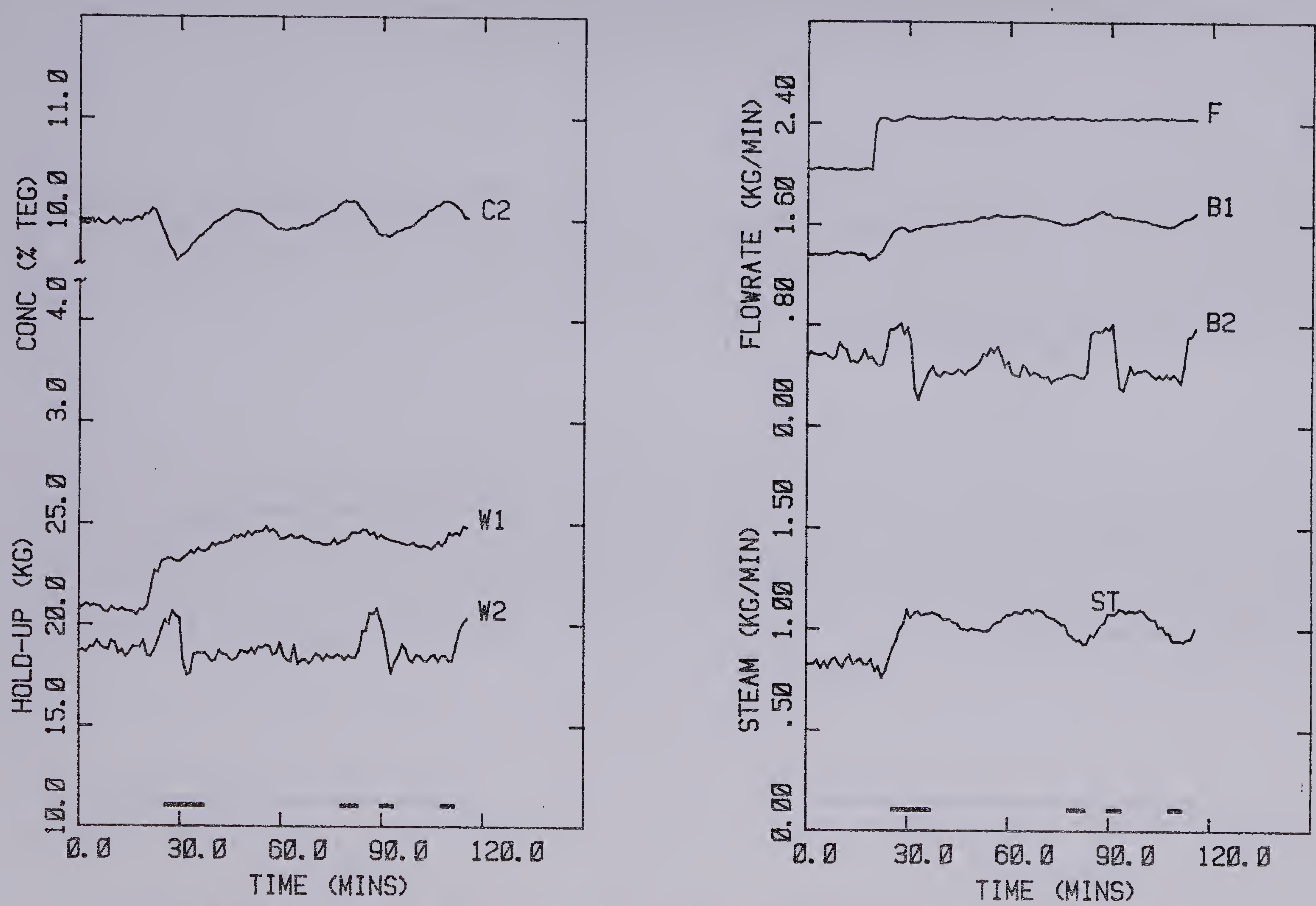


FIGURE 6.19 Evaporator Response Using SFC in PID with Large Disturbance Bound
(SFC/RR2018/ITDM/T64/M2/C1/D0.015/P1/Q0/ 20%FD/ DISTURBANCE BOUND)

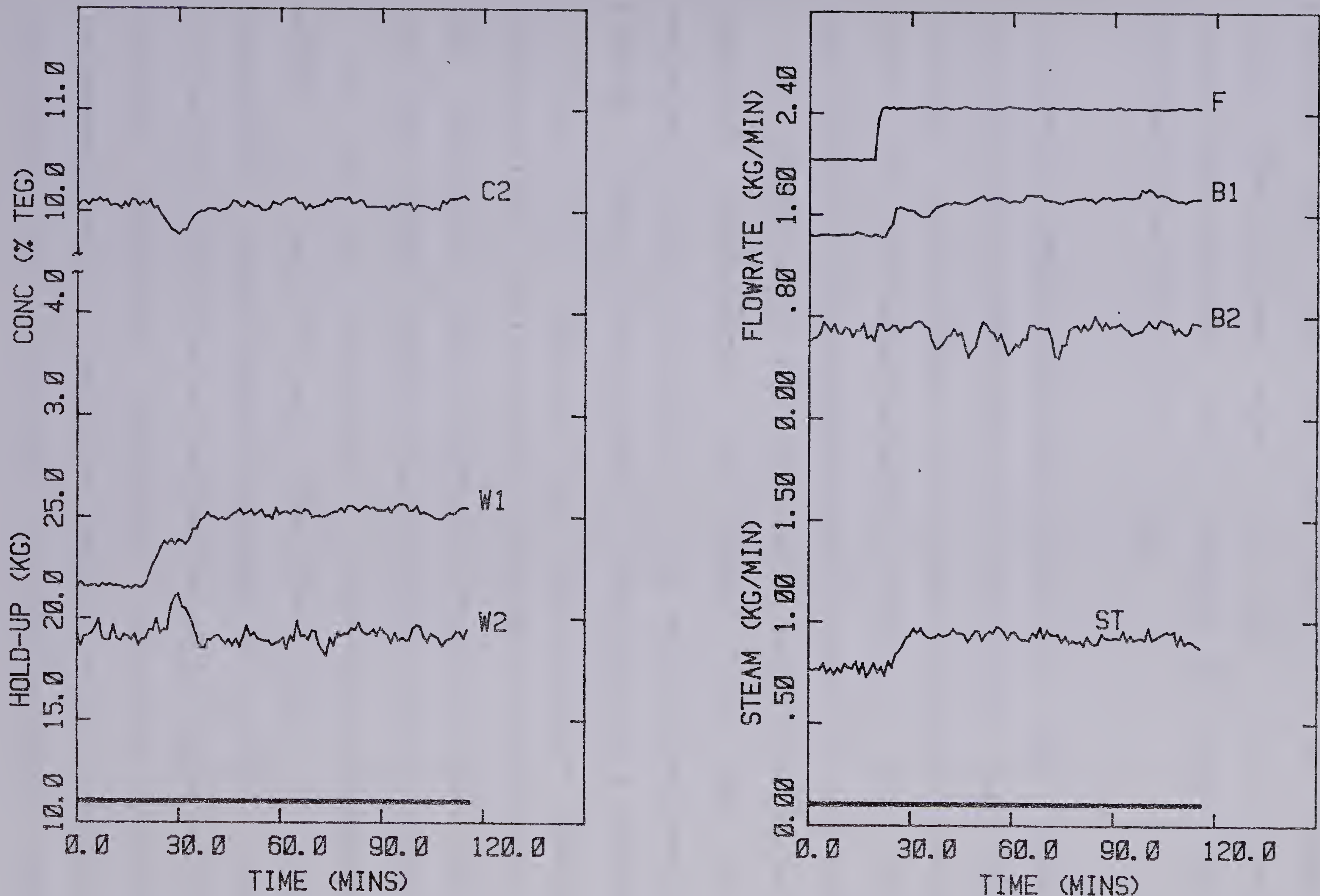


FIGURE 6.20 Evaporator Response Using SFC in PID with Zero Disturbance Bound
(SFC/RR2017/ITDM/T64/M2/C1/D0.0/P1/Q0/ 20%FD/ SFC ZERO BOUND cf. RR2004)

oscillatory due to large changes of the controller parameters. It is worthwhile to note that if the upper and the lower limits of a(k) are set to zero the SFC based on second order process model turns out to be a discrete, constant PID controller. This property has been used to doublecheck the SFC control program.

(2) **Bound on Unmeasurable Disturbance:** The upper bound on the unmeasurable and/or the modelling residual (perturbation variable), Δ_d, determines the parameter adaptation dead zone and hence the controller behaviour. Since the actual minimum upper bound for the evaporator was not known Δ_d was chosen to be 0.005, which is the value calculated from the evaporator noise model (cf. Figure 6.10). When Δ_d is increased to 0.015 in Figure 6.19 the control performance is similar to that of the discrete, conventional PID controller of Figure 6.11. There was not enough parameter adaptation (Figure 6.13(c)) at the initial stage and also during the load disturbance phase compared to Figure 6.13(a) where Δ_d was 0.005. Another extreme case is Figure 6.20 in which Δ_d is set to zero indicating a noise-free, deterministic process. As can be seen from the graph the output performance is very close to the case Δ_d is 0.005 in Figure 6.12 but the solid line at the bottom of the graph indicates continuous adaptation. Therefore, it can be concluded that the overestimation of the bound results in poor control with less computational effort while the underestimation requires

more parameter adaptation effort with no significant or noticable improvement of control as well as control parameter drifting. In practice a proper compromise has to be made on this bound.

6.7.3 Weighting Functions

(1) **P-Weighting:** For the regulatory control situation SFC without any weighting functions is shown to provide excellent control. However, some runs were made to illusrate the effect of weighting functions. In Figure 6.21 $P(z^{-1})$ was chosen to be a stable polynomial, $(1+.5z^{-1})$, without Q-weighting. The control performance was better than without P-weighting in Figure 6.10 in the sense that the control signal was noticably smoother. In a second experiment $P(z^{-1})$ was artificially selected to have critical value, i.e. ringing dynamics $(1+z^{-1})$ in Figure 6.22. As a result the output response in Figure 6.22 is oscillatory. Note that when the process dynamics are assumed to be of a second order type SFC ends up with an adaptive PID structure if $P(z^{-1})$ is unity and $Q(z^{-1})$ zero (Figure 6.10) but if $P(z^{-1})$ is other than unity and $Q(z^{-1})$ zero, the controller structure will still be a discrete PID acting on errors filtered by the P-polynomial (cf. Figure 6.21). $P(z^{-1})$ polynomial weighting can also be used to control the manipulative variable for the servo control problem as shown in the preceding simulation runs. Figure 6.23 shows the response to the setpoint change by SFC with a PID structure.

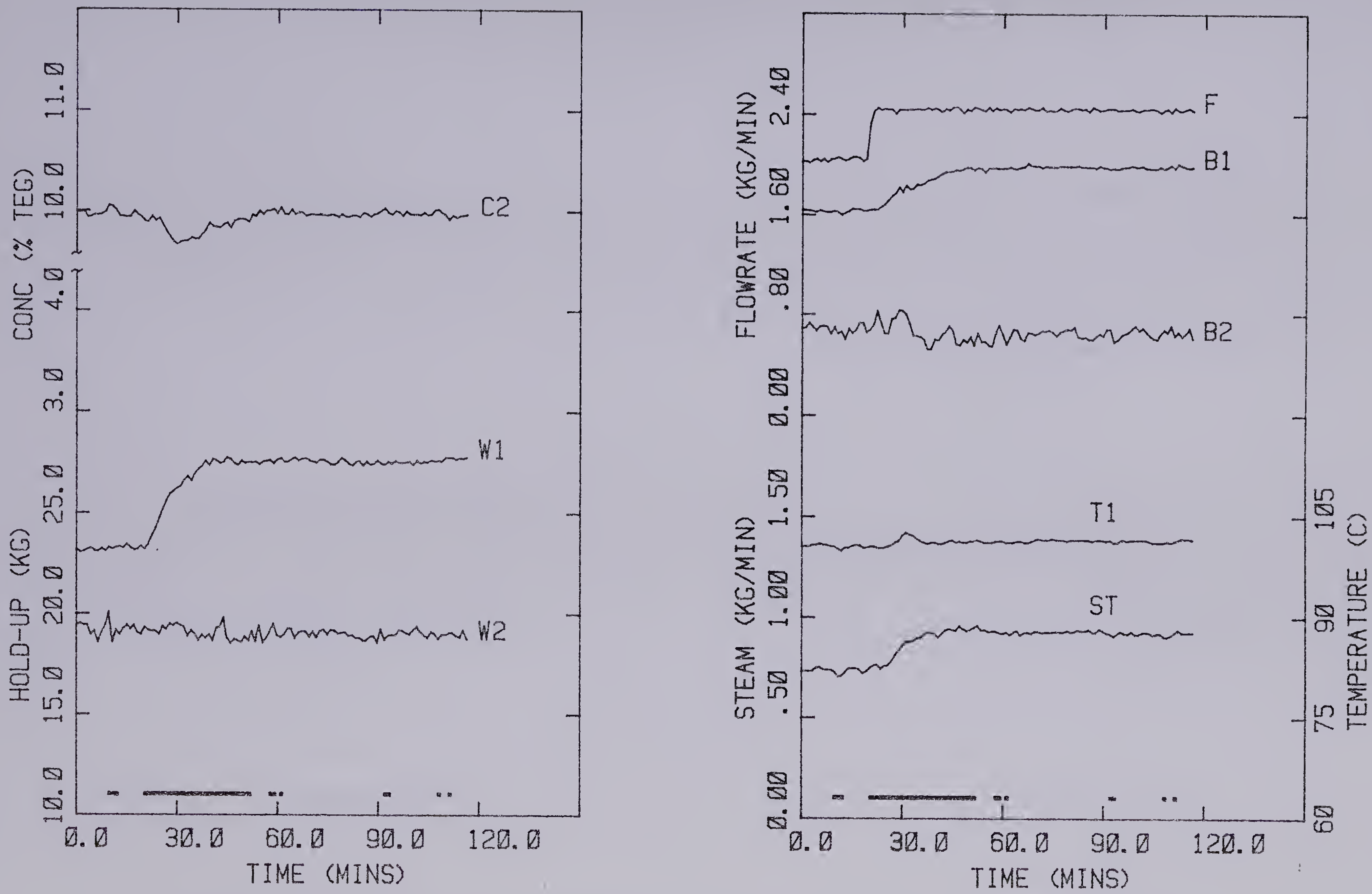


FIGURE 6.21 Evaporator Response Controlled by SFC with P-Weighting
(SFC/RR2006/.5*ITDM/T64/M2/C1/D.005/P(1+.5z)/Q0/ 20%FD/ P-WT cf.RR2005)

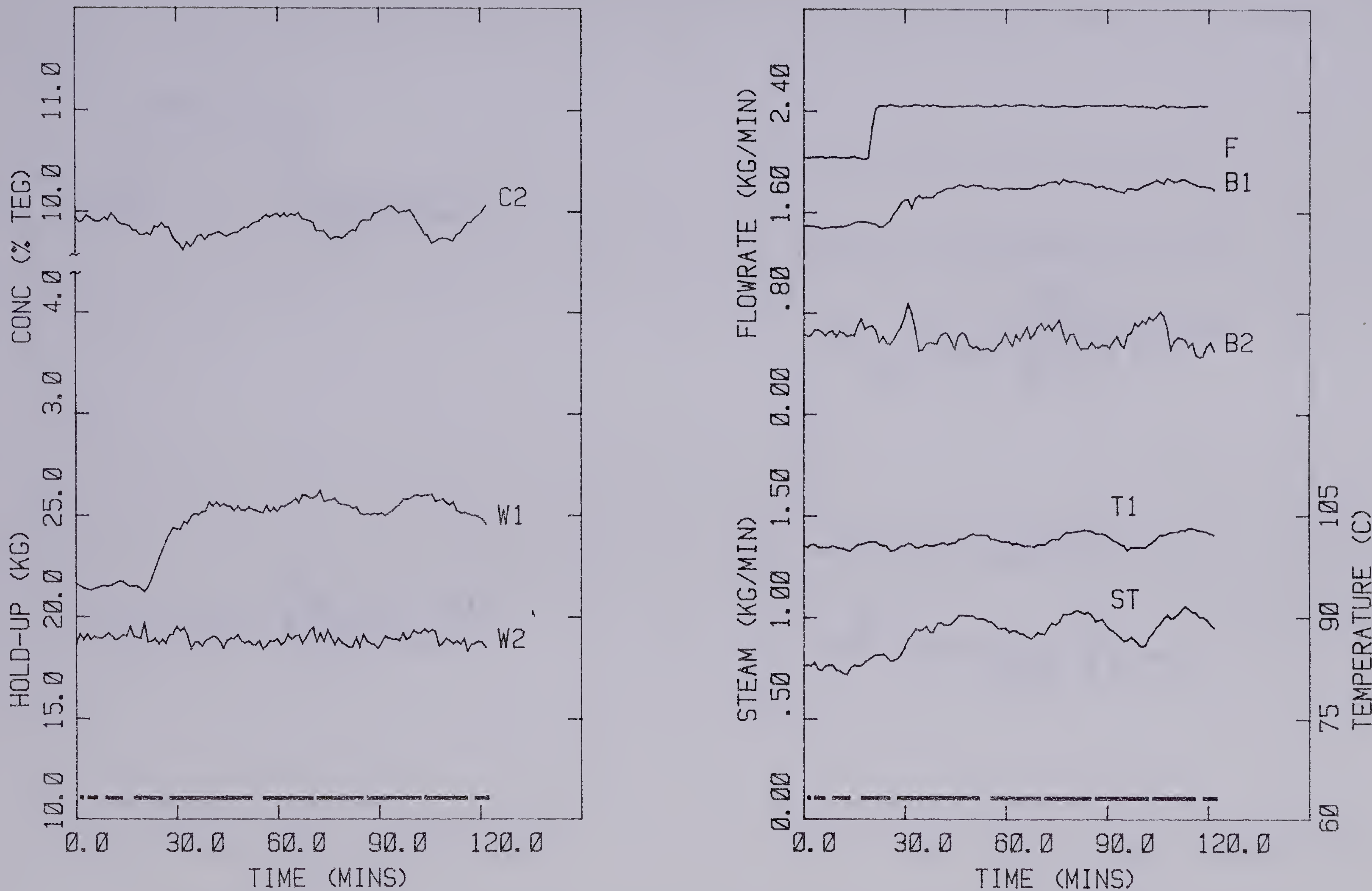


FIGURE 6.22 Evaporator Response Controlled by SFC with P-Weighting
(SFC/RR2005/.5*ITDM/T64/M2/C1/D0.005/P(1+z)/Q0/ 20%FD/ cf.RR2006)

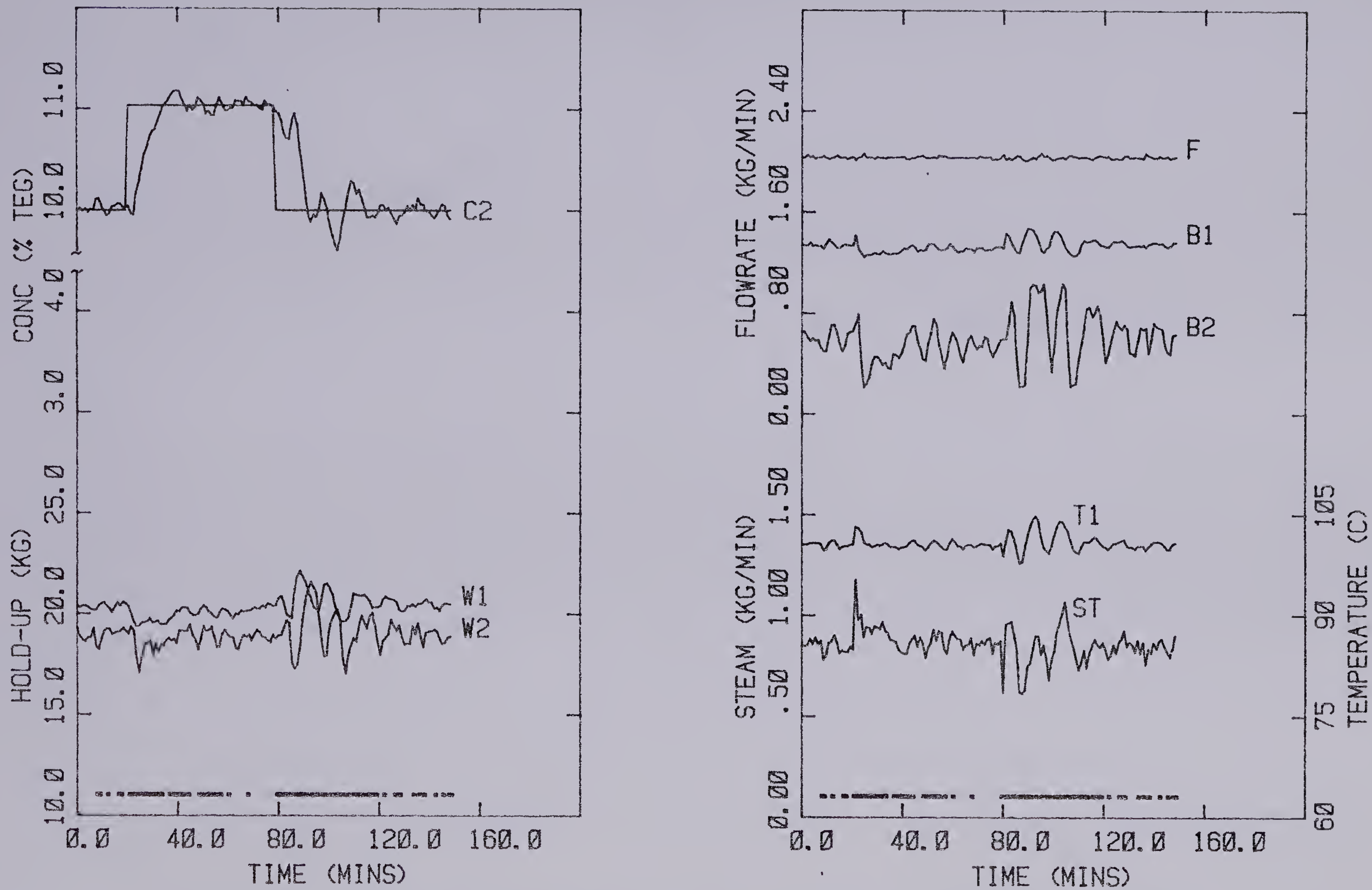


FIGURE 6.23 Evaporator Response Controlled by SFC to Setpoint Changes
(SFC/RS2001/ITDM/T64/M2/C1/D.005/P1/Q0/ 10%SP/ SETPOINT CHANGE)

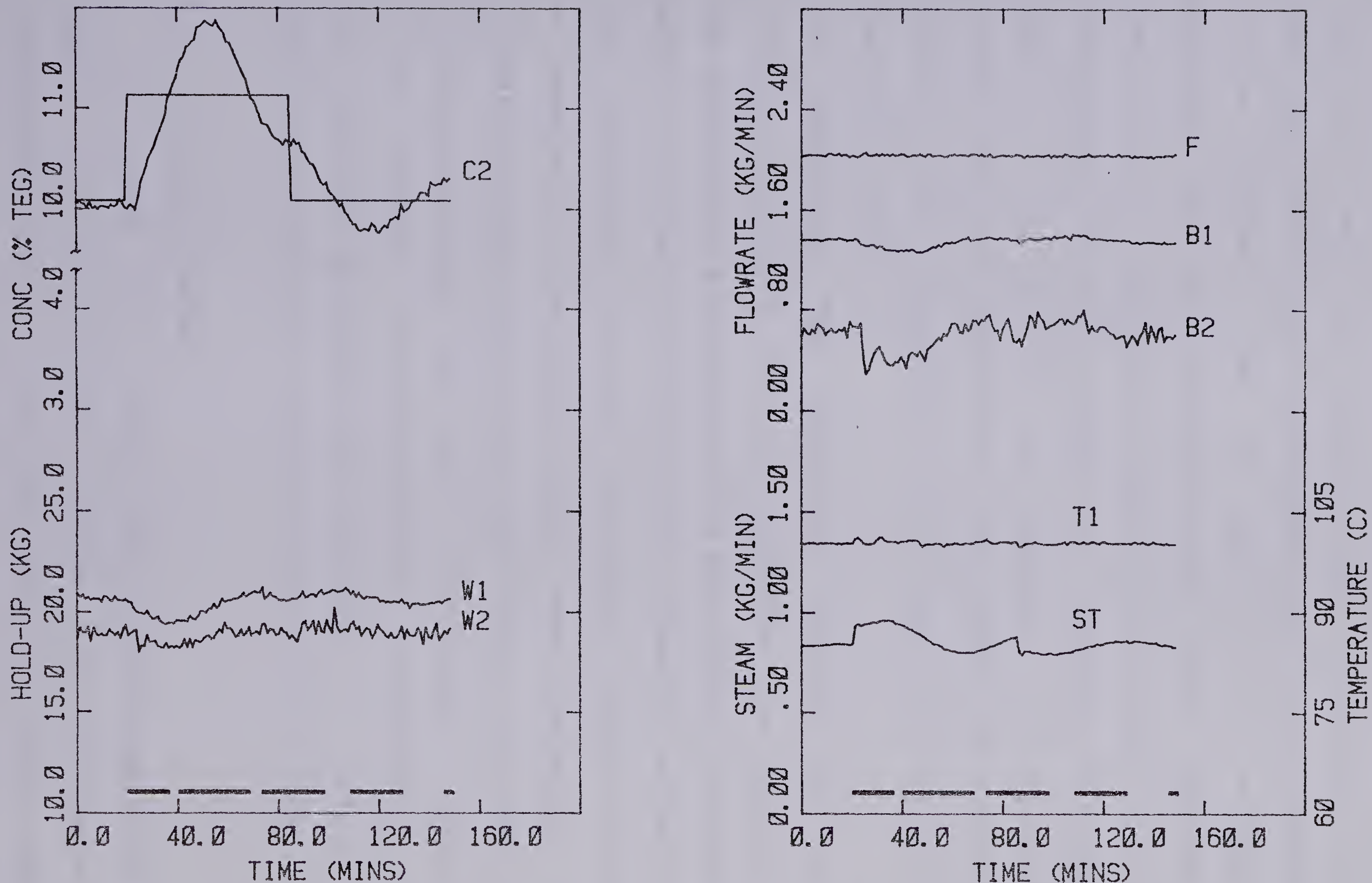


FIGURE 6.24 Evaporator Response by SFC with P-wt to Setpoint Changes
(SFC/RS2003/ITDM/T64/M2/C1/D0.005/P.2(1-.5z)/Q0/ 10%SP/ cf.RS2001)

The second setpoint change (showing the nonlinearity of the evaporator) makes the control signal oscillatory. It may cause the closed loop to be unstable due to the interactions of the evaporator when the setpoint change is larger in magnitude. The excessive control signal can be smoothed by use of a $P(z^{-1})$ polynomial weighting as in Figure 6.24. The P-weighting in 6.24 was chosen to indicate the dramatic effect it can have on the variance of the manipulated variable. Better control of C2 could probably be obtained by choosing the P polynomial to give less filtering action on the control error.

(2) **Q-Weighting:** The Q-weighting function acts on the auxillary signal $\eta(k)$ and also introduces the dynamics of the unmeasurable disturbances into the control law design of SFC (cf. equation (6.22) and (6.23)). Thus the estimator has more parameters that need to be updated. Figure 6.25 shows an example of Q-weighting where seven parameters were estimated at each sampling time. The control performance was slightly oscillatory. The effect of the number of parameters to be estimated is more significant in Figure 6.26, where nine controller parameters were estimated. In general the higher order controller requires longer tuning period.

6.7.4 Controller Order

SFC based on a higher order process models was also applied to the evaporator. Figure 6.27 represents the performance using a third order model and can be compared

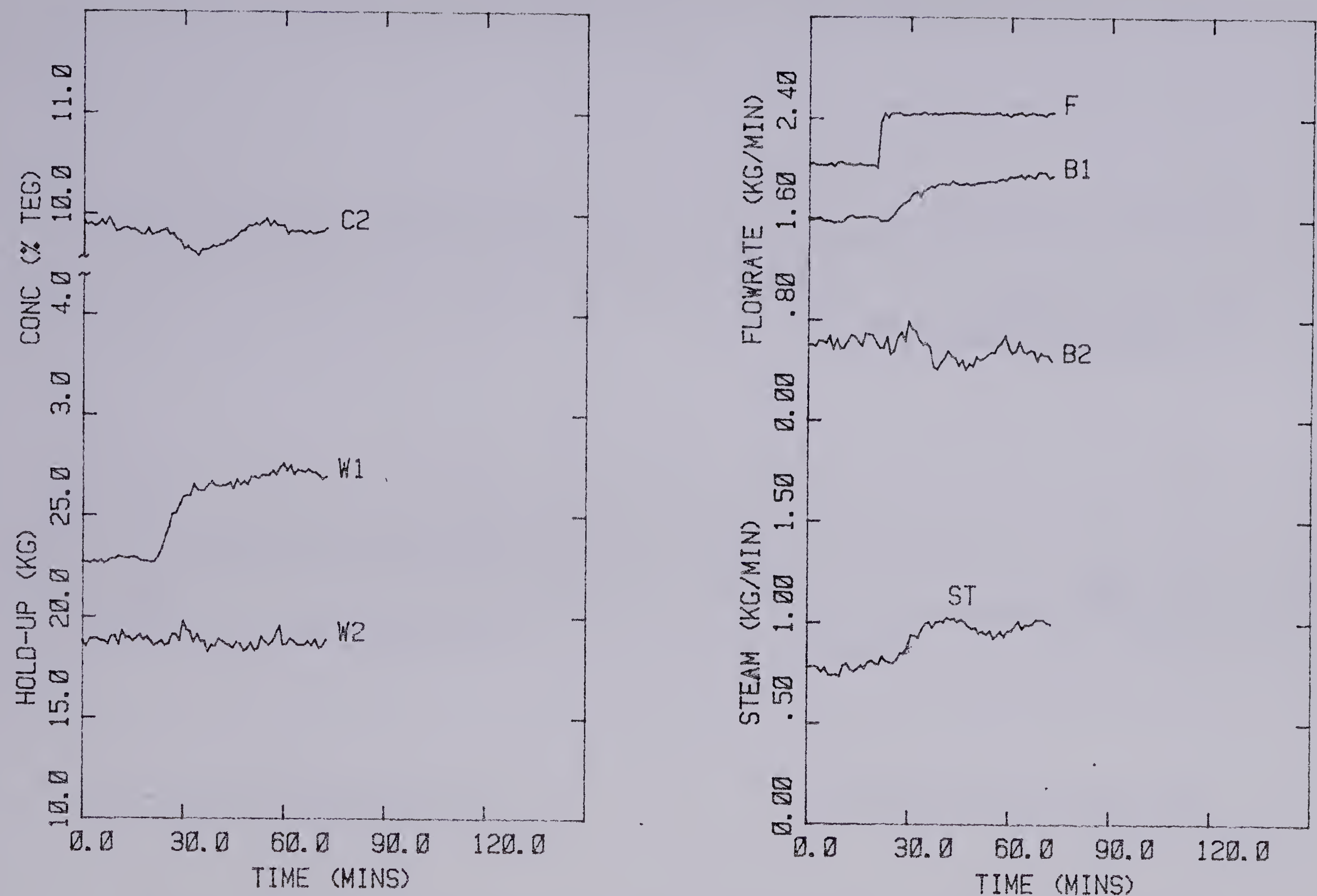


FIGURE 6.25 Evaporator Response Controlled by SFC with Q-Weighting
(SFC/RR2007/ITDM/T64/M2/C1/D.005/P1/Q1/ 20%FD/ Q-WT cf.RR3005)

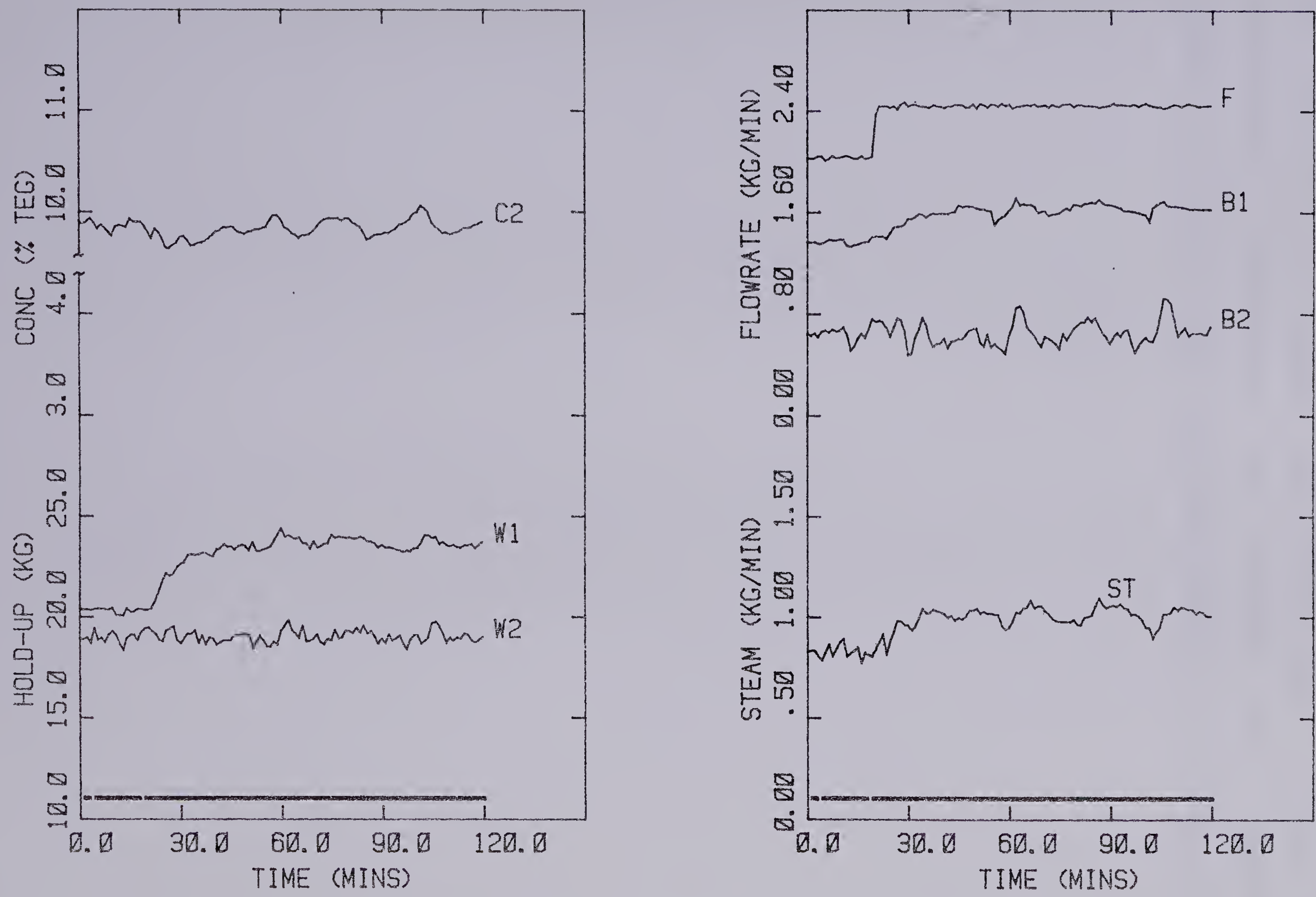


FIGURE 6.26 Evaporator Response using SFC with Q-wt and 3rd Order Model
(SFC/RR3005/ITDM/T64/M3/C1/D0.005/P1/Q1/ 20%FD/ Q-WT cf.RR2007)

with Figure 6.10 where the second model order is used. Another example is in Figure 6.28 where the same P-weighting is used as in Figure 6.21. Both cases indicate the oscillatory performance and require more parameter adaptation effort, which has been observed in Q-weighting.

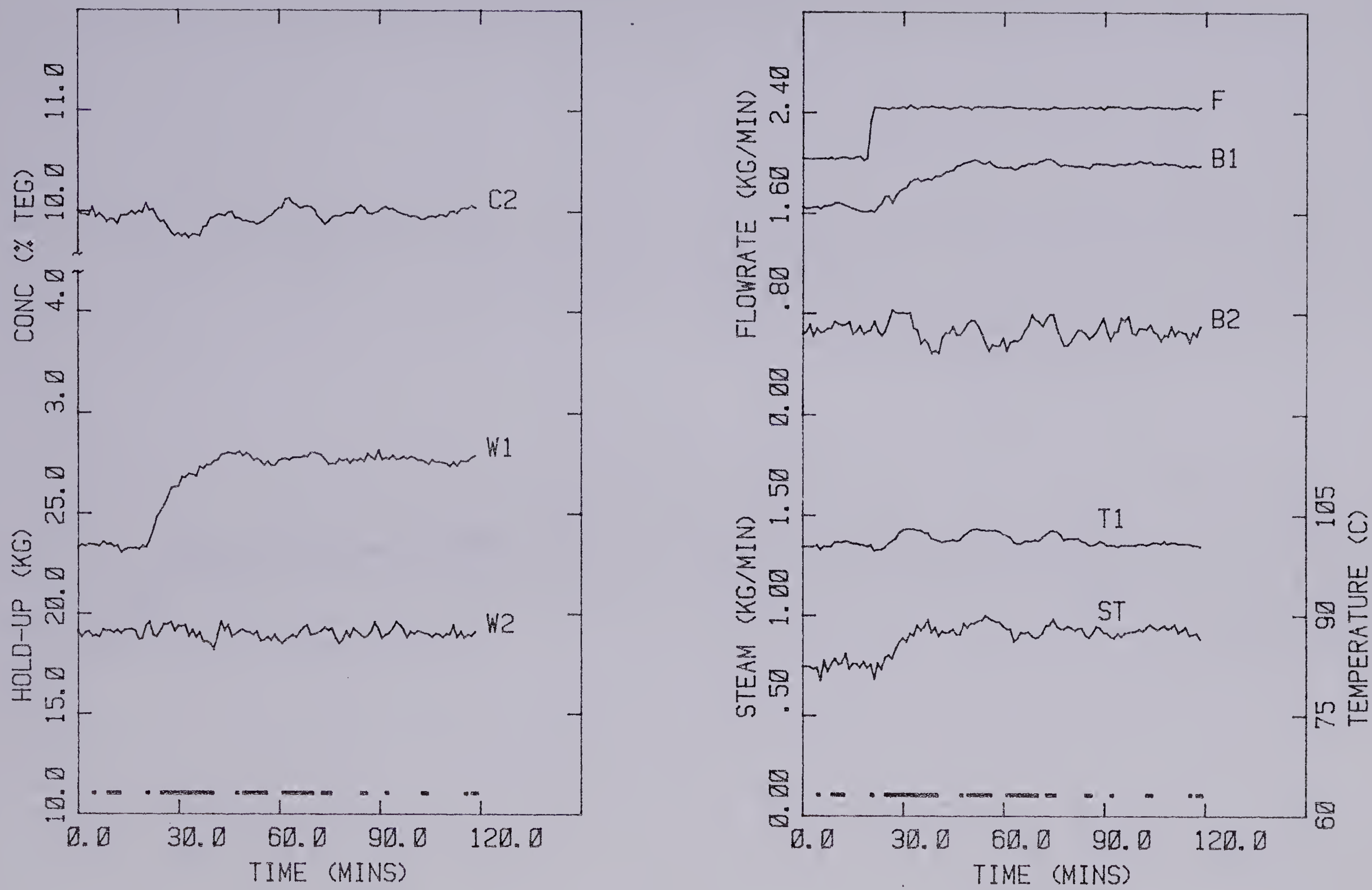


FIGURE 6.27 Evaporator Response Controlled by SFC with Higher Order Model
(SFC/RR3001/ITDM/M3/C1/D0.005/P1/Q0/ 20%FD/ HIGHER ORDER cf. RR2004)

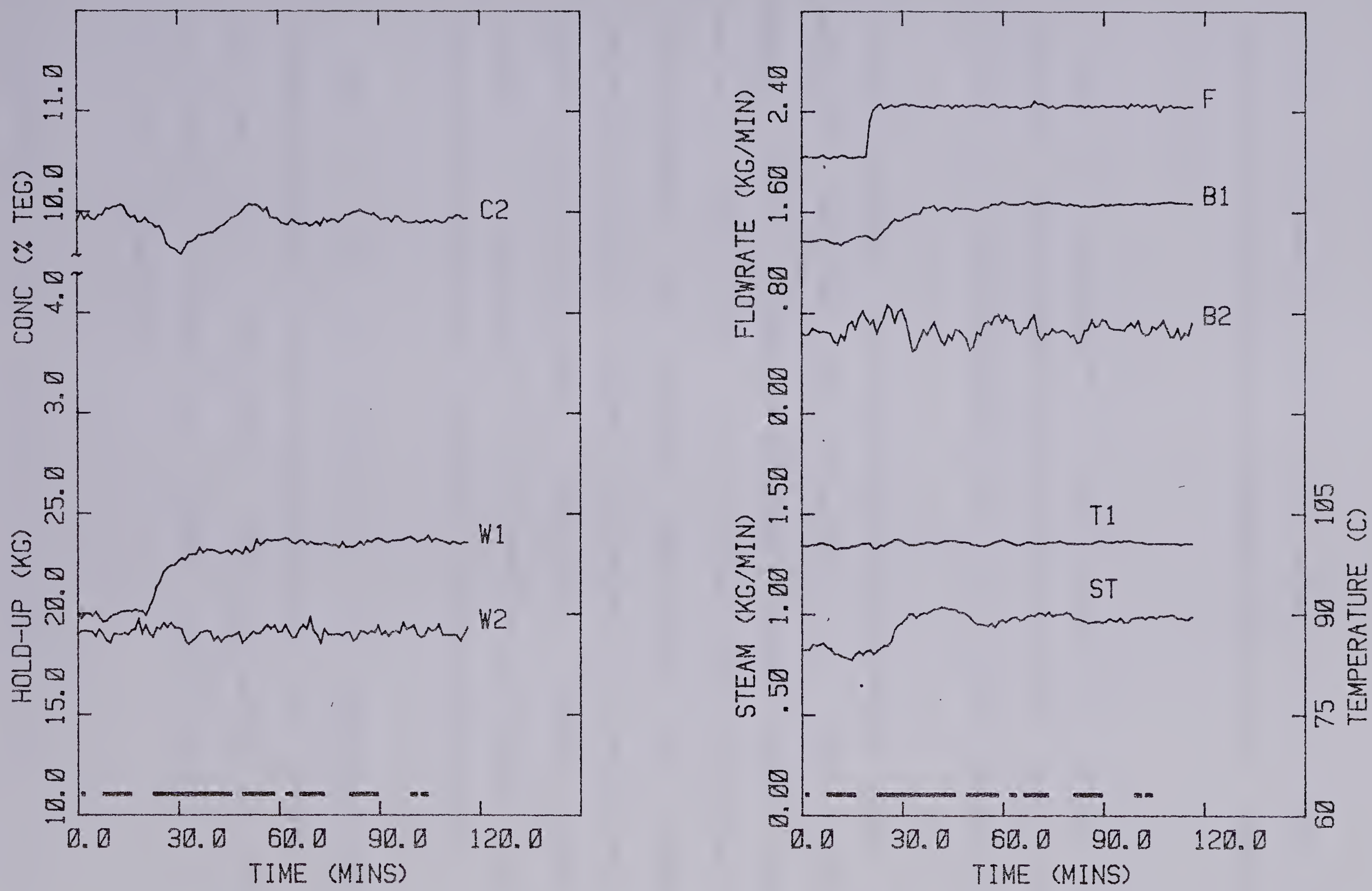


FIGURE 6.28 Evaporator Response by SFC with P-wt and 3rd Order Model
(SFC/RR3004/ITDM/T64/M3/C1/D0.005/P(1+.5z)/Q0/ 20%FD/ P-WT cf.RR2006)

6.8 Conclusions

1. SFC is a globally stable robust adaptive controller that operates in the presence of bounded noise and/or unmeasured disturbances.

2. The 'robust structure' of the SFC controller ensures that asymptotic tracking and regulation is achieved even in the presence of finite perturbations in the system parameters.

3. SFC minimizes a user-specified performance index. The weighting functions give flexibility in the control law design of SFC. In general, Q-weighting makes the SFC control law more complicated, i.e. a higher order controller with more parameters to estimate. The higher order adaptive controllers due to either inclusion of Q-weighting or a higher order process model, required a longer period to estimate the parameters and showed an oscillatory control response for short term regulation of the evaporator.

4. SFC, in what is essentially its simplest form, is mathematically and structurally equal to the discrete PID algorithm. Thus conventional PID applications in industry can be easily extended to include 'self-tuning' of the controller parameters.

5. The adaptive part of SFC can be stopped at any time and

is normally inoperative during steady state operation (zero control error). This prevents practical problems such as drift and parameter windup but still permits asymptotic tracking and regulation (point (2) above).

6. Simulation results show that the performance of SFC-PID is satisfactory and at least comparable to the results obtained by STC and APCS with the PI type Q-weighting.

7. The application of SFC to the double effect evaporator shows that SFC can use conventional PID constants as initial parameters and tune these constants to a better set of gains. Thus SFC in adaptive PID form can be used in tuning conventional PID controllers.

8. The experimental application of SFC shows that its performance is comparable to or even better than that of STC or APCS. SFC in its adaptive PID form outperforms the conventional, PID controller and in addition is a logical choice for application to real industrial processes where the conventional, PID controller is being used and retuning of the parameters is consistantly required.

7. Comparison of Adaptive Controllers

Chapters four, five and six describe the STR/C, APCS and SFC respectively. Each of these chapters is fairly independent of the others and this organization of thesis has proven convient for reference and educational purposes. However, there is an alternative way of looking at the same information and that is to take a single feature, such as the type of parameter estimation law used, performance of the evaporator with step feed disturbances etc., and examine all the controllers of interest relative to this single feature. This is the approach taken in this chapter.

Many of the similarities and differences, advantages and disadvantages of STR/C, APCS and SFC were brought out in the discussion in chapters four through six and will not be repeated here. Thus this chapter assumes a knowledge of the preceding chapters and is not intended to be read independently. The overall organization of the chapter is outlined in Table 7.1. The three controllers (columns) are compared based on features grouped into the categories of 'parameter estimation law', 'controller design' and 'internal model' (rows). The subsection titles in 7.2 and 7.3 correspond to the topics listed in Table 7.1.

To assist the reader in collating all the experimental and simulation data related to a single factor such as 'the effect of model order', the relevant run numbers have been collected into Tables such as 7.2. The detailed discussion

Table 7.1 Comparison of Adaptive Controllers

	STR/C	APCS	SFC
Parameter Estimation Law	• Model order and sampling time • Initial model parameters • Covariance matrix • Forgetting factor	• Model order and sampling time • Initial model parameters • Upper & lower limits of estimator gains • Disturbance bound	• Model order and sampling time • Initial controller parameters • Upper & lower limits of estimator gains • Disturbance bound
Controller Design	• Objective STR: $E\{y^2\}$ STC: $E\{(py-Rw)^2+(Qu)^2\}$ • Design factors STR: b_0 STC: P Q R	• Objective APCS: $E\{(y-y_d)\}^2$ APCS(w): $E\{(Py-Qy_d)^2+(Qu)^2\}$ • design Factors APCS: θ_0 APCS(W): P Q R	• Objective SFC: $E\{(Pe)^2+(Qu)^2\}$ • Design factors P Q
Internal Model			$D(z^{-1})$

of each run is in the chapter dealing with the type of controller used, e.g. SFC runs are discussed in chapter six. This chapter takes a higher-level viewpoint and attempts to make broader, more general conclusions. This task of making specific comparisons and general conclusions about different adaptive controllers proved to be very difficult. Some of the reasons are discussed in section 7.1.

7.1 Difficulty of Comparing Adaptive Controllers

One of the objectives of this thesis was to compare the performance of different adaptive controllers. Ideally such a comparison would be carried out over a long period of time on the actual application of interest. For example in industry two parallel production units subjected to the same product specifications, raw materials, disturbances, operators etc. would be ideal. However, for preliminary evaluation of a wide range of parameters and operating conditions a faster more convenient means of comparison is desirable. This proved very difficult to achieve for the reasons described below.

1. The computer controlled pilot plant evaporator used in this study is a convenient vehicle. However, it must be realized that the objective is not to find the best adaptive controller for this particular evaporator but rather to try to predict how the different adaptive controllers would perform in other applications. A brief consideration of the possible effect of a single factor such as modelling error (whether model structure, model order, parameter values, time delays or nonlinearities) indicates that the objective is difficult, if not impossible, to achieve. However, at this stage in the development of adaptive controllers any experimental comparisons are valuable, even if they are application and procedure dependent.

2. Adaptive controllers differ significantly in their basic

structure, e.g. direct methods, indirect methods, model-reference techniques etc. Many methods have a specific design objective, e.g. stability, minimum variance control, setpoint tracking, control of non-minimum phase systems etc. Unfortunately, one controller does not incorporate all the desirable characteristics and therefore it is frequently a question of 'selection' rather than 'comparison' for any given application. For example, is a controller with a (theoretical) stability guarantee better than a comparable controller without such a guarantee? What is the value of the simplicity, e.g. the number of design parameters that must be set by the user? In most cases there is no agreed method of trading off one factor vs. another nor is there a widely accepted quantitative performance criterion that can be used as a basis of comparison.

3. All adaptive controllers have parameters that are initialized by the user and/or self-initialized during a 'learning period'. Consider the case where the initial parameters are initialized by the user. Controllers like STR/C and APCS are initialized with estimates of the **process** parameters (or direct control parameters calculated from the process parameter estimates). However, SFC must be initialized with **controller** parameters (In the SFC-PID case these are explicit functions of the familar proportional, integral and derivative gains.). How can one say that a given set of **process** parameters (e.g. to initialize STR/C or APCS) is equivalent to a set of **controller** parameters (e.g.

to initialize SFC)?

Consider the case where the controllers are initialized during a 'learing period' and/or by actual operation 'under closely-supervised conditions' and then subjected to some standard tests, e.g. setpoint changes or disturbances. The performance of each controller will depend on its 'state' at the beginning of the test period and this 'state' will be a function of the previous operating history. (The manner in which the state at time k varies with the operating history will also depend on parameters such as forgetting factors, convergence factors etc. but let us ignore these effects.). If the standard performance test is a step change in setpoint should the learing period include a series of step setpoint changes or some semi-random disturbances? If an external period of steady state operation is included just prior to the test period then it is possible that controllers like STC with an ordinary RLS will 'windup' whereas controllers like APCS and SFC would turn off the parameter estimation until there was a larger estimation error. Thus such a history would not provide a 'fair' basis for test comparison.

For a given application a good control engineer could initialize an adaptive controller so that the performance would be satisfactory for that particular application. However, to select a basis for comparing two controllers is difficult. Even identical learning and test periods would not be an adequate basis for general conclusions.

4. Adaptive systems are **nonlinear** and in most cases the particular convergence point, or optimal set of parameters, is non-unique. For example, consider a single adaptive controller that starts with an initial parameter estimate $\theta_1(0)$ and minimizes a performance index J. The same controller initialized with a different set, $\theta_2(0)$, could reach the same minimum values of the performance index J at time k. However, the parameter values $\theta(k)$ and the shape of the time domain responses to a standard test signal could be different. The performance index J seldom defines the true desired results (in fact it is often used as a means of introducing design parameters into the formulation) and hence we are again left with a qualitative judgement about which is 'best'.

Controllers such as APCS and SFC, in the general case, guarantee only that the control error is reduced to within a certain bound. Hence the 'state' of the controller at time k when the control error enters this bound can differ depending on the initial parameters and/or performance history.

In both of the above cases the performance of the controller for times greater than k could differ because the state of the controller at time k would be different and the overall system is nonlinear. Hence comparison of controllers is difficult.

7.1.1 Conclusions

Direct comparison of adaptive controllers is difficult because they are application, procedure and operator (user) dependent. However, it is still worthwhile. In most cases it will be necessary to document the procedure and operator input as well as the process performance. Future users will then be faced with the task of making their own judgement about how this data can help them in their particular design and/or operating problems. At this point, it is doubtful that application studies can lead to definite conclusions such as 'controller A is always better than controller B'.

The difficulty of deriving general conclusions from application studies can be contrasted with the demonstration or proof of particular properties such as stability, convergence, robustness etc. Although these properties do not always carry over to specific applications because the theoretical conditions cannot always be guaranteed, history suggests that they are both desired and used by the control commumity.

7.2 Comparison of Parameter Estimation Algorithms

The parameter estimation laws compared in the following section are the APCS estimation law (which is similar to the 'learning method' of Nagumo and Noda (1967) and the 'vector projection algorithm') and an ordinary RLS with and without a forgetting factor.

7.2.1 Model Order and Sampling Time

Three different types of model were considered to describe the evaporator dynamics and used to implement the STR/C, APCS and SFC adaptive controllers. The models are a first order with or without time delay, a second order and a third order evaporator representation (cf. chapter three). Table 7.2 summarizes the results obtained using these models.

Table 7.2 Effect of Model Order

	STR/C		APCS		SFC	
	simul.	exp.	simul.	exp.	simul.	exp.
First order	4.4	4.27	5.2 5.7	5.23	6.4	
Second order	4.5 4.17	4.20 4.25 4.30 4.32	5.3 5.12	5.18 5.21 5.24 5.25	6.3	6.10
Third order		4.26	5.4 5.15	5.22	6.8	6.25 6.27

Simulation study showed that the first order model without time delay gave very oscillatory responses which were worse than any other model and therefore it was not used in the experimental study. Note that the first order model without time delay is a special case of the second order model without time delay (cf. equation (3.2) vs (3.4)). The first order model with time delay,

equation(3.3), and the third order model also resulted in oscillatory responses but they were not as severe as with the simple first order model. For all adaptive controllers the second order model, equation (3.4) or (3.5), gave the most satisfactory closed loop evaporator performance. Note that the second order model has four parameters to be estimated whereas the first order model with delay and the third order model have five and six model parameters to be estimated. The number of parameters to be estimated appears to have a strong influence on the performance of the parameter estimation algorithms and also influences the effect of other factors such as the initial parameter estimates, process noise etc.

The effect of sampling time on discretization of the process model was discussed in section 3.3 and an example on an evaporator model showed that a smaller sampling interval resulted in a zero nearer to the unit circle and b_0 getting close to zero both of which can cause problems in adaptive control. The following table summarizes the experimental runs using different sampling times.

In the applications of STR and APCS it was observed that the control response was highly oscillatory with a 64sec sampling time which is the one normally used for the evaporator. A longer sampling time frequently improves oscillatory responses (cf. example 3.1) in adaptive systems. However, in the case of STR and APCS experiments using a

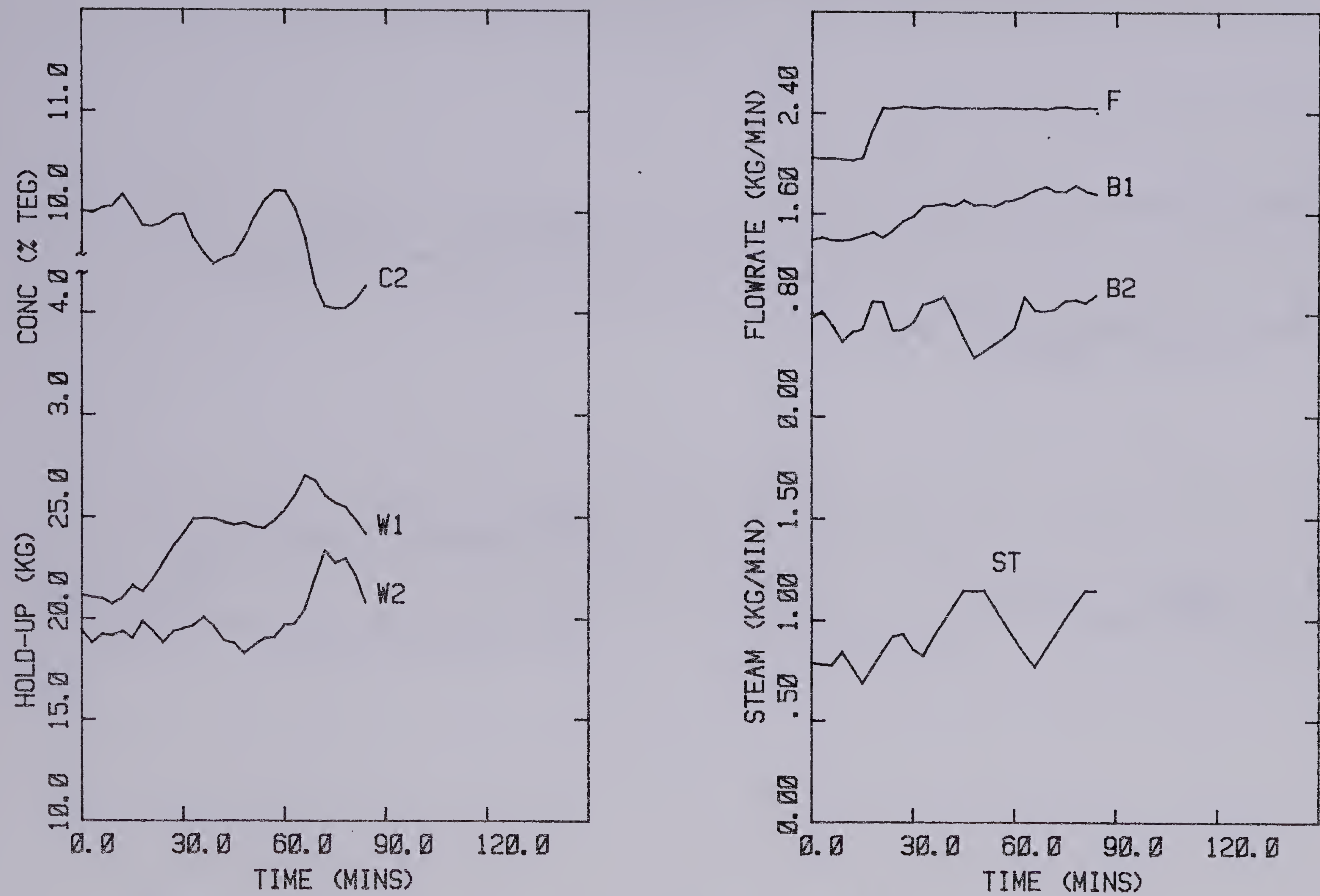


FIGURE 7.1 Evaporator Response Using STR with Three Minute Sampling Time
(STC/RT2023/ITDM/T180/M2/C.1/F1/P1/Q0/ 20%FD/ LONGER SAMPLING TIME)

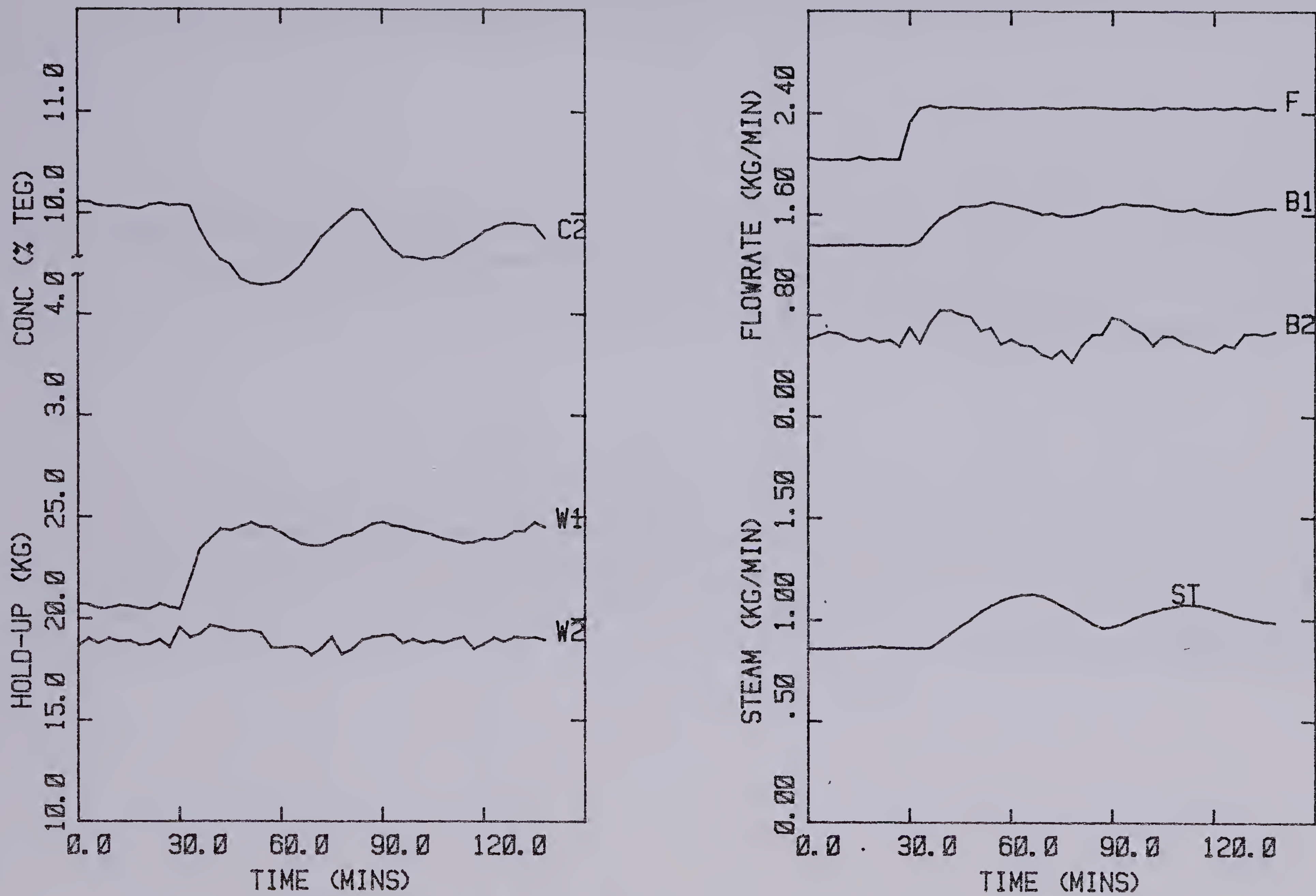
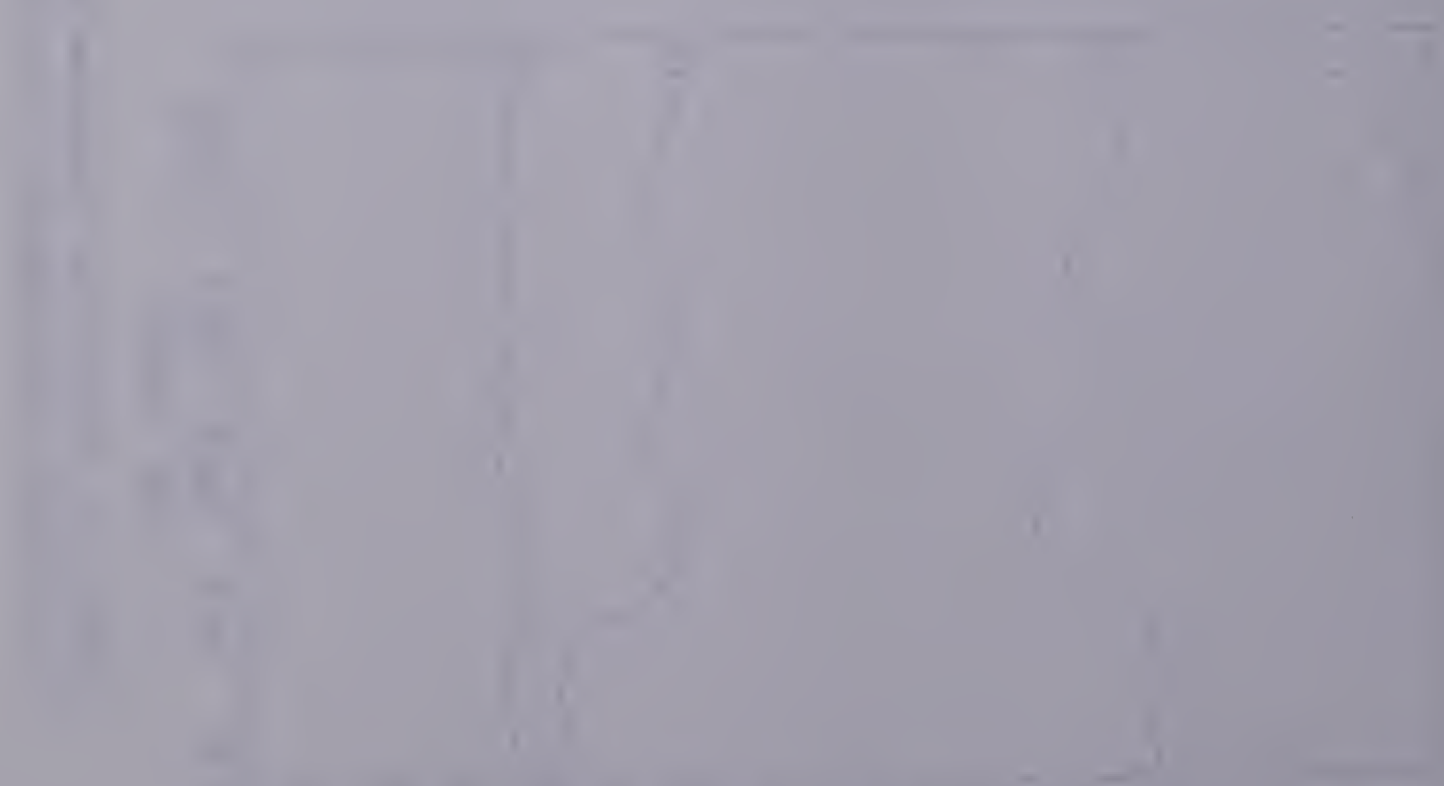


FIGURE 7.2 Evaporator Response Using STC with Q-wt and Three minute Ts
(STC/RT2005/ITSM/T180/M2/C.1/F1/P1/Q(.8-.8z)/ 20%FD/ Ts=3MIN cf. RR2003)



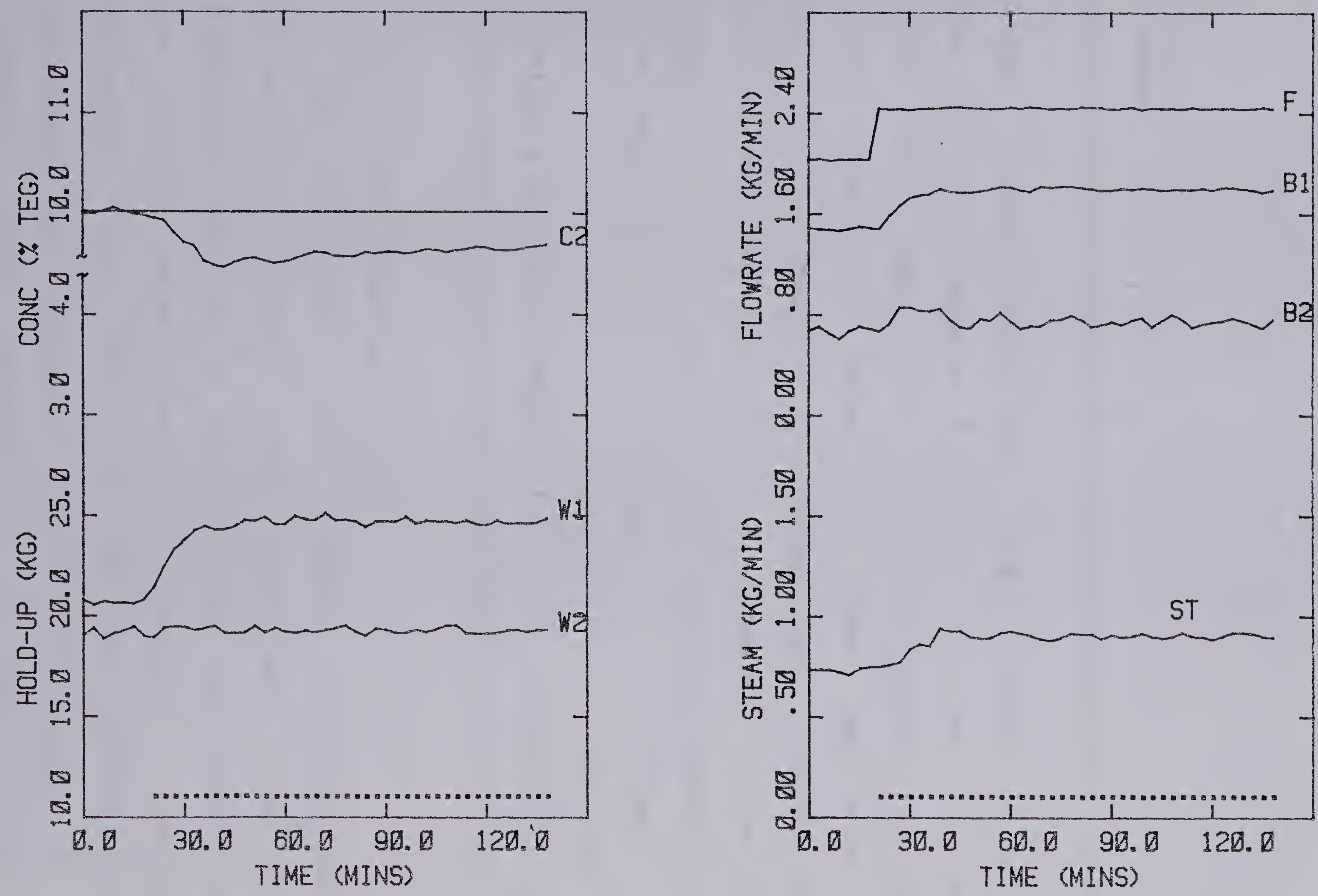


FIGURE 7.3 Evaporator Response using APCS with Three Minute Sampling Time
(APCS/RP2002/ITSM/T180/M2/C.1/D.005/P1/Q0/ 20%FD/ LONGER SAMPLING TIME)

Table 7.3 Effect of Sampling Time

	STR/C		APCS		SFC	
	t=64s	180s	t=64s	180s	t=128s	180s
Figure number	4.20 4.23	7.1 7.2	5.19	7.3	6.17	6.18

longer sampling time (180sec) were not helpful in reducing the oscillatory response. Figures 7.1, 7.2 and 7.3 use a three minute sampling time and can be compared with Figures 4.20, 4.23 and 5.19 respectively. These comparisons show that a longer sampling time (180sec) gave sluggish control without reducing oscillation as compared to the 64sec sampling time. A similar effect was also observed in SFC experiments (cf. Figure 6.17(64sec) vs 6.18(128sec)). The selection of a 64 second sampling time is consistent with previous work done by Newell (1971) who recommended a 64 second sampling time for LQG state feedback controllers.

7.2.2 Initial Parameter Estimates

The choice of initial parameter estimates strongly influences the resulting control performance. The initial parameters required by STR/C and APCS are coefficients of the 'process' model whereas SFC requires 'controller' parameters. For STR/C and APCS the initial process parameters were calculated by discretization of the evaporator models, equation (3.2), (3.3) and (3.4) and also

directly from the discrete evaporator models, equation (3.5) and (3.6). The initial controller parameters for SFC were calculated from the open loop evaporator model, equation (3.3), using a classical design procedure which generated PID parameters that minimized the IAE of the process [Miller et al. 1967]. In this manner the three adaptive controllers were assumed to have comparable initial parameters. Table 7.4 contains the figure numbers which can be used to compare the performance of each adaptive controller (column 1) when different sets of initial parameters are used.

For STR and APCS (no control weighting) the leading coefficient of the numerator polynomial of the model seemed to be the most important parameter (cf. Figure 4.20 vs 5.18; 4.22 vs 5.19). A small leading coefficient resulted in a high controller gain and hence the control action was excessive. For simulation studies the large control signals were acceptable. However, when applied to the actual evaporator STR and APCS without weighting resulted in unacceptable oscillation due to the high gain and the interacting nature of the evaporator. Both STR and APCS required controller weighting for satisfactory experimental performance. After introducing a quadratic performance index with well-tuned PI type Q-weighting, STC and APCS gave very similar responses. The performance was comparable even when three different sets of initial parameters were used, e.g. based on equation (3.3), (3.4) and (3.5). For SFC, the base set of initial parameter estimates was varied from 50% up to

Table 7.4 Effect of the Initial Parameter Estimates

	Initial parameters based on							
	zero		eqn (3.3)		eqn (3.4)		eqn(3.5)	
	sim	exp	sim	exp	sim	exp	sim	exp
STR	4.2		4.4		4.5	4.20	4.6	4.21 4.22
STC				4.27	4.17 4.16	4.25		4.30
APCS	5.8		5.7		5.3	5.18	5.6	5.19
APCS(W)				5.23	5.12 5.13	5.21		5.24
SFC			6.3 6.4	6.10 6.14 6.15 6.16				

150% in proportional gain and up to 5 times in the basic integral time (cf. Figure 6.10, 6.14, 6.15 and 6.16). Satisfactory performance was still achieved in all cases. Quantitative determination of the best set of initial parameter values for the evaporator was not the concern of this study. For actual applications of adaptive controllers the initial parameter estimates could be selected based on a priori, off-line or background identification studies.

7.2.3 RLS: Covariance Matrix and Forgetting Factor

The rate of parameter convergence is a function of the covariance matrix in RLS. Table 7.5 summarizes the runs using a RLS estimator.

Table 7.5 Runs Using the RLS Estimator

	covariance matrix	forgetting factor
simulated runs	4.2 4.8 4.9 4.10 4.11 4.12	4.13 4.14 4.15
experimental runs	4.28 4.29 4.30 4.27 4.25	4.31 4.25

The effect of the initial covariance matrix is demonstrated in two ways: i) for a poor set of initial parameters in Figure 4.2, 4.8 and 4.9, ii) for a good set of initial parameters in Figure 4.10, 4.11 and 4.12. These simulations show that a large initial covariance is good for poor initial parameters but bad for good initial estimates.

It can be noticed in the above simulations that as the estimation proceeds the elements of the covariance matrix get smaller and hence, the estimator will not adapt the process parameters as rapidly (assuming no forgetting factor is used.) For example in Figure 4.2 the initial covariance was 1000I but after 3000 iterations its diagonal elements became

$$\text{Diag. } P_t(3000)=[36.1;\ 25.0;\ .0051;\ .0036]$$

This self extinguishing feature of RLS is acceptable for time-invariant processes but not good for time-varying processes. Frequently, a forgeting or discounting factor is

introduced to prevent the covariance matrix from shrinking. However, use of a constant forgetting factor may cause estimator windup when applied to systems with low excitation. The effect of the forgetting factor was illustrated through simulation studies. As in Figure 4.2 a 1000I initial covariance matrix (but with a .99 forgetting factor) was used. After 850 iterations the diagonal elements of the covariance were inflated to

$$\text{Diag. } P_t(850)=[168{,}840.;\ 48{,}491.;\ 47.;33.]$$

Figure 7.4 shows the corresponding parameter variations. The parameter estimates are extremely biased and drifted because of the large elements of the covariance matrix especially the corresponding components for 'A' parameters. Figure 4.31, where the forgetting factor was .98, is comparable to Figure 4.25 with a unity forgetting factor.

In conclusion, RLS without a forgetting factor is not good for time-varying processes and RLS with a forgetting factor is good for time-varying processes but may cause estimator windup problems during periods of low noise or no disturbances. For the control of the evaporator, RLS without a forgetting factor was preferred. Note that the evaporator has low magnitude noise and that the process parameters are time invariant over the duration of a typical run.

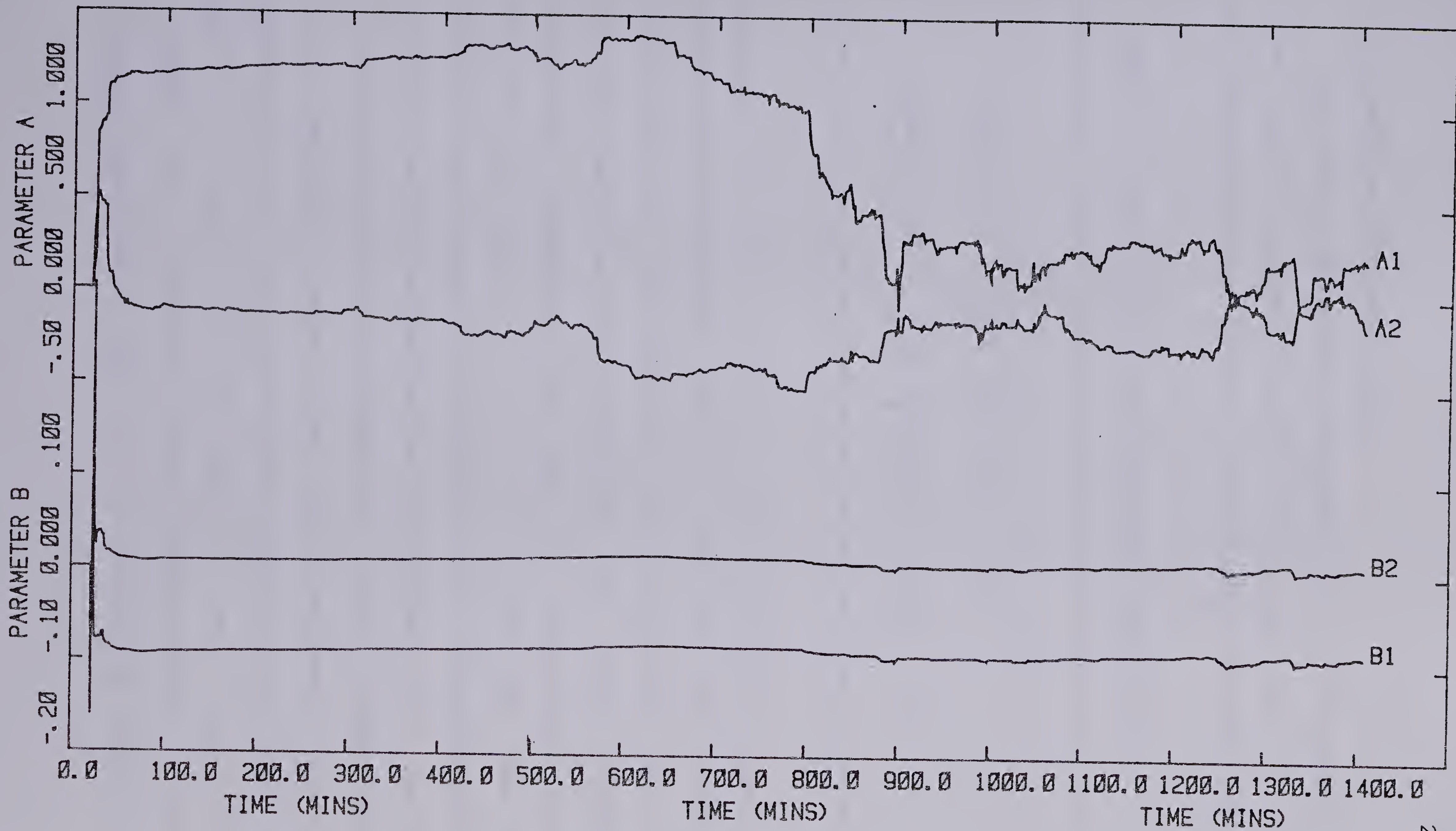


FIGURE 7.4 Parameter Trajectory when RLS with Forgetting Factor .95 is used for Parameter Estimation

7.2.4 APCS/SFC: Estimator Gain

The rate of parameter convergence can be influenced to some extent by proper selection of the scalar quantity a(k) and the perturbation bound Δ_d in the APCS estimation law.

Table 7.6 Evaluation of APCS Estimator Parameters

	a(k)	Δ_d
simulated runs	5.5 5.8 5.9 5.10 (5.11)	5.5
experimental runs	5.19 5.24 4.27 4.25 (4.30)	6.10 6.19 6.20

Figures 5.5, 5.8, 5.9 and 5.10 were intended to illustrate the effect of the upper limit of a(k) on the parameter convergence. It was observed that a large a(k), say 1000, was not good for even simulated runs. The parameter variations, especially θ_0, were too large and too rapid. Note that the upper limit of a(k) is a scalar and does not contain any process I/O information (cf. covariance matrix in RLS). The APCS estimation law is compared with the RLS (no forgetting factor) in Figure 5.10 and 5.11 respectively. This comparison shows that RLS achieves parameter convergence (to the true values) faster than the APCS law. On the other hand the APCS adaptive law has an on/off property and no estimator windup problems due to increasing estimator gains as in RLS with a forgetting factor. In

experimental runs an upper limit less than two was good, but $a_1>5$ resulted in oscillatory responses. In simulation runs it was possible to use values of a_1 up to 100.

The upper bound on the perturbation variable, Δ_d in equation (5.11) must be specified before starting the APCS adaptive law. It affects the rate of parameter convergence and determines the parameter adaptation dead zone. The effect of this variable is demonstrated through experimental runs. In Figures 6.10, 6.19 and 6.20 the upper limit was .005,.015 and 0.0 respectively. When the bound was zero, the parameter adaptation was continuous but the overall control performance was no better than when a value of .005 was used. When the value was too large, i.e. 0.015, the parameter estimates were not adapted properly and the control performance was poor. Therefore, this bound should be chosen carefully depending on the control objective. The ideal value is the minimum upper bound on the perturbation variable (cf. equation 5.12).

7.2.5 Computation Time

The computation time required to update the parameter estimates depends on the number of computer (arithmetic) operations required as well as the specific computer being used. Table 7.7 shows the number of operations required to estimate N coefficients using RLS and APCS estimators. The data are calculated based on FORTRAN code and the CPU time required by an HP 21X E minicomputer. Table 7.8 compares the

total execution time required to estimate N parameters. Note that for evaporator control at least four parameters were estimated.

Table 7.7 Operation Counts of RLS and APCS Algorithms with HP CPU Times

operation	execution time(μs)	RLS # of operation	APCS law # of operation
=	1.995	$2N^2 + 5N + 4$	$3N + 6$
+	13.300	$N^2 + 2N + 1$	$2N + 4$
-	14.000	$N^2 + N$	$N + 2$
*	25.655	$2N^2 + 3N$	$3N + 12$
÷	34.195	$2N^2 + N$	4
Jump	0.735	$2N^2 + 3N + 1$	$3N$
Compare	1.330		2

Table 7.8 Execution Time Required to Estimate N Parameters

Parameters

	total operation time (μs)	# of parameters to be estimated: 2	4	8
RLS	$152.5N^2+164N+20$	958.0	3,116.0	11,092.0
APCS law	$125.8N+540$	791.6	1,043.2	1,546.4
ratio RLS/APCS		1.2	3.0	7.2

These numbers can be different depending upon the program coding and the computer used. However, it is clear from Table 7.6 that the computation time for RLS increases as the square of the number of parameters while the APCS estimation

law is a linear function of the number of parameters. Further, the APCS adaptive law shuts off parameter estimation when the control error is small, e.g. at steady state. Thus the computation time for the APCS adaptive law is much less than that for the RLS. The constant term for the total APCS execution time in Table 7.8 allows for all the calculations and checks required for the APCS stability proof. In practice this could probably be reduced significantly.

7.3 Comparison of Controller Designs

7.3.1 Controller Design Objectives

The design objective for each adaptive controller used in this work is included in Table 7.1. Here, as mentioned earlier, design of the driver block for APCS is not considered. Instead, to facilitate comparison, a performance index similar to the one used in STC was introduced into the APCS algorithm.

7.3.2 Choice of Weighting Functions

The minimum variance type adaptive controllers, STR and APCS without weighting, could not be made to work experimentally even with different sets of design parameters and several weeks of effort. The main reasons were the high gain due to small b_0 or θ_0 and the highly interacting nature of the evaporator. The conclusion was that control weighting

was required for STR and APCS.

Table 7.9 A Summary of Runs Using Weighting Functions

	STR/C		APCS		SFC	
	sim	exp	sim	exp	sim	exp
(I)		4.23		5.20		
		4.24			6.9	6.25
(PI)		4.28		5.23		
Q	4.17	4.25	5.12	5.21		6.26
	4.19	4.30		5.24		
		4.32		5.25		
(PID)	4.16		5.13			
	4.18		5.16		6.6	6.21
P	4.19					6.24
					6.7	6.28

In this work the adaptive controllers were evaluated based mainly on regulatory control. The main effort thus directed towards finding suitable Q-weighting for the STC and the weighted APCS. Since there are no explicit guidelines for the design of Q-weighting various weighting functions, e.g. constant weighting, integral weighting, PI or PID type weighting, were tried. Table 7.9 summarizes the runs made to evaluate weighting functions.

For evaporator control integral Q-weighting resulted in very oscillatory responses (cf. Figure 4.23, 4.24, 5.20). Hence a more complicated Q-weighting was considered which has a PID form (PID compensation weighting). The gain constants were tuned by trial and error based on settings obtained from a classical PID design technique. It was found

that PID type Q-weighting was very sensitive to gain changes and difficult to tune. On the other hand PI type Q-weighting was rather easy to tune and resulted in satisfactory control performance for both the STC and the APCS.

SFC in its adaptive PID form (P=1 and Q=0) was mainly applied to explore its auto-tuning PID properties. However, several runs were made to demonstrate the effect of P and Q weightings. $P(z^{-1})$ was effective in controlling the error dynamics as well as filtering noise contained in the error signal (cf. Figure 6.21 and 6.28). Q-weighting was useful in reducing the variance of the control signal but increased the controller order and hence the number of parameters that had to be estimated. (In general, more parameters to estimate often means poorer overall performance.)

Figure 7.5 compares the 'best' experimental runs of each adaptive controller with a well-tuned conventional PID result. The SFC response appears slightly noisier than the others but this is probably due to the fact that the weighting functions $P(z^{-1})=(1+.5z^{-1})$ and $Q(z^{-1})=0$ were not tuned well. P and Q weighting improve the overall response in many cases but destroys the PID structure of SFC.

7.3.3 SFC: Choice of Internal Model

The SFC is designed based on the internal model principle. $D(z^{-1})$, which represents the dynamics of the external inputs, must be chosen based on the particular external disturbances that are expected to influence the

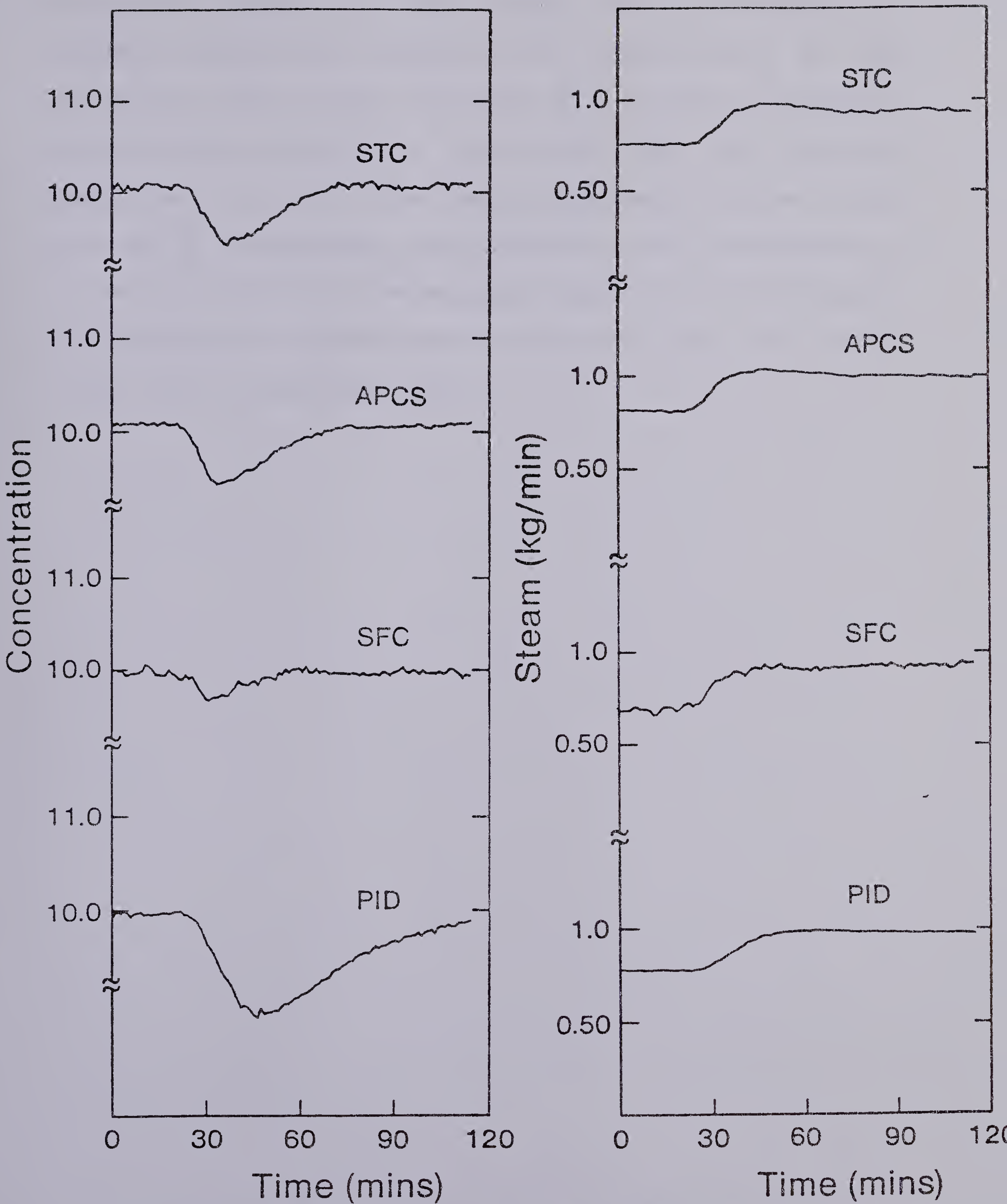


FIGURE 7.5 Comparison of the 'Best' Experimental Performance Obtained from STC, APCS, SFC and Conventional PID control of the plot plant evaporator.

controlled system. In this study feed disturbances and setpoint changes were assumed to be approximated by step functions which means that $D(z^{-1})$ is a simple integrator. This was done primarily so that the final SFC structure would have the PID form. More complicated internal models are easily incorporated into SFC and can be interpretted as a generalization of the integral action of PID controllers, i.e. asymptotic tracking can be achieved even with higher order and/or unbounded inputs.

7.4 Conclusions

1. Comparison of adaptive controllers can be based on structural (theoretical) properties, such as stability, convergence and robustness, and/or performance factors as determined in specific applications. Structural properties provide a quantitative basis for comparison but industrial applications do not always meet all the conditions required to guarantee the structural properties. Conclusions based on specific applications are difficult to generalize. However, inspite of the difficulties there is a continuing need for structural and experimental comparisons.

2. Sampling time and model order are important design parameters for all adaptive controllers. Higher order models will not necessarily produce better performance. For the evaporator, a second model and a 64 second sampling time were best.

3. The RLS parameter estimator may converge to constant values before the parameters are close enough to the true values to give good control. The use of a constant forgetting factor overcomes this limitation but can lead to parameter blowup.

4. The APCS parameter adaptive law is simpler, requires less computation time and has the advantage of continuing

adaptation (when required) with no estimator windup or shrinkage. However, it suffers from slower parameter convergence relative to the RLS.

5. The leading coefficient of the numerator of the process model has a dominant influence on the control performance of the unweighted STR and the APCS. For the evaporator the value was very small and caused severe oscillatory responses.

6. The most important design factor for the STC and the weighted APCS was Q-weighting which is useful in reducing excessive control action. For the double effect evaporator application PI type Q-weighting resulted in stable and robust design.

7. Modification of the SFC to include another parameter estimation scheme such as RLS in place of the APCS projection algorithm would result in improved performance. In fact it is possible that one estimation scheme, e.g. RLS, might be best during the initial startup phase when significant adjustment of the parameter estimates, $\hat{\theta}(0)$, is required while a second estimation algorithm, such as the APCS projection algorithm, would be best for continuing operation.

8. The choice of initial parameter estimates is important.

Therefore, the use of SFC is particularly convenient in applications where a change is being made from conventional PID control to an adaptive algorithm because the conventional PID constants will provide good initial estimates for SFC.

8. Conclusions and Recommendations

8.1 Conclusions

1. The self-tuning feedback controller (SFC) was successfully developed and evaluated. It has the following inherent characteristics (cf.dsection 6.5):

- global stability in the presence of unmeasured bounded, external inputs.
- an error-driven, feedback structure which meets the the conditions of Francis, Wonham and Davison for robust control (This guarantees asymptotic tracking and regluation even in the presence of model errors and/or perturbations).
- an internal model of the external inputs (setpoint and disturbances) which guarantees asymptotic tracking or regulation of bounded or unbounded disturbances of the assumed class.
- in its simplest form, a discrete PID structure.
- a quadratic performance index with explicit, user-specified, P-weighting on the control error and Q-weighting on the manipulated variable.
- a simple parameter estimation law which automaticlly turns off when the error is small and will track slowly time varying parameters.

2. The evaluation of SFC (and the other controllers)

consisted of two parts:

1) theoretical or structural features such as stability and robustness which are inherent characteristics of the controller itself, and
2) performance factors (objective functions) which are application and procedure dependent.

The theoretical and structural features have been formally stated in theorems and/or lemmas. The performance factors are based on over 88 simulation runs (cf. Tables 4.1, 5.1, 6.1) and 64 experimental runs (cf. Table 4.2, 5.2, 6.2). Experimental studies are an essential followup to analytical and/or simulation evaluations. For example, STR and APCS (without weighting) gave satisfactory control of the simulated evaporator but did not perform satisfactorily on the real evaporator. Also design parameters determined by simulation were often an order of magnitude different than those that were best in the experimental applications.

3. The SFC performance when applied to the evaporator pilot plant was equal to, or better than, results achieved using STR/C, APCS and well-tuned PID controllers. It is particularly attractive for industrial applications because it can be made identical to a standard discrete PID controller and, when required, can be extended to include:

- adaptation of the control parameters.
- weighting on the controlled and manipulated variables.
- a more complex disturbance and/or process model.

- different parameter adaptation algorithms (stability has only been proven for the 'projection algorithm').

4. All adaptive controllers require that a number of design parameters and/or initial conditions be set before the adaptive controller can be made fully operational. They are important and should be chosen carefully. This can be done analytically, by experience, by simulation, by a priori experimental trials, etc. For the PID form of SFC the initial parameters can be expressed as an explicit function of the conventional PID controller parameters that would be used for the same loop. Thus any method that can be used to estimate PID controller parameters can also be used to estimate initial values for SFC.

5. SFC uses the same estimation algorithm and approach to the stability proof as APCS. However, the basic form of APCS could not be made to perform satisfactorily on the real evaporator and was therefore extended to include a quadratic performance index which allowed P and Q weighting of the process input/output variables. This modified form of APCS gave satisfactory performance.

6. Self-tuning controllers (STC) using a recursive least squares (RLS) estimation algorithm appeared to give faster parameter convergence than SFC in some applications. However, the covariance matrix and hence parameter

adaptation sometimes decreased to zero before the parameters converged close enough to their true values. Use of a constant forgetting factor solved this problem but introduced the possibility of 'parameter bursting' (covariance blowup) during periods of low excitation.

7. The software developed to implement computer control of the pilot plant evaporator and to perform data handling (e.g. filing and plotting) performed well. It is flexible enough to be used in future experimental evaluations of adaptive and/or fixed parameter controllers.

8.2 Recommendations

Some recommendations for future work in this field are:

1. Derivation of a MIMO version of the SFC algorithm. Once the internal model matrix is chosen, the derivation of the algorithm should be similar to the SISO case but it may not have the adaptive PID structure.

2. Re-designing or developing an algorithm to choose the APCS error correcting factor a(k) to improve the speed of parameter convergence without destroying the stability analysis.

3. Development of a SFC with other adaptive mechanisms such as RLS, RML, etc. This includes stability and convergence

analysis (which would be complicated) and also ways of improving parameter convergence.

4. Further evaluation and development of design guidelines for the weighting functions of SFC. For example, the polynomials, P and Q, provide the means to design adaptive classical controllers such as Smith predictor, Dahlin algorithm, etc.

5. Application of MIMO adaptive controllers to the evaporator. MIMO adaptive controllers may be better suited to the interactive nature of the evaporator.

6. Comparison of the SFC adaptive PID properties with other self-tuning PID algorithms such as those by Gawthrop, Isermann, Seborg, etc.

7. Investigation of how low level controllers like SFC, PID, etc. can be integrated into a more general control hierarchy to provide overall process supervison and/or optimization.

Apart from the above academic work, some improvements are required on the pilot plant evaporator to correct the refractometer fouling and feed tank rust problems (this work has been started).

9. Nomenclature

9.1 Technical Abbreviations

APCS	Adaptive Predictive Control System
ARMAS	Autoregressive Moving Average with Stochastic input
DISCO	Distributed Simulation and Control
ELS	Extended Least Squares
GLS	Generalized Least Squares
LQG	Linear Quadratic Gaussian
MV	Minimum Variance
MKY	Meyer-Kalman-Yacubovitch lemma
PRBS	Pseudo-Random Binary Sequence
RIV	Recursive Instrumental Variable
RLS	Recursive Least Squares
R(A)ML	Recursive (Approximate) Maximum Likelihood
SFC	Self-Tuning Feedback Controller
STC	Self-Tuning Controllers
STR	Self-Tuning Regulators

9.2 Nomenclature for chapter three

<u>Alphabetic</u>

K	Process gain in dimensionless unit
KC	Controller gain
KD	Derivertive constant
KI	Integral constant

KP	Proportional constant
T_1	Process time constant in minutes
T_2	Process time constant in minutes
T_d	Process time-delay in minutes
T_s	Sampling interval in minutes
T_{st}	Settling time in minutes

<u>Greek</u>

τ	Dominant time constant
τ_i	Integral time constant of PID
τ_d	Derivertive constant of PID
ω	Noise frequency
ω_n	Eigenfrequency or natural frequency

9.3 Nomenclature for chapter four

<u>Alphabetic</u>

$A(z^{-1})$	Polynomial corresponding to the process output
$B(z^{-1})$	Polynomial corresponding to the process input
$C(z^{-1})$	Polynomial characterizing stochastic noise
d	Discrete time-delay for the process input (integer multiple of T_s, an approximation to T_d)
$E\{\}$	Statistical expectation operator
I	Identity matrix
$K(k)$	Parameter estimator gain

$L(z^{-1})$	Characteristics of deterministic disturbance
ni	Order of $F(z^{-1})$ polyomial
nj	Order of $E(z^{-1})$ polynomial
nk	Number of weighted predicted output
$n\theta$	Number of parameters to be estimated
$P(z^{-1})$	Rational polynomial for output weighting
P_d	Denominator of polynomial P
P_n	Numerator of polynomial P
$P_t(k)$	Covariance matrix at time k
$Q'(z^{-1})$	Rational polynomial for input weighting
q	Time-delay of measurable disturbance
$R(z^{-1})$	Rational polynomial for setpoint weighting
u(k)	Control input at time k
v(k)	Deterministic disturbance
w(k)	Setpoint or reference value
y(k)	Process output at time k
Z{}	Z-transformation operator

Greek

$\epsilon(k)$	Estimation error
Θ_0	True system parameter vector
$\theta(k)$	Parameter estimates vector
λ	Positive scalar constant
$\xi(k)$	Stochastic disturbance
ρ	Forgetting or discounting factor
σ^2	Noise variance
Φ	Auxiliary controller output function

X Process input and output vector

<u>Superscripts</u>

*	Predicted value
t	Matrix transpose
^	Estimated value

9.4 Nomenclature for chapter five

<u>Alphabetic</u>

$a_i(k)$	Error-correcting factor for ith component
a_{io}	Lower limit of $a_i(k)$
a_{i1}	Upper limit of $a_i(k)$
d	Discrete time-delay for the process input (integer multiple of T_s, an approximation to T_d)
$e(k\|k-1)$	A priori estimation error
$e(k+d\|k)$	Prediction error
$nu(k)$	Noise component of $u(k)$
$nw(k)$	Noise component of $w(k)$
$ny(k)$	Noise component of $y(k)$
$nz(k)$	Noise component of $z(k)$
$u(k)$	Control input at time k
$w(k)$	Measurable deterministic disturbance
$y(k)$	Process output at time k
$y_d(k)$	Desired output or the output of driver block
$z(k)$	External input

Greek

α_1	Finite positive constant
α_2	Finite positive constant
$\Delta(k)$	Perturbation vector
Δ_d	Upper bound on the absolute value of $\Delta(k)$
Δ_m	Minimum upper bound of Δ_d
$\epsilon(k)$	Control error at time k
Θ_{io}	True system parameter matrix
Θ_1	Parameter vector corresponding to the input
$\theta(k)$	Parameter matrix to be estimated
λ	Finite real number
$\xi(k)$	Unmeasured disturbance
Φ	Process input and output matrix
Ψ	Augmented process input output matrix

Superscripts

t	Matrix transpose
^	Estimated value
~	Error between true and estimated values

Subscripts

0	Lower limit
1	Upper limit
d	Desired value
i	ith component of a vector or matrix
s	True system variable

9.5 Nomenclature for chapter six

Alphabetic

$a(k)$	Error-correcting factor in parameter estimator
a_0	Lower limit of $a(k)$
a_1	Upper limit of $a(k)$
$A_m(z^{-1})$	Polynomial corresponding to model output
$B_m(z^{-1})$	Polynomial corresponding to model input
$D(z^{-1})$	Polynomial describing external input
d	Discrete time-delay for the process input (integer multiple of T_s, an approximation to T_d)
$E\{\}$	Statistical expectation operator
$H_m(z^{-1})$	Polynomial characterizing stochastic noise
$L_m(z^{-1})$	Polynomial for deterministic disturbance
np	Order of polynomial $P(z^{-1})$
nq	Order of polynomial $Q(z^{-1})$
$n\theta$	Dimension of vector θ
$P(z^{-1})$	Polynomial for control error weighting
$Q'(z^{-1})$	Polynomial for regulating signal weighting
q	Time-delay of measurable disturbance
$u(k)$	Control input at time k
$v(k)$	Measurable disturbance
$w(k)$	Unmeasurable deterministic disturbance
$y(k)$	Process output at time k
$y_d(k)$	Desired output value

Greek

$\gamma(k)$	Modelling residuals

$\gamma'(k)$	Unmeasurable, stochastic disturbance
Δ_d	Bound on unmeasurable disturbance
Δ_m	Suprimum of unmeasrable disturbance
$\epsilon(k)$	Control error
$\hat{\epsilon}(k)$	Filtered ϵ by $P(z^{-1})$
$\eta(k)$	Auxiliary regulating signal
$\hat{\eta}(k)$	Filtered η by $Q(z^{-1})$
Θ_o	True controller parameter vector
$\theta(k)$	Parameter vector to be estimated
$\tilde{\theta}(k)$	Parameter error vector
$\xi(k)$	Estimation error
Φ	Auxiliary system output function
Ψ	Process input and output vector

Superscripts

j	Order of polynomial $D(z^{-1})$
t	Matrix transpose
*	Best predicted value
^	Estimated value
~	Error between true and estimated value

Subscripts

0	Lower limit
1	Upper limit
m	Indicating model

Note: Annotation for the figure captions (in parenthesis)

(1 / 2 / 3 / 4 / 5 / 6 / 7 / 8 / 9 / 10 / 11)

1 : name of the controller, e.g. STC etc.

2 : run number, e.g. ST3008 in Figure 4.1

3 : initial parameters (for PID controller constant)

e.g. I0 : zero initial parameters

ITDM: time domain curve-fitted model

ITSM: time series model

4 : sampling time, e.g. T64: 64 sec sampling time, etc.

5 : model order (for PID control mode)

e.g. M1+d: first order model with time delay

M2 : second order model, etc.

6 : covariance matrix or error correcting factor

e.g. C1000: 1000I initial covariance matrix for RLS

1000 upper limit of a(k) for APCS law

7 : forgetting factor or bound on perturbation variable

e.g. F1 : unity forgetting factor

d.005: .005 upper bound on disturbance

8 : $P(z^{-1})$-weighting polynomial

e.g. P(1-.8z): $P(z^{-1}) = (1-.8z^{-1})$

9 : $Q(z^{-1})$-weighting polynomial

e.g. Q PI : $Q(z^{-1}) = (1-z^{-1})/(q_0+q_1z^{-1})$

Q PID: $Q(z^{-1}) = (1-z^{-1})/(q_0+q_1z^{-1}+q_2z^{-2})$, etc.

10 : external disturbance

e.g. 20%FD: 20% change in feed flowrate

10%SP: 10% change in setpoint

11 : comments

N.B.: Because of the limitations of the TEXTFORM system used for producing the hard copy of this thesis some modifications have been made in what is regarded as widely accepted or 'standard' nomenclature. For example,

X'	vs	X^T	To indicate transpose
nq	vs	n_q	To indicate order of $Q(z^{-1})$ polynomial

etc

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11. Appendices

11.1 Appendix A : Evaporator Model and Steady State Values

1. Normal Steady State Operating Conditions

$\underline{x}$ Five element state vector

W1	First effect holdup	20.10	KG
C1	First effect concentration	4.31	wt.% glycol
H1	First effect solution enthalpy	441.40	KJ/KG
W2	Second effect holdup	18.81	KG
C2	Second effect concentration	10.00	wt.% glycol

$\underline{u}$ Three element control vector

S	Steam flowrate to first effect	0.91	KG/MIN
B1	First effect bottoms	1.50	KG/MIN
B2	Second effect bottoms	0.70	KG/MIN

$\underline{d}$ Three element disturbance vector

F	Feed flowrate	2.27	KG/MIN
CF	Feed concentration	3.00	wt.% glycol
HF	Feed enthalpy	376.30	KJ/KG

$\underline{y}$ Three element output vector

$$\underline{y} = \left[W1 \;\; W2 \;\; C2 \right]$$

2. The Fifth Order Discrete Evaporator Model (based on T_s=64sec)

$$x(k+1) = \underline{\Phi}x(k) + \underline{\Delta}u(k) + \underline{\Theta}d(k) \tag{A.1}$$

$$y(k) = \underline{C}x(k) + D^t\xi(k) \tag{A.2}$$

$$\underline{\Phi} = \begin{bmatrix} 1.0 & -0.0008 & -0.0912 & 0.0 & 0.0 \\ 0.0 & 0.9223 & 0.0871 & 0.0 & 0.0 \\ 0.0 & -0.0042 & 0.4377 & 0.0 & 0.0 \\ 0.0 & -0.0009 & -0.1052 & 1.0 & 0.0001 \\ 0.0 & 0.0391 & 0.1048 & 0.0 & 0.9603 \end{bmatrix}$$

$$\underline{\Delta} = \begin{bmatrix} -0.0119 & -0.0817 & 0.0 \\ 0.0116 & 0.0 & 0.0 \\ 0.1568 & 0.0 & 0.0 \\ -0.0137 & 0.0847 & -0.0406 \\ 0.0137 & -0.0432 & 0.0 \end{bmatrix}$$

$$\underline{\Theta} = \begin{bmatrix} 0.1182 & 0.0 & -0.0050 \\ -0.0351 & 0.0785 & 0.0049 \\ -0.0135 & -0.0002 & 0.0662 \\ 0.0012 & 0.0 & -0.0058 \\ -0.0019 & 0.0016 & 0.0058 \end{bmatrix}$$

$$\underline{C} = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.0 & 0.0 & 1-1.780z^{-1}+0.8031z^{-2} \end{bmatrix}$$

11.2 Appendix B : Derivation of SFC Control Law

The control law of SFC is based on the minimization of equation (6.21). Equation (6.21) is rewritten here for convience.

$$J = E\{[\epsilon^*(k+d)]^2 + [Q'(\eta(k) + P\epsilon(k))/D]^2\} + [E\{\xi(k+d)\}]^2 + \sigma^2 \qquad (B.1)$$

The minimization of the performance index results in an infinite number of terms and solution of Riccati equation is not practical for the design of the adaptive controller. Instead, single-step optimization is introduced to find a practical adaptive control law, i.e.

$$J' = \{[\epsilon^*(k+d)]^2 + [Q'(\eta(k) + P\epsilon(k))/D]^2\} + [E\{\xi(k+d)\}]^2 + \sigma^2 \qquad (B.2)$$

At each sampling instance the performance index is minimized with respect to $\eta(k)$, i.e.

$$\frac{\partial J'}{\partial \eta(k)} = 0,$$

or, since the last two terms are uncorrelated with $\eta(k)$,

$$2\epsilon^*(k+d)\frac{\partial \epsilon^*(k+d)}{\partial \eta(k)} + 2\frac{q_0'Q'}{D}[\eta(k) + P\epsilon(k)] = 0 \qquad (B.3)$$

From equation (6.18)

$$\frac{\partial \epsilon^*(k+d)}{\partial \eta(k)} = \frac{g_o \cdot b_o}{c_o} \tag{B.4}$$

where b_o, g_o and c_o are the first coefficient of polynomials $B(z^{-1})$, $G(z^{-1})$ and $C(z^{-1})$ respectively. Combining equations (B.3) and (B.4) yields;

$$\epsilon^*(k+d) + \frac{q_o' Q'}{b_o D}[\eta(k) + P\epsilon(k)] = 0 \tag{B.5}$$

Note that c_o and g_o are unity from equations (6.10) and (6.15). This is the control law equation (6.22).

Remark: In this derivation the auxiliary signal $\eta(k)$, and hence u(k), is selected such that the d-step-ahead forcast of the controller output is driven to zero subject to constraints on the present control action. In a sense this is a 'short-sighted' control solution since no account is taken of the fact that $\eta(k)$ also influences the output at times greater than (k+d). In other words this is a suboptimal solution to the original minimization problem. However, the solution achieved through the above appoach is practical for implementation of the controller. [MacGregor and Tidwell, 1977].

11.3 Appendix C : Proof of Theorem 6.1

This appendix contains the proof of theorem 6.1 and related lemmas. First of all, let $\Phi_1^*(k)$ be the a posteriori prediction of the controller output in relation with equation (6.31) and defined as follows:

$$\Phi_1^*(k) = \theta^t(k)\Psi(k-d) \tag{C.1}$$

Subtracting (C.1) from equation (6.31) yields

$$\Phi(k) - \Phi_1^*(k) = [\theta(k)-\theta(k-d)]^t\Psi(k-d) + \xi(k) \tag{C.2}$$

To simplify the analysis equation (C.2) can be rewritten more compactly as:

$$S(k) + \Delta(k) = \Omega(k) \tag{C.3}$$

where

$$\begin{aligned} S(k) &= \Phi(k) - \Phi_1^*(k) \\ \Delta(k) &= [\theta(k)-\theta(k-d)]^t\Psi(k-d) \\ \Omega(k) &= \xi(k) \end{aligned} \tag{C.4}$$

The a priori estimation error $\delta(k)$ of equation (6.32) and a posteriori estimation error $S(k)$ can now be written as

$$\delta(k) = \Phi(k) - \theta^t(k-d)\Psi(k-d) \tag{C.5}$$

$$S(k) = \Phi(k) - \theta^t(k)\Psi(k-d) \tag{C.6}$$

Subtracting (C.5) from (C.6) and substituting the adaptive law equation (6.32) gives

$$S(k) = \frac{\delta(k)}{1 + a(k)\Psi^t(k-d)\Psi(k-d)} \tag{C.7}$$

Note that $\delta(k)=\Phi(k)$ since $\theta^t(k-d)\Psi(k-d)$ is zero due to the control law. Combining equation (C.7) and (6.32), the adaptive algorithm can thus be expressed as:

$$\theta(k) = \theta(k-d) + a(k)S(k)\Psi(k-d) \tag{C.8}$$

Subtracting the true parameter Θ_0 from both sides of equation (C.8) and letting $\theta(k)$ be the difference between the estimated and the true parameters, i.e. $(\theta(k)-\Theta_0)$, then equation (C.8) becomes as

$$\theta(k) = \theta(k-d) + a(k)S(k)\Psi(k-d) \tag{C.9}$$

The most important property of the parameter error vector is summarized in the following lemma.

Lemma A.1: The parameters $\theta(k)$ are adapted by the adaptive mechanism, equations (6.32) to (6.39), such that the norm of the vector $\theta(k)$ is a nonincreasing function and converges.

Proof: From equation (C.9) the norm of parameter error

vector $\theta(k)$ is calculated as follows:

$$||\theta(k)||^2 = ||\theta(k-d)||^2 + 2a(k)S(k)\theta^t(k-d)\Psi(k-d) + a(k)^2S(k)^2\Psi^t(k-d)\Psi(k-d) \quad (C.10)$$

From equations (C.3) and (C.4) and using (C.9) S(k) can be expressed in the following form:

$$\begin{aligned} S(k) &= -\theta^t(k)\Psi(k-d) + \Omega(k) \\ &= -[\theta(k-d)+a(k)S(k)\Psi(k-d)]^t\Psi(k-d) + \Omega(k) \end{aligned} \quad (C.11)$$

Solving for $\theta^t(k-d)\Psi(k-d)$, and substituting into equation (C.10) gives the following relationship:

$$||\theta(k)||^2 - ||\theta(k-d)||^2 = 2a(k)S(k)\Omega(k) - a(k)S(k)^2[2 + a(k)\Psi^t(k-d)\Psi(k-d)] \quad (C.12)$$

The RHS of equation (C.12) will be (i) zero if $a(k)=0$ and (ii) less than or equal to zero if $a(k)\neq 0$ and the following condition is satisfied.

$$|S(k)| \geq \frac{2|\Omega(k)|}{2 + a(k)\Psi^t(k-d)\Psi(k-d)} \quad (C.13)$$

Combining equations (C.7) and (C.13) gives the following inequality:

$$|\delta(k)| \geq \lambda^2 |\Omega(k)| \tag{C.14}$$

where,

$$\lambda^2 = \frac{2 + 2a(k)\Psi^t(k-d)\Psi(k-d)}{2 + a(k)\Psi^t(k-d)\Psi(k-d)}$$

To prove inequality (C.14), it is sufficient to show that $|\delta(k)| \geq \lambda^2 \Delta_d$ since $\Delta_d \geq |\Omega(k)|$ (cf. equation (6.36)). According to condition ii) of the adaptive law, i.e. equations (6.37) to (6.39),

1. For the case $a_d(k)=a_1$

$$|\delta(k)| > \Delta_d'(a_1, \Delta_d, k) \geq \Delta_d'(a(k), \Delta_d, k) = \lambda^2 \Delta_d$$

Note that Δ_d' increases as $a(k)$ increases.

2. For the case where $a_d(k)$ is defined by equation (6.39), solving for $|\delta(k)|$ yields,

$$|\delta(k)| = \Delta_d'(a_1, \Delta_d, k) \geq \lambda^2 \Delta_d$$

where $a(k) \leq a_d(k)$ is used to derive the inequality.

Therefore, the following inequality is true for all $a(k) \neq 0$ specified by equations (6.37) to (6.39),

$$|\delta(k)| \geq \lambda^2 \Delta_d \geq \lambda^2 |\Omega(k)|, \qquad \forall\ k \tag{C.15}$$

and the norm of $\theta(k)$ is a nonincreasing function, i.e.

$$||\theta(k)||^2 - ||\theta(k-d)||^2 \leq 0\ , \qquad \forall\ k \tag{C.16}$$

Now, the index k can be replaced by nd+i, $0 \le i < d$, $n=0,1,2,\cdots$ and then $||\theta(nd+i)||^2$ is bounded for each i by the norm of initial parameter error, $||\theta(i)-\Theta_0||^2$, and below by zero. Note that d sets of initial parameters are given. Hence, for each i the following equality is established.

$$\lim_{n\to\infty} [||\theta(nd+i)||^2 - ||\theta(nd-d+i)||^2] = 0 \qquad (C.17)$$

This completes the proof of lemma A.1. theorem 6.1.

<u>**Lemma A.2**</u>: For the adaptive law equation (6.32) and the system (6.31) if there exists a subsequence $\{k_n\}$ within a sequence $\{k\}$ such that

$$\lim_{k_n\to\infty} ||\Psi(k_n-d)|| = \infty$$

then

1) $\lim_{k_n\to\infty} ||\theta(k_n) - \theta(k_n-d)|| = 0 , \qquad \forall\ a(k_n)$

2) $\lim_{k_n\to\infty} |S(k_n)| = 0$, for those k_n for which $a(k_n)\neq 0$

$|S(k_n)| \le \Delta_d'(a_0,\Delta_d,k) < 2\Delta_d$, for those k_n for which $a(k_n)=0$

<u>**Proof**</u>: i) When $a(k_n) = 0$, $\theta(k_n)$ is equal to $\theta(k_n-d)$ from the recursive law. Also, $S(k_n)=\delta(k_n)$ and the second part of property 2) is true along the sequence $\{k_n\}$ form adaptive condition (6.34).

ii) When $a(k_n) \neq 0$, from the definition of $\theta(k_n)$ and the triangle inequality, within this subsequence $\hat{\theta}(k_n)$ is given as

$$||\hat{\theta}(k_n)|| \leq ||\Theta_0|| + ||\theta(k_n)||, \qquad \forall\ k_n \tag{C.18}$$

Furthermore, $||\theta(k)||$ is a nonincreasing function and bounded above by $||\theta(k_i)||$, for $0 \leq i < d$, hence (C.18) can be written as

$$||\hat{\theta}(k_n)|| \leq ||\Theta_0|| + ||\theta(k_i)|| \leq \gamma_0, \qquad \forall\ k_n \tag{C.19}$$

where γ_0 is a positive scalar constant. Now, the norm of vector $(\hat{\theta}(k_n)-\hat{\theta}(k_n-d))$ can be written as

$$||\hat{\theta}(k_n) - \hat{\theta}(k_n-d)|| \leq$$
$$||\hat{\theta}(k_n)|| + ||\hat{\theta}(k_n-d)|| \leq \gamma_1, \qquad \forall\ k_n \tag{C.20}$$

where γ_1 is a positive scalar constant.

On the other hand the adaptive law (C.8) can be expressed as

$$||\hat{\theta}(k_n)-\hat{\theta}(k_n-d)||^2 = a(k_n)^2 S(k_n)^2 \Psi^t(k_n-d)\Psi(k_n-d) \tag{C.21}$$

combining equations (C.20) and (C.21) gives the following inequality.

$$S(k_n)^2\Psi^t(k_n-d)\Psi(k_n-d) \le \frac{\gamma_1^2}{a(k_n)^2} < \infty \tag{C.22}$$

Therefore, along the sequence $\{k_n\}$, if

$$\lim_{k_n\to\infty} ||\Psi(k_n-d)|| = \infty$$

then

$$\lim_{k_n\to\infty} S(k_n)^2 = 0 \tag{C.23}$$

This proves property 2) of the lemma.

Now, to prove the first part, taking the limit on both sides of equation (C.12), then the LHS is equal to zero because of equation (C.17) and the RHS is as follows;

$$\lim_{k_n\to\infty} \{2a(k_n)S(k_n)[S(k_n)-\Omega(k_n)]+a(k_n)^2S(k_n)^2\Psi^t(k_n-d)\Psi(k_n-d)\}$$
$$= \lim_{k_n\to\infty} \{2a(k_n)S(k_n)[S(k_n)-\Omega(k_n)]+||\theta(k_n) - \theta(k_n-d)||^2\}$$

Equation (C.21) is used in the rearrangement. By recalling the result of equation (C.23) the following limit is obtained.

$$\lim_{k_n\to\infty} ||\theta(k_n) - \theta(k_n-d)|| = 0 \tag{C.24}$$

This completes the proof of lemma A.2.

Proof of Theorem 6.1

Part (i): The norm of the I/O vector is finite.

To prove the boundedness of the I/O vector, assume that the sequence $\{\Psi(k)\}$ is unbounded, which implies that there exists a subsequence $\{k_n\}$ such that

$$\lim_{k_n \to \infty} ||\Psi(k_n)|| = \infty \quad \text{and} \quad ||\Psi(k)|| \le ||\Psi(k_n)||, \quad \text{for } k \le k_n$$

Subtracting equation (C.6) from (C.5) gives the following equation.

$$\delta(k_n) = S(k_n) + [\theta(k_n)-\theta(k_n-d)]^t\Psi(k_n-d) \tag{C.25}$$

Now, using the Schwartz inequality and the triangular rule equation (C.25) can be written as

$$|\delta(k_n)| \le |S(k_n)| + ||\theta(k_n)-\theta(k_n-d)|| \; ||\Psi(k_n-d)|| \tag{C.26}$$

Along this sequence the stable inverse condition, equation (6.41), is still applicable.

$$|\Phi(k_n)| \ge \alpha_1 ||\Psi(k_n-d)|| - \alpha_2 \tag{C.27}$$

From equations (6.31) ,(6.33) and the assumption that the desired controller output function value is zero, i.e. $\Phi^*=0$.

$$|\delta(k_n)| = |\Phi(k_n)| \tag{C.28}$$

Combining equation (C.26), (C.27) and (C.28) gives the following inequality.

$$[\alpha_1 - ||\theta(k_n)-\theta(k_n-d)||]\ ||\Psi(k_n-d)|| \le |S(k_n)| + \alpha_2 \tag{C.29}$$

Since α_1 and α_2 are positive constants and from the result of Lemma 2, i.e.

$$\lim_{k_n\to\infty} ||\theta(k_n) - \theta(k_n-d)||=0 \quad \text{when} \quad \lim_{k_n\to\infty} ||\Psi(k_n)|| = \infty$$

thus, inequality (C.29) holds only if $\lim_{k_n\to\infty} |S(k_n)| = \infty$.

This contradicts the property of $S(k_n)$ described in Lemma A.2. Hence, the assumed sequence $\{k_n\}$ cannot exist and $||\Psi(k)||$ must be bounded for all k.

Part(ii): The norm of the parameter error vector is a non-increasing function and the tracking error is bounded.

The first property is proved in lemma A.1. This section establishes the boundedness of the tracking or control error. Let's assume that there exists a subesquence $\{k_n\}$ within a sequence $\{k\}$ such that $a(k_n)\neq 0$ for all k_n, then from equations (C.7) and (6.37) and property i) of this theorem $S(k_n)$ is given as:

$$|S(k_n)| = \frac{|\delta(k_n)|}{1 + a(k_n)\Psi^t(k_n-d)\Psi(k_n-d)}$$
$$\geq \frac{\lambda^2\Delta_d}{1 + a(k_n)\Psi^t(k_n-d)\Psi(k_n-d)} > \beta_o \qquad (C.30)$$

where β_o is a finite positive constant. Further, substituting λ^2, $S(k_n)$ can be expressed in the following inequality:

$$|S(k_n)| \geq \frac{2\Delta_d}{2 + a(k_n)\Psi^t(k_n-d)\Psi(k_n-d)} \qquad (C.31)$$

Recalling equation (C.12) and rewriting it within this subsequence gives as:

$$||\theta(k_n)||^2 - ||\theta(k_n-d)||^2 = 2a(k_n)S(k_n)\Omega(k_n) -$$
$$a(k_n)S(k_n)^2[2+a(k_n)\Psi^t(k_n-d)\Psi(k_n-d)] \qquad (C.32)$$

Combining (C.31) and (C.32) yields

$$||\theta(k_n)||^2 - ||\theta(k_n-d)||^2 \leq$$
$$-2a(k_n)|S(k_n)|(\Delta_d-|\Omega(k_n)|) \qquad (C.33)$$

Recalling assumption i) of the theorem and equation (C.30), equation (C.33) can be written as:

$$||\theta(k_n)||^2 - ||\theta(k_n-d)||^2 \leq -\beta \tag{C.34}$$

where $\beta = 2a_0 \cdot \beta_0 \cdot (\Delta_d - \Delta_m)$, a positive constant. By successive substitution equation (C.34) can be expressed in terms of the initial parameter error.

$$||\theta(nd+i)||^2 \leq ||\theta(i)||^2 - \sum^{n} \beta, \qquad 0 \leq i \leq d \tag{C.35}$$

where k_n is replaced by $nd+i$, $n=0,1,2,\cdots$. Therefore the norm of parameter error vector is decreased from its initial deviation by at least β at each iteration. If it is assumed that the norm of the initial parameter error is finite, equation (C.35) implies that n or equivalently k_n is finite. In other words for a finite number, k_1, $a(k) \neq 0$ for $k \leq k_1$ and $a(k)=0$ for $k>k_1$. Recalling adaptive law or equation (C.8) this means $\theta(k) = \theta(k-d)$ for $k>k_1$ and also according to equation (6.34) $|\delta(k)|$ is bounded.

$$|\delta(k)| \leq \Delta_d'(a_0, \Delta_d, k) \leq 2\Delta_d < \infty$$

This completes the proof of theorem 6.1.